

Unit 8: Finance

Introduction with Technology: TVM Solver

- Compound Interest:** - interest is incurred on the **existing** balance.
 - usually **interest incurs at each term**. (A term can be annually, semi-annually, quarterly, monthly, or daily.)
 - **interest rate is always quoted per annum unless otherwise stated**.
 - it is used to calculate interest on different types of savings and loans.

- Saving:** - putting money away to earn interest for a future purpose.
 - there are three types of saving.

Three Types of Saving:

- 1) **Lump Sum:** - an amount of money is put away initially with no additional contributions.
- 2) **Annuity:** - no initial principal but a specific amount of money is saved every period for a larger sum.
- 3) **Combination:** - putting away an initial amount of money at first and make addition contributions every period.

Lump Sum Saving/Loan

$$A = P (1 + r)^n$$

A = Final Amount P = Initial Investment (Principal)
 r = rate per term n = total number of terms

Compound Term	r	n
Annually	$\frac{\text{Annual Interest Rate}}{1}$	Number of Years \times 1
Semi-annually	$\frac{\text{Annual Interest Rate}}{2}$	Number of Years \times 2
Quarterly	$\frac{\text{Annual Interest Rate}}{4}$	Number of Years \times 4
Monthly	$\frac{\text{Annual Interest Rate}}{12}$	Number of Years \times 12
Daily	$\frac{\text{Annual Interest Rate}}{365}$	Number of Years \times 365

There are many **purposes for saving**:

- Educational:** College Fund or RESP (Registered Education Saving Plan).
- Insurance:** Term Life and Disability Insurance.
- Retirement:** RRSP (Registered Retirement Saving Plan), RSP (Registered Saving Plan), Company Pension Plan.
- Personal Savings:** (for a down payment of a purchase, a travel trip, or wedding expenses).

Using the TVM Solver (Time Value Money) by TI-83 Plus

To access the TVM Solver

1. Press **APPS**

```
APPLICATIONS
1 Finance...
```

2. Select Option 1

```
NAME VARS
1 TVM Solver...
2: tvn_Pmt
3: tvn_I%
4: tvn_PV
5: tvn_N
6: tvn_FV
7: tvn_PV<
```

3. Select Option 1 again

```
N=0
I%=0
PV=0
PMT=0
FV=0
P/Y=1
C/Y=1
PMT: [END] BEGIN
```

N= Number of Years \times Number of Payments per year

I%= Interest Rate per year

PV= Principal Value (Present Value)

PMT= Payment Amount per Period

FV= Future Value

P/Y= Number of Payments per Year (minimum value is 1)

C/Y= Number of Compound Period per Year

PMT: **[END]** **BEGIN** (payment made at the end or beginning of the period)

For the TVM Solver:

Money is **POSITIVE** when it is **GOING INTO** the pocket.

Money is **NEGATIVE** when it is **LEAVING** the pocket.

To solve for any parameters using the TVM Solver

- Enter all other known parameters.
- Take the cursor to the parameter that needs to be solved.

3. Press

ALPHA

ENTRY SOLVE

ENTER

Example 1: Using the lump sum formula and the TVM Solver, calculate the final amount and the interest earned on the followings.

- a. Investment of \$2400 at 6%/a compounded annually for 5 years.

$P = \$2400$ $r = 0.06$ $n = 5$

$A = P(1 + r)^n$
 $A = \$2400(1 + 0.06)^5$ $A = \$3211.74$

Interest Earned = Final Amount – Principle
 $= \$3211.74 - \2400
Interest Earned = \$811.74

TVM Solver

```

N=5
I%=6
PV=-2400
PMT=0
FV=3211.741386
P/Y=1
C/Y=1
PMT: [ ] [ ] [ ] BEGIN
    
```

Press ALPHA
ENTRY SOLVE
ENTER
after entering all other parameters.

- b. Investment of \$2400 at 6%/a compounded semi-annually for 5 years.

$P = \$2400$ $r = \frac{0.06}{2}$ $n = 5 \times 2 = 10$

$A = P(1 + r)^n$
 $A = \$2400 \left(1 + \frac{0.06}{2}\right)^{10}$ $A = \$3225.40$

Interest Earned = Final Amount – Principle
 $= \$3225.40 - \2400
Interest Earned = \$825.40

TVM Solver

```

N=5
I%=6
PV=-2400
PMT=0
FV=3225.39931
P/Y=1
C/Y=2
PMT: [ ] [ ] [ ] BEGIN
    
```

- c. Investment of \$2400 at 6%/a compounded quarterly for 5 years.

$P = \$2400$ $r = \frac{0.06}{4}$ $n = 5 \times 4 = 20$

$A = P(1 + r)^n$
 $A = \$2400 \left(1 + \frac{0.06}{4}\right)^{20}$ $A = \$3232.45$

Interest Earned = Final Amount – Principle
 $= \$3232.45 - \2400
Interest Earned = \$832.45

TVM Solver

```

N=5
I%=6
PV=-2400
PMT=0
FV=3232.452016
P/Y=1
C/Y=4
PMT: [ ] [ ] [ ] BEGIN
    
```

- d. Investment of \$2400 at 6%/a compounded monthly for 5 years

$P = \$2400$ $r = \frac{0.06}{12}$ $n = 5 \times 12 = 60$ TVM Solver

$A = P(1+r)^n$

$A = \$2400 \left(1 + \frac{0.06}{12}\right)^{60}$ $A = \$3237.24$

Interest Earned = Final Amount – Principle
 $= \$3237.24 - \2400

Interest Earned = \$837.24

N=5
 I%=6
 PV=-2400
 PMT=0
 FV=3237.240366
 P/Y=1
 C/Y=12
 PMT: [] [] [] BEGIN

Example 2: Using the TVM Solver, calculate the final amount and the interest earned on the followings.

- a. Investment of \$480 at the **end** of each year for 5 years at 6%/a compounded monthly.

TVM Solver $N = 5 \text{ years} \times 1 \text{ payment /yr}$

N=5
 I%=6
 PV=0
 PMT=-480
 FV=2714.883492
 P/Y=1
 C/Y=12
 PMT: [] [] [] BEGIN

Final Amount = \$2714.88

Interest Earned = Final Amount – Principle
 $= \$2714.88 - (5 \times \$480)$

Interest Earned = \$314.88

- b. Investment of \$40 at the **beginning** of each month for 5 years at 6%/a compounded monthly.

TVM Solver $N = 5 \text{ years} \times 12 \text{ payments /yr}$

N=60
 I%=6
 PV=0
 PMT=-40
 FV=2804.755226
 P/Y=12
 C/Y=12
 PMT: [] [] [] END

Final Amount = \$2804.76

Interest Earned = Final Amount – Principle
 $= \$2804.76 - (5 \times 12 \times \$40)$

Interest Earned = \$404.76

Invested at the beginning of each month.

Present Value: - amount of money needed now for regular withdrawals in the future.

Example 3: A scholarship committee would like to set up a scholarship fund of \$1000 at the end of every half a year for the next ten years, how much money does the committee need to invest now if the present value gets 8%/a compound monthly. Determine the total interest this present value will accumulate over the ten years.

TVM Solver $N = 10 \text{ years} \times 2 \text{ payments /yr}$

```

N=20
I%=8
PV=-13509.73968
PMT=1000
FV=0
P/Y=2
C/Y=12
PMT: [ ] BEGIN
    
```

No money left in scholarship fund after 10 years

Scholarship paid at the end of each half a year.

Present Value = \$13,509.74

Interest Earned = Total Payouts – Present Value
 $= (2 \times 10 \times \$1000) - \$13,509.74$

Interest Earned = \$6490.26

Example 4: Mary won the LOTTO 649 jackpot of \$2.2 million. If she invested the entire sum in an account that earns 8%/a compound quarterly, how much can she withdraw at the end of each month for the next 70 years of her life? Determine the amount of interest she will earn.

TVM Solver $N = 70 \text{ years} \times 12 \text{ payments /yr}$

```

N=840
I%=8
PV=-2200000
PMT=14627.12348
FV=0
P/Y=12
C/Y=4
PMT: [ ] BEGIN
    
```

No money left in scholarship fund after 10 years

Each Withdrawal is at the end of each month.

Monthly Payment = \$14,627.12

Interest Earned = Total Payouts – Present Value
 $= (12 \times 70 \times \$14,627.12) - \$2,200,000$

Interest Earned = \$10,086,780.80

Rule of 72: - the number of year it takes to double a lump sum investment $\approx \frac{72}{\text{Annual Interest Rate}}$

Example 5: Determine the number of years it will take to double an initial investment of \$2400 at 6%/a compounded annually using the Rule of 72. Verify the answer with the TVM Solver.

$t \approx \frac{72}{6}$

$t \approx 12 \text{ years}$

```

N=11.89566105
I%=6
PV=-2400
PMT=0
FV=4800
P/Y=1
C/Y=1
PMT: [ ] BEGIN
    
```

TVM Solver Assignment: pg. 544 - 545 All

9-1: Earning Income

Gross Income: - the total amount of income earned within a period before any deductions.

Earning Period	Monthly	Semi-monthly	Biweekly	Weekly
Number of Pay Periods in a Year	12	24	26	52

Wage: - income is calculated using an hourly rate.

Overtime: - overtime pay is usually a time and a half of the regular wage.

- overtime rate is applied to the work period that **exceeds 8 hours a day** or **exceeds 40 hours per week**.

Salary: - a set income with no overtime given.

Gratuities: - tips given by customers common in the many industries.

Straight Commission: - income is based on sales only.

Salary and Commission: - income is based a base salary and amount of sales.

Graduated Commission: - commission is calculated by various commission rates depending on the amount of sales.

Piecework: - income is determined by the number of items produced or the amount of services provided.

Example 1: As an experienced engineer, Jane earns an annual salary of \$80,000. What is her gross income if she is paid

a. monthly?

$$\frac{\$80,000}{12}$$

\$6,666.67 / month

b. biweekly?

$$\frac{\$80,000}{26}$$

\$3076.92 / two weeks

c. What is Jane's hourly rate if she works on average 50 hours a week?

$$\frac{\$80,000}{52} = \$1538.46 / \text{week}$$

$$\frac{\$1538.46 / \text{week}}{50 \text{ hrs} / \text{week}} = \mathbf{\$30.77 / \text{hour}}$$

Example 2: Mark is a waiter at a popular restaurant. He earns \$8 / hour plus 75% of his tips, with a time and a half for overtime. In one week, he worked 6 days for 7 hours per day, and received \$400 in tips. How much did Mark earn?

$$\text{Total Hours} = 6 \times 7 = \mathbf{42 \text{ hours}} \Rightarrow 40 \text{ hrs Regular Time} + 2 \text{ hrs Overtime}$$

$$\begin{aligned} \text{Total Income} &= \text{Regular Time Income} + \text{Overtime Income} + 75\% \text{ of Tips} \\ \text{Total Income} &= (40 \text{ hrs} \times \$8 / \text{hr}) + (2 \text{ hrs} \times \$8 / \text{hr} \times 1.5) + (0.75 \times \$400) \end{aligned}$$

$$\mathbf{\text{Total Income} = \$644}$$

Example 3: Most realtors are paid by straight commission. The commission rate is 7% on the first \$100,000 and 3% on the remaining final price. In 2002, the price of an average home in Calgary was \$190,000. If a realtor sells on average 2 homes in a month, what is her monthly and annual gross income in 2002?

Commission on \$190,000 home = $(\$100,000 \times 0.07) + (\$90,000 \times 0.03) = \$9700$

Average Monthly Income = $2 \times \$9700 =$ **\$19,400 / month**

Average Annual Income = $12 \times \$19,400 =$ **\$232,800 / year**

Example 4: A salesclerk at an electronic store earns \$7.50/ hour plus 5% commission on his total sales. In a two-week period, he worked 30 hours a week and sold \$8000 of equipments. What is his gross income for the two-week period?

Biweekly Income = **Wage** + **Commission**

Biweekly Income = $(30 \text{ hrs / week} \times 2 \text{ weeks} \times \$7.50 / \text{hr}) + (0.05 \times \$8000)$

Biweekly Income = \$850

Example 5: Mary is a contracted proofreader for a technical manual publisher. She is paid \$300 for the first 100 pages and \$4 / page for the remaining part of a book. In one week, Mary has proofread 2 books. One book has 258 pages and the other has 354 pages. Calculate Mary’s earning for the week.

Income = **Piecework for 258 pages** + **Piecework for 354 pages**

Income = $(\$300 + 158 \text{ page} \times \$4 / \text{page}) + (\$300 + 254 \text{ page} \times \$4 / \text{page})$

Income = \$2248

Exchange Rate: - the conversion rate between currencies of different countries.

Example 6: John would like to buy \$500 US in traveller cheques. How much would it cost him in Canadian dollars if the exchange rate were 1.576 CAD/ USD?

To get CAD as the unit for the answer, we must **multiply** the exchange rate.

$\$500 \text{ USD} \times \frac{1.576 \text{ CAD}}{\text{USD}} =$ **\$788 CAD**

(USD will cancel out with unit analysis)

Example 7: Margaret would like to convert \$650 CAD to British Pound for her trip to London. Calculate the amount of British Pounds she will get if the exchange rate were 2.537 CAD/GBP.

To get GBP as the unit for the answer, we must **divide (multiply reciprocal)** the exchange rate.

$\$650 \text{ CAD} \div \frac{2.537 \text{ CAD}}{\text{GBP}} =$ **£256.21 GBP**

$\$650 \text{ CAD} \times \frac{1 \text{ GBP}}{2.537 \text{ CAD}} =$ **£256.21 GBP**

(CAD will cancel out with unit analysis)

9-1 Assignment: pg. 526 All, pg. 530 – 531 #7, 8, 11, 13, 16, 17, 19, 21, 23, 24, 26

9-2: Net Income

Net Income: - income after deductions.

Canada Pension Plan (CPP): - a federal government program to ensure Canadians will earn a retirement income at a qualifying age.

Basic CPP Exemption: - CPP contribution is exempted for the first \$3500 of a person's income.

<u>CPP Contributions</u>	
Textbook Figures:	3.2% of Gross Income (between \$3500.01 to \$36,900) (Maximum Annual CPP Contribution = \$1068.80)
2003 CPP Deduction:	4.95% of Gross Income (between \$3500.01 to \$39,900) (Maximum Annual CPP Contribution = \$1801.80)

Employment Insurance (EI): - a federal government insurance program that all income earners pay into for the event of sudden unemployment.

<u>EI Premium</u>	
Textbook Figures:	2.7% of Gross Income (between \$0.01 to \$39,000) (Maximum Annual EI Premium = \$1053)
2003 EI Deduction:	2.1% of Gross Income (between \$0.01 to \$39,000) (Maximum Annual EI Contribution = \$819)

Example 1: For the questions below, determine the CPP contribution and EI premium each pay period and annual deductions.

- a. Jack is paid \$1405 biweekly (textbook figures).

Annual Gross Income = $\$1405 \times 26 = \$36,530$ (Gross income is LESS than Max EI and CPP Limits)
ANNUAL CPP and EI are calculated FIRST by using the appropriate rate. The annual deductions are then divided by the number of pay period for amounts deducted per pay period.

Annual CPP = (Annual Gross Income – CPP Exemption) \times Rate
Annual CPP = $(\$36,530 - \$3500) \times 0.032$

Annual CPP = \$1056.96

Biweekly CPP = $\frac{\text{Annual CPP}}{26} = \frac{\$1056.96}{26}$

Biweekly CPP = \$40.65

Annual EI = Annual Gross Income \times Rate
Annual EI = $\$36,530 \times 0.027$

Annual EI = \$986.31

Biweekly EI = $\frac{\text{Annual EI}}{26} = \frac{\$986.31}{26}$

Biweekly EI = \$37.94

b. Jill is paid \$4200 monthly (2003 Tax Guide).

Annual Gross Income = $\$4200 \times 12 = \$50,400$ (Gross income EXCEEDS Max EI and CPP Limits) MAXIMUM ANNUAL CPP and EI are USED. The amounts deducted per pay period are calculated by multiplying the rates with the Gross Income Per Period. (In case she quits before the year is up.)

Annual CPP = Maximum Annual CPP Contribution
 Monthly CPP = $\$4200 / \text{month} \times 0.495$

Annual CPP = \$1801.80
Monthly CPP = \$207.90

(CPP Deductions From Jan to Aug = \$207.90 / month; Sept = \$138.60; No CPP for Oct to Dec)

Annual EI = Maximum EI Contribution
 Monthly EI = $\$4200 \times 0.021$

Annual EI = \$819.00
Monthly EI = \$88.20

(EI Deductions From Jan to Sept = \$88.20 / month; Oct = \$25.20; No CPP for Nov and Dec)

Tax-Exempted Deductions: - deductions that are taken off the gross earning **BEFORE** the tax rate is applied to it.

Examples: Professional or Union Dues, Childcare Expenses, Pension Contributions, RRSP (Registered Retirement Saving Plan), and RPP (Registered Pension Plan)

Taxable Income: - the amount of earning that the tax rate is applied to it.

$\text{Annual Taxable Income} = \text{Annual Gross Earning} - \text{Annual Tax-Exempted Deductions}$

Total Tax Credits: - tax deductions that are taken off the federal tax

(Lowest Tax Rate \times Applicable Tax Credits)

Examples: CPP (Canada Pension Plan), EI (Employment Insurance), and Basic Personal Tax Credit

Basic Personal Tax Credit: - basic tax credit for each person.

Basic Personal Tax Credit	
(Textbook Figure = \$6456)	2003 Tax Guide = \$7756

Federal and Provincial Tax Rate

Federal and Provincial Tax (Textbook Figures)

Annual Taxable Income	Federal Tax Rate	Province	Provincial Tax Rate
First \$29,590	17%	Alberta	45.5% of basic federal tax
Portion \$29,590.01 to \$59,180	26%	British Columbia	50.5% of basic federal tax
Amount Over \$59,180.01	29%	Manitoba	52% of basic federal tax
		Saskatchewan	50% of basic federal tax

To Calculate Net Annual Income

- Personal Tax Credits** = Basic Personal Tax Credit + CPP + EI + Other Tax Credits
- Annual Taxable Income** = Gross Annual Income – Tax-Exempted Deductions
- Basic Federal Tax** = Federal Tax Rates on Annual Taxable Income – (Lowest Tax Rate × Personal Tax Credits)
- Provincial Tax** = Provincial Tax Rate × Basic Federal Tax
- Net Annual Income** = Gross Annual Income – All Deductions

Example 2: As a beginning chemical engineer, Jane works for an alternate fuel company in Lethbridge, Alberta and makes an annual salary of \$55,000. She is paid biweekly and has the following biweekly tax-exempted deductions: \$20 for professional dues and \$120 for Retirement Saving Plan. She also contributes \$5 biweekly for her medical plan. Using the textbook figures,

- Determine Jane’s annual CPP and EI deductions.
- Calculate the total amount of income tax that Jane will have to pay annually.
- What is Jane’s net annual income?
- Using the answers above, fill out her biweekly pay stub below.

- a. Gross income (\$55,000) EXCEEDS Max EI and CPP Gross Incomes. Use Maximum Annual CPP and EI.

$$\text{Annual CPP} = \$1068.80 \quad \text{Annual EI} = \$1053$$

- b. Annual Income Tax

$$\begin{aligned} 1. \text{ Personal Tax Credits} &= \text{Basic Personal Tax Credit} + \text{CPP} + \text{EI} + \text{Other Tax Credits} \\ \text{Personal Tax Credits} &= \quad \quad \quad \$6456 \quad \quad \quad + \$1068.80 + \$1053 \quad \text{Personal Tax Credits} = \$8577.80 \end{aligned}$$

$$\begin{aligned} 2. \text{ Annual Taxable Income} &= \text{Gross Annual Income} - \text{Tax-Exempted Deductions} \\ \text{Annual Taxable Income} &= \$55,000 - (\$20 \times 26) - (\$120 \times 26) \quad \text{Annual Taxable Income} = \$51,360 \\ &\quad \quad \quad \text{Professional Due} \quad \text{Retirement Saving} \end{aligned}$$

$$\begin{aligned} 3. \text{ Basic Federal Tax} &= \text{Federal Tax Rates on Annual Taxable Income} - (\text{Lowest Tax Rate} \times \text{Personal Tax Credits}) \\ \text{Basic Federal Tax} &= (\text{First } \$29,590 \times 0.17) + (\text{Next } \$21,770 \times 0.26) - (0.17 \times \$8577.80) \end{aligned}$$

$$\text{Basic Federal Tax} = \$9232.27$$

$$\begin{aligned} 4. \text{ Alberta Provincial Tax} &= \text{Alberta Provincial Tax Rate} \times \text{Basic Federal Tax} \\ \text{Alberta Provincial Tax} &= 0.455 \times \$9232.27 \quad \text{Alberta Provincial Tax} = \$4200.68 \end{aligned}$$

$$\begin{aligned} \text{Total Income Tax Deduction} &= \text{Federal Income Tax} + \text{Alberta Provincial Income Tax} \\ \text{Total Income Tax Deduction} &= \quad \quad \quad \$9232.27 \quad \quad \quad + \quad \quad \quad \$4200.68 \end{aligned}$$

$$\text{Total Annual Income Tax Deduction} = \$13,432.95$$

c. Net Annual Income

Net Annual Income = Gross Annual Income – All Deductions

Net Annual Income = Gross Income – Income Tax – CPP – EI – Professional Dues – Retirement – Medical

Net Annual Income = \$55,000 – \$13,432.95 – \$1068.80 – \$1053 – (\$20 × 26) – (\$120 × 26) – (\$5 × 26)

$$\text{Net Annual Income} = \$35,675.25$$

$$\text{Percentage of Annual Net Income} = \frac{\text{Annual Net Income}}{\text{Annual Gross Income}} \times 100\% = \frac{\$35,675.25}{\$55,000} \times 100\%$$

$$\text{Percentage of Annual Net Income} = 64.86\%$$

$$\text{Percentage of Annual Deductions} = 100\% - \text{Percentage of Annual Net Income} = 100\% - 64.86\%$$

$$\text{Percentage of Annual Deductions} = 35.14\%$$

(For every dollar Jane earns annually, the deductions are \$0.35 and she takes home \$0.65!)

d. Jane’s Biweekly Pay Stub

	Amount	Calculations
Gross Biweekly Income	\$2115.38	\$55,000 / 26
CPP Contribution	\$67.69	\$2115.38 × 0.032 (Gross Annual Income ≥ Max CPP Gross)
EI Premium	\$57.12	\$2115.38 × 0.027 (Gross Annual Income ≥ Max EI Gross)
Income Tax	\$516.65	\$13,432.95 / 26
Registered Retirement Plan	\$120.00	
Professional Dues	\$20.00	
Medical Plan	\$5.00	
Net Biweekly Income	\$1328.92	\$2115.38 – \$67.69 – \$57.12 – \$516.65 – \$120 – \$20 – \$5

$$\text{Percentage of Biweekly Net Income} = \frac{\text{Biweekly Net Income}}{\text{Biweekly Gross Income}} \times 100\% = \frac{\$1328.92}{\$2115.38} \times 100\%$$

$$\text{Percentage of Biweekly Net Income} = 62.82\%$$

$$\text{Percentage of Biweekly Deductions} = 100\% - \text{Percentage of Biweekly Net Income} = 100\% - 62.82\%$$

$$\text{Percentage of Biweekly Deductions} = 37.18\%$$

Federal and Provincial Tax (2003 Tax Guide)

Annual Taxable Income	Federal Tax Rate	Province	Provincial Tax Rate
First \$32,183	16%	Alberta	10% of taxable income*
Portion \$32,183.01 to \$64,368	22%	British Columbia	see table below**
Portion \$64,368.01 to \$104,648	26%	Manitoba	see table below***
Amount \$104,648.01 and over	29%	Saskatchewan	see table below****

*Alberta has a basic personal tax credit of \$13,525 (The method of calculating Alberta Provincial Tax is the same as the Federal Tax WITHOUT the tax brackets).

** British Columbia has a basic personal tax credit of \$8,307

*** Manitoba has a basic personal tax credit of \$7,634

**** Saskatchewan has a basic personal tax credit of \$8,000

(The method of calculating B.C., Manitoba and Saskatchewan Provincial Taxes is the same as the Federal Tax with their own tax brackets below).

****British Columbia Provincial Tax (2003 Tax Guide)**

Annual Taxable Income	British Columbia Provincial Tax Rate
First \$31,653	6.05%
Portion \$31,653.01 to \$63,308	9.15%
Portion \$63,308.01 to \$72,685	11.7%
Portion \$72,685.01 to \$88,260	13.7%
Amount \$88,260.01 and over	14.7%

*****Manitoba Provincial Tax (2003 Tax Guide)**

Annual Taxable Income	Manitoba Provincial Tax Rate
First \$30,544	10.9%
Portion \$30,544.01 to \$65,000	14.9%
Amount \$65,000.01 and over	17.4%

******Saskatchewan Provincial Tax (2003 Tax Guide)**

Annual Taxable Income	Saskatchewan Provincial Tax Rate
First \$35,000	11%
Portion \$35,000.01 to \$100,000	13%
Amount \$100,000.01 and over	15%

Example 3: As a beginning math teacher, Gary works for a local school board in Calgary and makes an annual salary of \$40,000. He is paid monthly and has the following monthly tax-exempted deductions: \$77 for union dues and \$231 for Retirement Saving Plan. He also contributes \$77 for his long-term disability plan. Using the 2003 Tax Guide,

- Determine Gary's annual CPP and EI deductions.
- Calculate the total amount of income tax that Gary will have to pay annually.
- What is Gary's net annual income?
- Using the answers above, fill out his monthly pay stub below.

a. Gross income (\$55,000) EXCEEDS Max EI and CPP Gross Incomes. Use Maximum Annual CPP and EI.

$$\text{Annual CPP} = \$1801.80 \quad \text{Annual EI} = \$819$$

b. Annual Income Tax

1. **Federal Personal Tax Credits = Federal Basic Personal Tax Credit + CPP + EI + Other Tax Credits**
 Federal Personal Tax Credits = \$7756 + \$1801.80 + \$819 **Federal Personal Tax Credits = \$10,376.80**

2. **Annual Taxable Income = Gross Annual Income – Tax-Exempted Deductions**
 Annual Taxable Income = \$40,000 – (\$77 × 12) – (\$231 × 12) **Annual Taxable Income = \$36,304**
 Union Due Retirement Saving

3. **Basic Federal Tax = Federal Tax Rates on Annual Taxable Income – (Lowest Tax Rate × Personal Tax Credits)**
 Basic Federal Tax = (First \$32,183 × 0.16) + (Next \$4121 × 0.22) – (0.16 × \$10,376.80)

$$\text{Basic Federal Tax} = \$4395.61$$

4. **Alberta Provincial Tax**

i. **Alberta Personal Tax Credit = Alberta Basic Personal Tax Credit + CPP + EI + Other Tax Credits**
 Alberta Personal Tax Credits = \$13,525 + \$1801.80 + \$819 **Alberta Personal Tax Credits = \$16,145.80**

ii. **Annual Taxable Income = Gross Annual Income – Tax-Exempted Deductions**
 Annual Taxable Income = \$40,000 – (\$77 × 12) – (\$231 × 12) **Annual Taxable Income = \$36,304**
 Union Due Retirement Saving

iii. **Basic Alberta Tax = Alberta Tax Rate on Annual Taxable Income – (Lowest Tax Rate × Personal Tax Credits)**
 Basic Alberta Tax = (\$36,304 × 0.10) – (0.10 × \$16,145.80) **Basic Alberta Tax = \$2015.82**

Total Income Tax Deduction = Federal Income Tax + Alberta Income Tax
 Total Income Tax Deduction = \$4395.61 + \$2015.82

$$\text{Total Annual Income Tax Deduction} = \$6411.43$$

c. Net Annual Income

Net Annual Income = Gross Annual Income – All Deductions

Net Annual Income = Gross Income – Income Tax – CPP – EI – Union Dues – Retirement – Disability

Net Annual Income = \$40,000 – \$6411.43 – \$1801.80 – \$819 – (\$77 × 12) – (\$231 × 12) – (\$77 × 12)

$$\text{Net Annual Income} = \$26,347.77$$

$$\text{Percentage of Annual Net Income} = \frac{\text{Annual Net Income}}{\text{Annual Gross Income}} \times 100\% = \frac{\$26,347.77}{\$40,000} \times 100\%$$

$$\text{Percentage of Annual Net Income} = 65.87\%$$

$$\text{Percentage of Annual Deductions} = 100\% - \text{Percentage of Annual Net Income} = 100\% - 65.87\%$$

$$\text{Percentage of Annual Deductions} = 34.13\%$$

(For every dollar Gary earns annually, the deductions are \$0.34 and he takes home \$0.66!)

d. Gary's Monthly Pay Stub

	Amount	Calculations
Gross Monthly Income	\$3333.33	\$40,000 / 12
CPP Contribution	\$165.00	\$3333.33 × 0.0495 (Gross Annual Income ≥ Max CPP Gross)
EI Premium	\$70.00	\$3333.33 × 0.021 (Gross Annual Income ≥ Max EI Gross)
Income Tax	\$534.29	\$6411.43 / 12
Registered Retirement Plan	\$231.00	
Union Dues	\$77.00	
Long-Term Disability Plan	\$77.00	
Net Monthly Income	\$2179.04	\$3333.33 – \$165 – \$70 – \$534.29 – \$231 – \$77 – \$77

$$\text{Percentage of Monthly Net Income} = \frac{\text{Monthly Net Income}}{\text{Monthly Gross Income}} \times 100\% = \frac{\$2179.04}{\$3333.33} \times 100\%$$

$$\text{Percentage of Monthly Net Income} = 65.37\%$$

$$\text{Percentage of Monthly Deductions} = 100\% - \text{Percentage of Monthly Net Income} = 100\% - 65.37\%$$

$$\text{Percentage of Monthly Deductions} = 34.63\%$$

Example 4: Jane works as a nurse at a hospital in British Columbia and makes an annual salary of \$60,000. She is paid biweekly and has the following biweekly tax-exempted deductions: \$50 for union dues and \$175 for Retirement Saving Plan. She also pays \$10 for parking. Using the 2003 Tax Guide,

- a. Determine Jane’s annual CPP and EI deductions.
- b. Calculate the total amount of income tax that Jane will have to pay annually.
- c. What is Jane’s net annual income?
- d. Using the answers above, fill out his biweekly pay stub below.

a. Gross income (\$60,000) EXCEEDS Max EI and CPP Gross Incomes. Use Maximum Annual CPP and EI.

$$\text{Annual CPP} = \$1801.80 \quad \text{Annual EI} = \$819$$

b. Annual Income Tax

$$\begin{aligned} 1. \text{ Federal Personal Tax Credits} &= \text{Federal Basic Personal Tax Credit} + \text{CPP} + \text{EI} + \text{Other Tax Credits} \\ \text{Federal Personal Tax Credits} &= \$7756 + \$1801.80 + \$819 \quad \text{Federal Personal Tax Credits} = \$10,376.80 \end{aligned}$$

$$\begin{aligned} 2. \text{ Annual Taxable Income} &= \text{Gross Annual Income} - \text{Tax-Exempted Deductions} \\ \text{Annual Taxable Income} &= \$60,000 - (\$50 \times 26) - (\$175 \times 26) \quad \text{Annual Taxable Income} = \$54,150 \\ &\qquad \qquad \qquad \text{Union Due} \quad \text{Retirement Saving} \end{aligned}$$

$$\begin{aligned} 3. \text{ Basic Federal Tax} &= \text{Federal Tax Rates on Annual Taxable Income} - (\text{Lowest Tax Rate} \times \text{Personal Tax Credits}) \\ \text{Basic Federal Tax} &= (\text{First } \$32,183 \times 0.16) + (\text{Next } \$21,967 \times 0.22) - (0.16 \times \$10,376.80) \end{aligned}$$

$$\text{Basic Federal Tax} = \$8321.73$$

4. British Columbia Provincial Tax

$$\begin{aligned} \text{i. B.C. Personal Tax Credit} &= \text{B.C. Basic Personal Tax Credit} + \text{CPP} + \text{EI} + \text{Other Tax Credits} \\ \text{B.C. Personal Tax Credits} &= \$8307 + \$1801.80 + \$819 \quad \text{B.C. Personal Tax Credits} = \$10,927.80 \end{aligned}$$

$$\begin{aligned} \text{ii. Annual Taxable Income} &= \text{Gross Annual Income} - \text{Tax-Exempted Deductions} \\ \text{Annual Taxable Income} &= \$60,000 - (\$50 \times 26) - (\$175 \times 26) \quad \text{Annual Taxable Income} = \$54,150 \\ &\qquad \qquad \qquad \text{Union Due} \quad \text{Retirement Saving} \end{aligned}$$

$$\text{iii. Basic B.C. Tax} = \text{B.C. Tax Rate on Annual Taxable Income} - (\text{Lowest Tax Rate} \times \text{Personal Tax Credits})$$

$$\begin{aligned} \text{Basic B.C. Tax} &= (\text{First } \$31,653 \times 0.0605) + (\text{Next } \$22,497 \times 0.0915) - (0.0605 \times \$10,927.80) \\ \text{Basic B.C. Tax} &= \$3312.35 \end{aligned}$$

$$\begin{aligned} \text{Total Income Tax Deduction} &= \text{Federal Income Tax} + \text{B.C. Income Tax} \\ \text{Total Income Tax Deduction} &= \quad \quad \quad \$8321.73 \quad \quad + \quad \quad \quad \$3312.35 \end{aligned}$$

$$\text{Total Annual Income Tax Deduction} = \$11,634.08$$

c. Net Annual Income

Net Annual Income = Gross Annual Income – All Deductions

Net Annual Income = Gross Income – Income Tax – CPP – EI – Union Dues – Retirement – Parking

Net Annual Income = \$60,000 – \$11,634.08 – \$1801.80 – \$819 – (\$50 × 26) – (\$175 × 26) – (\$10 × 26)

$$\text{Net Annual Income} = \$39,635.12$$

$$\text{Percentage of Annual Net Income} = \frac{\text{Annual Net Income}}{\text{Annual Gross Income}} \times 100\% = \frac{\$39,635.12}{\$60,000} \times 100\%$$

$$\text{Percentage of Annual Net Income} = 66.06\%$$

Percentage of Annual Deductions = 100% – Percentage of Annual Net Income = 100% – 66.06%

$$\text{Percentage of Annual Deductions} = 33.94\%$$

(For every dollar Jane earns annually, the deductions are \$0.34 and she takes home \$0.66!)

d. Jane's Biweekly Pay Stub

	Amount	Calculations
Gross Biweekly Income	\$2307.69	\$60,000 / 26
CPP Contribution	\$114.23	\$2307.69 × 0.0495 (Gross Annual Income ≥ Max CPP Gross)
EI Premium	\$48.46	\$2307.69 × 0.021 (Gross Annual Income ≥ Max EI Gross)
Income Tax	\$447.46	\$11,634.08 / 26
Registered Retirement Plan	\$175.00	
Union Dues	\$50.00	
Parking	\$10.00	
Net Biweekly Income	\$1462.54	\$2307.69 – \$114.23 – \$48.46 – \$447.46 – \$175 – \$50 – \$10

$$\text{Percentage of Biweekly Net Income} = \frac{\text{Biweekly Net Income}}{\text{Biweekly Gross Income}} \times 100\% = \frac{\$1462.54}{\$2307.69} \times 100\%$$

$$\text{Percentage of Biweekly Net Income} = 63.38\%$$

Percentage of Biweekly Deductions = 100% – Percentage of Biweekly Net Income = 100% – 63.38%

$$\text{Percentage of Biweekly Deductions} = 36.62\%$$

9-2 Assignment: pg. 536 – 537 #1 to 27 (odd), 31, 32 (Use Textbook Figures)

9-3: Interest and Annuities

Compound Interest: Lump Sum Saving/Loan

$$A = P(1 + r)^n$$

$A =$ Final Amount $P =$ Initial Investment (Principal) $r =$ rate per term $n =$ total number of terms

Example 1: Using the compound interest formula, determine the final balance if \$10,000 is invested in a term deposit account for 5 years. The first 2 years pays 6%/a compound monthly, and 7%/a compound monthly for the next 3 years. Verify your answer using the TVM Solver.

<p>First Two Years $P = \\$10,000$ $r = \frac{0.06}{12}$ $n = 2 \times 12 = 24$ $A = P(1 + r)^n$ $A = \\$10,000 \left(1 + \frac{0.06}{12}\right)^{24}$ $A = \\$11,271.60$</p>	<p>Next Three Years $P = \\$11,271.60$ $r = \frac{0.07}{12}$ $n = 3 \times 12 = 36$ $A = P(1 + r)^n$ $A = \\$10,000 \left(1 + \frac{0.07}{12}\right)^{36}$ $A = \\$13,897.04$</p>	<p>TVM Solver</p> <table border="1"> <tr> <td>N=2 I% = 6 PV = -10000 PMT = 0 FV = 11271.59776 P/Y = 1 C/Y = 12 PMT: [] BEGIN</td> <td>N=3 I% = 7 PV = -11271.6 PMT = 0 FV = 13897.04405 P/Y = 1 C/Y = 12 PMT: [] BEGIN</td> </tr> </table> <p>C/Y = 12 (Compound Monthly) P/Y = 1 (Lump Sum) After 5 years, the final balance is \$13,897.04</p>	N=2 I% = 6 PV = -10000 PMT = 0 FV = 11271.59776 P/Y = 1 C/Y = 12 PMT: [] BEGIN	N=3 I% = 7 PV = -11271.6 PMT = 0 FV = 13897.04405 P/Y = 1 C/Y = 12 PMT: [] BEGIN
N=2 I% = 6 PV = -10000 PMT = 0 FV = 11271.59776 P/Y = 1 C/Y = 12 PMT: [] BEGIN	N=3 I% = 7 PV = -11271.6 PMT = 0 FV = 13897.04405 P/Y = 1 C/Y = 12 PMT: [] BEGIN			

Example 2: Johnny Cash is 18 years old and he would like to have \$1 million by the time he is 55. He has found a saving bond that pays 4%/a compound monthly. Calculate algebraically the amount he needs now to reach his financial goal. Verify your result with the TVM Solver.

<p>$A = \\$1,000,000$ $r = \frac{0.04}{12}$ $n = (55 - 18) \times 12 = 444$ $A = P(1 + r)^n$ Solve for P $P = \frac{A}{(1 + r)^n} = \frac{\\$1,000,000}{\left(1 + \frac{0.04}{12}\right)^{444}}$ A lump sum investment of \$228,198.64 is needed to reach a final balance of \$1 million in 37 years at 4%/a compound monthly.</p>	<p>TVM Solver</p> <table border="1"> <tr> <td>N=37 I% = 4 PV = -228198.64 PMT = 0 FV = 1000000 P/Y = 1 C/Y = 12 PMT: [] BEGIN</td> <td>N = 55 - 18</td> </tr> </table> <p>$P = \\$228,198.64$</p>	N=37 I% = 4 PV = -228198.64 PMT = 0 FV = 1000000 P/Y = 1 C/Y = 12 PMT: [] BEGIN	N = 55 - 18
N=37 I% = 4 PV = -228198.64 PMT = 0 FV = 1000000 P/Y = 1 C/Y = 12 PMT: [] BEGIN	N = 55 - 18		

Example 3: All inflation rates are stated compounded annually. Suppose the average inflation rate over the years is about 3.5%. Using the TVM Solver, how long will it take for prices to double in Canada?

<p>Assume $PV = \\$100$ (Current Price) Then $FV = \\$200$ (Doubles)</p>	<table border="1"> <tr> <td>N=20.14879168 I% = 3.5 PV = -100 PMT = 0 FV = 200 P/Y = 1 C/Y = 1 PMT: [] BEGIN</td> <td>N = 20.15 years</td> </tr> </table> <p>At inflation rate of 3.5 %/a, it will take 20.15 years for prices to double.</p>	N=20.14879168 I% = 3.5 PV = -100 PMT = 0 FV = 200 P/Y = 1 C/Y = 1 PMT: [] BEGIN	N = 20.15 years
N=20.14879168 I% = 3.5 PV = -100 PMT = 0 FV = 200 P/Y = 1 C/Y = 1 PMT: [] BEGIN	N = 20.15 years		

Annuity: - making equal payments at regular time intervals.

Schedule: - a table showing deposits, withdrawals, interests, and balances at each time intervals.

Example 4: Mark is saving for a trip to go to the Maritimes in a year. On January 1st of this year, he deposited \$200. He makes the same \$200 deposit at the beginning of each month for the entire year. Determine his balance at the end of the year if his account pays 4.8% interest compound monthly by using

a. Algebraic Method

$$N = 1 \text{ yr} \times 12 \text{ P/Y}$$

b. TVM Solver

Jan 1 Feb 1 March 1 Nov 1 Dec 1
 Interest for 12 months, 11 months, 10 months, ..., 2 months, 1 month.

$$\$200\left(1 + \frac{0.048}{12}\right)^{12} + \$200\left(1 + \frac{0.048}{12}\right)^{11} + \dots + \$200\left(1 + \frac{0.048}{12}\right)^1 = \text{\$2463.32}$$

TVM Solver:
 N=12
 I%=4.8
 PV=0
 PMT=-200
 FV=2463.324418
 P/Y=12
 C/Y=12
 PMT: END

P/Y = 12 payments/year
 (Beginning of the month)

c. EXCEL Spreadsheet

	A	B	C	D	E
1	Months	Opening Balance	Monthly Deposit (Begin)	Interest Earned	Closing Balance
2	1	0	200	=(B2+C2)*0.048/12	=B2+C2+D2
3	=A2+1	=E2	200	=(B3+C3)*0.048/12	=B3+C3+D3
4	=A3+1	=E3	200	=(B4+C4)*0.048/12	=B4+C4+D4
5	=A4+1	=E4	200	=(B5+C5)*0.048/12	=B5+C5+D5
6	=A5+1	=E5	200	=(B6+C6)*0.048/12	=B6+C6+D6
7	=A6+1	=E6	200	=(B7+C7)*0.048/12	=B7+C7+D7
8	=A7+1	=E7	200	=(B8+C8)*0.048/12	=B8+C8+D8
9	=A8+1	=E8	200	=(B9+C9)*0.048/12	=B9+C9+D9
10	=A9+1	=E9	200	=(B10+C10)*0.048/12	=B10+C10+D10
11	=A10+1	=E10	200	=(B11+C11)*0.048/12	=B11+C11+D11
12	=A11+1	=E11	200	=(B12+C12)*0.048/12	=B12+C12+D12
13	=A12+1	=E12	200	=(B13+C13)*0.048/12	=B13+C13+D13

	A	B	C	D	E
1	Months	Opening Balance	Monthly Deposit (Begin)	Interest Earned	Closing Balance
2	1	\$ -	\$ 200.00	\$ 0.80	\$ 200.80
3	2	\$ 200.80	\$ 200.00	\$ 1.60	\$ 402.40
4	3	\$ 402.40	\$ 200.00	\$ 2.41	\$ 604.81
5	4	\$ 604.81	\$ 200.00	\$ 3.22	\$ 808.03
6	5	\$ 808.03	\$ 200.00	\$ 4.03	\$ 1,012.06
7	6	\$ 1,012.06	\$ 200.00	\$ 4.85	\$ 1,216.91
8	7	\$ 1,216.91	\$ 200.00	\$ 5.67	\$ 1,422.58
9	8	\$ 1,422.58	\$ 200.00	\$ 6.49	\$ 1,629.07
10	9	\$ 1,629.07	\$ 200.00	\$ 7.32	\$ 1,836.39
11	10	\$ 1,836.39	\$ 200.00	\$ 8.15	\$ 2,044.53
12	11	\$ 2,044.53	\$ 200.00	\$ 8.98	\$ 2,253.51
13	12	\$ 2,253.51	\$ 200.00	\$ 9.81	\$ 2,463.32

9-3 Assignment
 pg. 542 – 543
 #1 to 27 (odd),
 29 to 34, 36, 37
 (Use TVM Solver
 for #31 and 34)

Finance Activity: Retirement Planning

Purpose: To calculate the amount of contribution to make **every month** for your retirement.

Procedure: **(Use the TVM Solver for all your calculations)**

1. Basic Information

At what age do you want to start saving?	
At what age do you want to retire?	
At what age do you think you will die?	
What is the average annual inflation rate? (Select from 2%/a to 5%/a compound annually)	
What is the average interest rate on your savings? (Choose between 6%/a to 15%/a) Compound Period Per Year? (Annually, Semi-annually, Quarterly, or Monthly)	
What is the annual gross income you would like to retire with in today's value? (Should be above \$30,000 per year)	

2. Total Withdrawal Amount of your Retirement

- a. Using the average annual inflation rate, calculate how much you will have to earn a year when you **start retirement**.
- b. Using the average annual inflation rate, calculate how much you will have to earn a year **just before you die**.
- c. Average the two results above. **This is your average annual income during your retirement.**
- d. Use the result in part c. above; multiply by the number of years you will spend retiring. **This is your Total Withdrawal Amount of your Retirement.**

3. Retirement Goal (amount needed at the beginning of the retirement)

- Using the above result in **2c**, and the **average interest rate for your savings**, calculate how much you will need at the beginning of your retirement. **This is your Retirement Goal.**

4. Monthly Contribution for your Retirement

- Using the **average interest rate on your savings**, **the number of months you have to save for your retirement goal** and the **retirement goal** itself calculate the **Monthly Contribution for your retirement**.

5. Final Analysis

- Determine the **total interest earned** from the time you started saving until the time you die.

Example 1: Mary is 18 years old, and she thinks that she will live till she is 80 years old. She would like to retire at the age of 55. Knowing the power of compound interest, she starts saving for her retirement now. From research, she knows that the average annual inflation rate is 2.5%. Her father suggested a RRSP mutual fund account that earns an average 8%/a compounded monthly. Mary would like to have the same living standard as someone who earns \$45,000 per year by the time she retires.

At what age does Mary want to start saving?	18
At what age does Mary want to retire?	55
At what age does Mary think she will die?	80
What is the average annual inflation rate? (Select from 2%/a to 5%/a compound annually)	2.5%
What is the average interest rate on your savings? (Choose between 6%/a to 15%/a) Compound Period Per Year? (Annually, Semi-annually, Quarterly, or Monthly)	8%/a Monthly
What is the annual gross income you would like to retire with in today's value? (Should be above \$30,000 per year)	\$45,000

a. Accounting for inflation, what should Mary be earning in the year at the beginning of her retirement?

TVM Solver (Age 55 – Age 18) This is similar to a lump sum saving problem, except we will use **the annual inflation rate of 2.5%/a (compound annually) as our interest rate**, and **the earning potential of \$45000 as our principal**.

N=37
I%=2.5
PV=-45000
PMT=0
FV=112200.6914
P/Y=1
C/Y=1
PMT: \square \square \square BEGIN

\$112,200.69 will have the same value 37 years from now as \$45,000 has today.

b. Accounting for inflation, what should Mary be earning in the very last year of her retirement (just before she die)?

TVM Solver (Age 80 – Age 18) Again, this is similar to a lump sum saving problem. We will use **the annual inflation rate of 2.5%/a (compound annually) as our interest rate**, and **the earning potential of \$45000 as our principal**.

N=62
I%=2.5
PV=-45000
PMT=0
FV=208013.8097
P/Y=1
C/Y=1
PMT: \square \square \square BEGIN

\$208,013.81 will have the same value 62 years from now as \$45,000 has today.

- c. Determine the total amount that Mary needs for her retirement using the average amount calculated for part a. and b. above.

$$\text{Average Annual Withdrawal During Retirement} = \frac{(\$112,200.69 + \$208,013.80)}{2}$$

$$\text{Average Annual Withdrawal During Retirement} = \$160,107.25$$

During 25 years of retirement (Age 55 to Age 80), she will need on average \$160,107.25 per year.

$$\text{Total Withdrawals} = 25 \text{ years} \times \$160,107.25 / \text{year}$$

$$\text{Total Withdrawals over 25 years of Retirement} = \$4,002,681.25$$

- d. Calculate Mary's retirement goal (the amount of money needed to be in the account at the beginning of her retirement).

Assuming ONE payment per year during retirement

TVM Solver

```
N=25
I%=8
PV=-1666212.263
PMT=160107.25
FV=0
P/Y=1
C/Y=12
PMT: [ ] BEGIN
```

RRSP account Interest Rate is 8%/a compound monthly

This is a Present Value problem. We expect the Retirement Goal (Present Value) is going to be much lower than the Total Withdrawals for the entire Retirement Period.

$$\text{Retirement Goal at Age 55} = \$1,666,212.26$$

At the end of her retirement (at age 80), all the money would have been used up.

- e. How much Mary needs to contribute into her RRSP account every month to reach her retirement goal.

Initially, there is NO Money in RRSP account. RRSP account Interest Rate is 8%/a compound monthly.

TVM Solver

N = 37 Working Years × 12 payments / yr

```
N=444
I%=8
PV=0
PMT=-613.38981...
FV=1666212.26
P/Y=12
C/Y=12
PMT: [ ] BEGIN
```

Number of Working Years = Age 55 – Age 18
Number of Working Years = 37 years

$$\text{RRSP Contribution} = \$613.39/\text{month}$$

Future Value is the Retirement Goal

- f. Calculate the amount of interest she will make on her retirement plan.

$$\text{Total Withdrawals by Age 80} = \$4,002,681.25$$

$$\text{Total RRSP Contributions} = \$613.39/\text{month} \times 37 \text{ Working Years} \times 12 \text{ months / year} = \$272,345.16$$

$$\begin{aligned} \text{Overall Interest Earned} &= \text{Total Withdrawals} - \text{Total RRSP Contributions} \\ &= \$4,002,681.25 - \$272,345.16 \end{aligned}$$

$$\text{Overall Interest Earned Between Age 18 to Age 80} = \$3,730,336.09$$

9-4: Effective Annual Rate of Interest

Nominal Interest Rate: - the stated interest rate that was advertised.

Average Annual Rate of Return (Effective Rate): - the actual interest rate if the interest were to calculate compound annually into of other type of compounding periods.

▶ **Eff (Effective Rate):** - returns Effective Interest Rate given the **Nominal Interest Rate in Percentage** and **Number of Compounding Terms Per Year**.

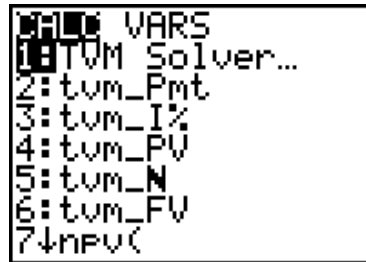
▶ **Eff (Nominal Rate in %, Number of Compound Terms Per Year)**

To access ▶ Eff:

1. Press **APPS**

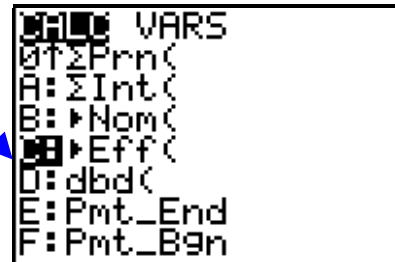


2. Select Option 1



3. Press

4. Select Option C



Example 1: Find the effective interest rate if an investment firm advertised at 10%/a compounded monthly

Method 1: Algebraic Method

Assume an initial investment of \$100. At 10%/a compound monthly for 12 months,

$$P = \$100, r = \frac{0.10}{12}, n = 12. \quad \text{The Final Balance is } A = 100 \left(1 + \frac{0.10}{12} \right)^{12} = \$110.47$$

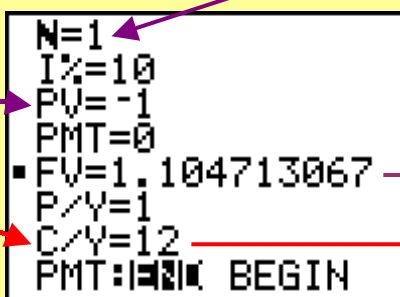
$$\text{Interest Earned} = \$110.47 - \$100 = \$10.47$$

To get the same balance compound annually, the effective rate must be 10.47%

Method 2: Using the TVM Solver

We can first calculate how much an investment of \$1 will be worth after 1 year at 10%/a compound monthly.

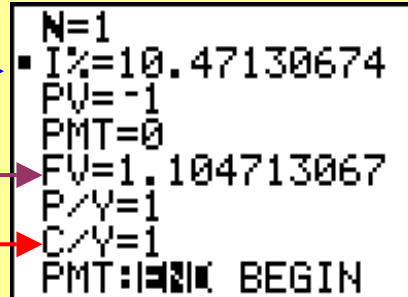
Then, assuming the same return for an investment compound annually, the rate will be the effective interest rate.



Solve for Effective Rate

SAME Return

Change to compound annually



Effective Rate 10.47%

Method 3: Using \blacktriangleright Eff

Nominal Rate = 10%
12 Compound Terms / Year

```

▶Eff(10,12)
10.47130674
    
```

Effective Rate = 10.47%

Example 2: Complete the table below. Find the average annual rate of return for the following portfolio.

2001 Investment Portfolio			
Investment	Amount Invested	Average Rate of Return	Amount of Return
Saving Account	\$1500	2%	\$30
GIC	\$2000	7%	\$140
Mutual Funds	\$3000	9%	\$270
Stocks	\$1000	(4%)	-\$40
Total	\$7500	Brackets mean a LOSS	\$400

TVM Solver

N = 1 Year

(Cannot use \blacktriangleright Eff because we do not know the nominal rate.)

```

N=1
I% = 5.33333333
PV = -7500
PMT = 0
FV = 7900
P/Y = 1
C/Y = 1
PMT: [ ] [ ] [ ] BEGIN
    
```

Effective Rate = 5.333%

Future Value = \$7500 invested + \$400 return

Assume Compound Annually for Effective Rate

Example 3: A line of credit offers by a major bank advertised the annual interest rate as 7% and the daily interest rate as 0.01917%.

- How are the two rates related? Calculate the effective rate if interest is compounded daily?
- What is the monthly rate? If the interest is compounded monthly, determine the effective rate.

a. Interest is Compounded Daily

The daily interest rate does NOT equal to 7% when it is multiplied by 365 days.

$$(0.01917\% \times 365 = 6.99705\%)$$

It is so closed to 7% that the bank rounded it up and advertised it as the nominal rate.

The actual nominal rate is 6.99705%

Using \blacktriangleright Eff

Nominal Rate = 6.99705%
365 Compound Terms / Year

```

▶Eff(6.99705,365)
7.246935092
    
```

Effective Rate = 7.247%

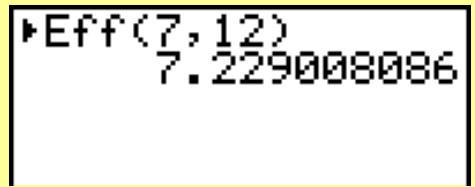
b. Interest is Compounded Monthly

$$\text{Monthly Rate} = \frac{7\%}{12} = 0.583\%$$

Using ▶ Eff

Nominal Rate = 7%

12 Compound Terms / Year



Effective Rate = 7.229%

Example 4: Mary invested \$5000 in a particular natural resource mutual fund at the beginning of 1997. It has the following performance over the next 5 years.

Year	1997	1998	1999	2000	2001
Average Rate of Return	5.2%	6.4%	(3.1%)	7.9%	15.3%

a. Complete the following table.

Year	Opening Balance	Average Rate of Return	Calculations	Closing Balance
1997	\$5000.00	5.2%	\$5000 (1 + 0.052)	\$5260.00
1998	\$5260.00	6.4%	\$5260 (1 + 0.064)	\$5596.64
1999	\$5596.64	(3.1%)	\$5596.64 (1 - 0.031)	\$5423.14
2000	\$5423.14	7.9%	\$5423.14 (1 + 0.079)	\$5851.57
2001	\$5851.57	15.3%	\$5851.57 (1 + 0.153)	\$6746.86

b. What is the average annual interest rate for the period from 1997 to 2001?

TVM Solver N = 5 Years (2001 to 1997 inclusive)

N = 5 → N = 5 Years (2001 to 1997 inclusive)
I% = 6.175994821 → Average Annual Interest Rate (Effective Rate) 6.176%
PV = -5000 → Initial Opening Balance
PMT = 0
FV = 6746.86 → Final Closing Balance
P/Y = 1
C/Y = 1 → Assume Compound Annually for Effective Rate
PMT:BEGIN

9-4 Assignment: pg. 549 - 550 #1 to 9 (odd), 11 to 17, 18a, 19

9-6: Housing Costs

Mortgage: - the amount of money borrowed from a financial institution for the purchase of a home

Maximum Mortgage Allowed: - the **maximum amount of money a person can borrow** from a financial institution based on the borrower(s)' incomes, expenses, and credit history.

This does NOT mean the borrower has to borrow the maximum amount.

Liabilities: - **fixed expenses** like credit card bills, student loans, personal credit line payments, car payments, insurance ...etc.

Amortization: - the number of years needed to pay off the mortgage. The most common amortization periods are 5, 10, 15, 20, 25 and 30 years.

Mortgage Rate: - the interest rate of the mortgage.

- **Fixed Rate:** - also called a **CLOSED Mortgage**, the **Rate is FIXED** for the **duration of the term**. A Mortgage term can be 6 months, 1, 2, 3, 4, 5, 7, 10, or 18 years. **When a mortgage term has passed, it needs to be re-negotiated.** (In Canada, a Close Mortgage is always calculated compounded semi-annually).
- **Variable Rate:** - also called an **OPEN Mortgage**, the **Rate can FLUCTUATE**. The borrower can convert to a fixed rate when interest rates start to climb due to a better economy. However, the Variable Rate is always LOWER than any kind of Fixed Rate. (In Canada, an Open Mortgage is always calculated compounded monthly).

Mortgage Payments: - the **amount** a borrower **pays** to the financial institution **every period** until the mortgage is paid off.

Payment Arrangements	Monthly	Semi-monthly	Biweekly	Weekly	Daily
Number of Payments per Year	12	24	26	52	365

Down Payment: - the amount of cash you want to put up as the **initial deposit** on the house.

- it guarantees your place during the negotiation of the final price of the house.
- the home buyer(s) may borrow up to \$20,000 from the RRSP account(s) with NO penalty! (Home Buyers Plan – HBP). It is a **single lifetime allowance**, and the home buyer(s) will have to pay back the borrowed amount in a form of decreasing the RRSP contribution limit for the next 15 years.
- **Minimum 5% of the final price.**
- **if the down payment is between 5% to 25%,** it is **subjected to an insurance charge** by **CMHC (Canada Mortgage and Housing Corporation <http://www.cmhc-schl.gc.ca>).** The reason is that most banks only accept mortgage application if the borrower puts down 25% down payment. Therefore, CMHC has to guarantee the bank that homebuyers with less than 25% down payment will not default on their mortgages. This insurance cost will then be added on to total mortgage amount.

Size of Down Payment of the House	5%	5% to 10%	10% to 15%	15% to 20%	20% to 25%
CMHC Insurance (expressed as % of the final house price)	3.75%	2.50%	2.00%	1.25%	0.75%

Calculating Mortgage Payments and Other Variables

- payment is always made at the **END** of a period.
- in Canada, only **Fixed Rate** is **compounded Semi-annually**, **Variable Rate** is **compounded Monthly**. (When the question did **NOT SPECIFIED** the rate type, **ALWAYS ASSUME** it is a **FIXED** rate).
- if the borrower decides to make **Semi-monthly or Biweekly Mortgage Payments** instead, the payment is **calculated by Dividing the Monthly Payment by 2**.

Example 1: Joseph and Mary would like to purchase a \$200,000 house. Their down payment is 25% of the purchase price. They have obtained a fixed mortgage rate of 6.375%/a over 25 years.

a. How much was their down payment? Determine the mortgage amount to be financed.

Down Payment = \$200,000 × 0.25

Down Payment = \$50,000

Mortgage Amount = Purchase Price – Down Payment

Mortgage Amount = \$200,000 – \$50,000

Mortgage Amount = \$150,000

b. Calculate the monthly payment.

Final Balance is \$0

Payments made at the END of the period

N = 25 years × 12

Mortgage Rate

Mortgage Amount

Monthly Mortgage Payment = \$993.40

12 Monthly Payments per Year

Compound Semi-annually (assume Fixed Rate)

```

N=300
I%=6.375
PV=150000
PMT=-993.396804
FV=0
P/Y=12
C/Y=2
PMT: [ ] [ ] BEGIN
    
```

c. Determine the total cost and the interest paid over 25 years.

Total Cost = Total Payments

Total Cost = 25 × 12 × \$993.40

Total Cost = \$298,020

Interest = Total Cost – Mortgage Amount

Interest = \$298,020 – \$150,000

Interest Paid = \$148,020

d. If the payment is made biweekly, calculate the biweekly payment?

Biweekly Payment = $\frac{\text{Monthly Payment}}{2} = \frac{\$993.40}{2}$

Biweekly Payment = \$496.70

e. What is the new amortization period?

Mortgage Rate

N = 542.1438072 Payments

Mortgage Amount

Biweekly Mortgage Payment

26 BIWEEKLY Payments per Year

Fixed Rate (Compound Semi-annually)

Amortization Period = 20.852 Years

$\frac{542.1438072 \text{ Payments}}{26 \text{ Payments per Year}}$

```

N=542.1438072
I%=6.375
PV=150000
PMT=-496.7
FV=0
P/Y=26
C/Y=2
PMT: [ ] [ ] BEGIN
    
```

f. What is the total interest paid over this new amortization period?

$$\begin{aligned}
 \text{Interest} &= (\text{Number of Payments} \times \text{Biweekly Payments}) - \text{Mortgage Amount} \\
 \text{Interest} &= 542.1438072 \text{ Payments} \times \$496.70 - \$150,000 \\
 \text{Interest} &= \$269282.83 - \$150,000 \\
 &= \mathbf{\$119,282.83}
 \end{aligned}$$

Property Tax: - a municipal tax applied to real estates owners (include residential and commercial properties) for the purpose of supporting police and fire protections, road maintenance, libraries, schools, garbage pickup...etc.

Mill Rate: - the annual property tax rate expressed per \$1000 of the assessed value of the property.

$$\text{Annual Property Tax} = \text{Assessed Value} \times \frac{\text{Mill Rate}}{1000}$$

Example 2: In Example 1, Joseph and Mary live in Calgary, and the total mill rate in 2002 was 8.9724. (City of Calgary Mill Rate 4.3935 + Calgary Catholic School Board Mill Rate 4.5789) That year, the city of Calgary assessed their house at \$212,000

- Calculate their annual and monthly property tax.
- Determine the total monthly housing cost.
- How much of the annual property tax did the Catholic School Board receive?

a. $\text{Annual Property Tax} = \$212,000 \times \frac{8.9724}{1000}$ $\text{Monthly Property Tax} = \frac{\$1902.15}{12}$

$\mathbf{\text{Annual Property Tax} = \$1902.15}$ $\mathbf{\text{Monthly Property Tax} = \$158.51}$

b. $\text{Total Monthly Housing Cost} = \text{Monthly Mortgage Payment} + \text{Monthly Property Tax}$
 $\text{Total Monthly Housing Cost} = \$993.40 + \$158.51$

$\mathbf{\text{Total Monthly Housing Cost} = \$1151.91}$

c. $\text{Annual School Tax} = \$212,000 \times \frac{4.5789}{1000}$ $\mathbf{\text{Annual School Tax} = \$970.73}$

9-6 Assignment: pg. 560 #1 to 25 (odd), 26 to 31, 33 to 36, 38

Finance Activity: Automobile Leasing and Financing

Taxes and Extra Charges

When purchasing a new vehicle, there are many other charges to consider along with the sales tax.

1. **GST**: - 7% of the final price of the vehicle
2. **Shipping & Freights**: - the cost of shipping the vehicle to Alberta from Ontario or BC (about \$850.00).
3. **Tires & Air Conditioning Tax**: - government charges for environmental purposes (about \$100).
4. **Document & Process Fee**: - the cost of processing documents related to the vehicle sale (about \$125).
5. **Additional Accessories**: - these are extra options that do not come with the standard features of the vehicle (example: underside coating and fabric protections, rear spoiler, hood deflector, running board, head lights protector).

Financing

- the buyer agreed to **pay it off** with equal payments per month over a number of years.
- **balance owing at the end** of the finance term is **\$0**.
- **all taxes and extra charges are added up into the final price of the vehicle before monthly payment is calculated**. Therefore, there is NO GST on the monthly payment.
- at the end of the finance term, **the car belongs to the buyer**.
- when we bargain with the dealer, we bargain on the final price of the vehicle.
- the **finance rate (compounds monthly) is advertised**, and monthly payment is then calculated after the bargaining is over.
- **payment is always made at the END of the month**.

Example 1: Mary wants to finance a new vehicle that has a final price of \$20000. She went with a finance term of 48 months at the interest rate of 4%/a. Suppose her extra charges are the same as outlined above with the additional accessories of \$950 that covers the underside coating and fabric protection.

- a. Calculate her monthly payment with no down payment.

Final Price	\$20000
GST (7% × \$20000)	\$1400
Shipping & Freights	\$850
Tire & AC Tax	\$100
Document & Process Fee	\$125
Additional Accessories	\$950
Finance Amount	\$23425

N = 48 months

Finance Amount
(Positive because she has the car loan)

N=48
 I%=4
 PV=23425
 PMT=-528.914355
 FV=0
 P/Y=12
 C/Y=12
 PMT: [] BEGIN

Monthly Payment
\$528.91

Loan is paid off at the end

b. Calculate the total cost of the vehicle.

$$\text{Total Cost} = \text{Number of Months} \times \text{Monthly Payment}$$

$$\text{Total Cost} = 48 \times \$528.91$$

$$\text{Total Cost} = \$25387.68$$

c. What is the total interest paid over 4 years?

$$\text{Interest Paid} = \text{Total Cost} - \text{Finance Amount}$$

$$\text{Interest Paid} = \$25387.68 - \$23425.00$$

$$\text{Interest Paid} = \$1962.68$$

Leasing

- the buyer agreed to pay off a portion of the final price with equal month payments over time.
- the **balance owing** at the end of the finance term called the **residual** (usually 45% to 55% of the final vehicle price).
- only extra charges are added up into the final price of the vehicle.** GST is added on the monthly payment and residual at the end of the term.
- at the end of the lease, the buyer has **two options**. He or she can pay the residual with GST and keep the car. He or she can walk away and the dealer will take the vehicle back.
- the monthly payment is like the monthly rental cost of the vehicle
- when we bargain with the dealer, we bargain on the monthly lease payment of the vehicle.
- the **lease rate (compounds monthly) is NEVER advertised**, and it is then calculated after the bargaining is over.
- payment is always made at the **beginning** of the month. **(On the TVM Solver, PMT is set to BEGIN).**

Example 2: John wants the same new vehicle as Mary’s, which has a final price of \$20000. He went with a lease term of 48 months with a monthly payment of \$350 plus tax. At the end of the lease, the residual amount is 48% of the final price of the vehicle. Suppose his extra charges are the same as outlined above with the additional accessories of \$950 that covers the underside coating and fabric protection.

a. Calculate his interest rate for the lease with no down payment.

Final Price	\$20000
Shipping and Freights	\$850
Tire and AC Tax	\$100
Document and Process Fee	\$125
Additional Accessories	\$950
Lease Amount	\$22025
Monthly Payment (before GST)	\$350
Residual before GST (48% × \$20000)	\$9600

N = 48 months

Leasing Rate 6.889%

Lease Amount

Monthly Payment

Residual Value

Leasing Payment is at the BEGINNING of the month.

b. Calculate the total cost of the vehicle including GST.

$$\text{Total Cost} = (\text{Number of Payments} \times \text{Monthly Payment with GST}) + (\text{Residual with GST})$$

$$\text{Total Cost} = (48 \times \$350 \times 1.07) + (\$9600 \times 1.07)$$

$$\text{Total Cost} = \$28248$$

c. What is the total interest paid over 4 years?

$$\begin{aligned} \text{Interest Paid} &= \text{Total Cost} - \text{Lease Amount with GST on the Final Price} \\ \text{Interest Paid} &= \$28248 - (\$20000 \times 1.07 + \$850 + \$100 + \$125 + \$950) \\ \text{Interest Paid} &= \$28248 - \$23425 \end{aligned}$$

Interest Paid = \$4823

Example 3: Mr. Tang bought a \$25000 car. His down payment included the GST and all other extra costs.

d. Calculate his monthly payment if he goes with 6%/a for 48 months with \$3000 down payment. Determine what he would pay in total and in interest.

$$\begin{aligned} \text{Finance Amount} &= \text{Final Price with GST} - \text{Down Payment} \\ \text{Finance Amount} &= \$25000 \times 1.07 - \$3000 \\ \text{Finance Amount} &= \$23750 \end{aligned}$$

```
N=48
I%=6
PV=23750
PMT=-557.76943...
FV=0
P/Y=12
C/Y=12
PMT:BEGIN
```

Monthly Payment = \$557.77

$$\begin{aligned} \text{Total Cost} &= \text{Total Payments} + \text{Down Payment} \\ \text{Total Cost} &= 48 \times \$557.77 + \$3000 \end{aligned}$$

Total Cost = \$29772.96

$$\begin{aligned} \text{Interest} &= \text{Total Cost} - \text{Final Price with GST} \\ \text{Interest} &= \$29772.96 - (\$25000 \times 1.07) \\ \text{Interest} &= \$29772.96 - \$26750 \end{aligned}$$

Interest Paid = \$3022.96

e. Calculate his leasing rate if he pays \$1250 for down payment (which covers all other extra costs), and \$365 + GST for 48 months with a 50% residual value. Determine what he would pay in total and in interest.

$$\begin{aligned} \text{Lease Amount} &= \text{Final Price} - \text{Down Payment} \\ \text{Lease Amount} &= \$25000 - \$1250 \\ \text{Lease Amount} &= \$23750 \\ \text{Monthly Lease Payment} &= \$365 \\ \text{Residual} &= 50\% \times \$25000 = \$12500 \end{aligned}$$

```
N=48
I%=8.613456937
PV=23750
PMT=-365
FV=-12500
P/Y=12
C/Y=12
PMT:END
```

Leasing Rate = 8.613%

$$\begin{aligned} \text{Total Cost} &= \text{Total Payments with GST} + \text{Down Payment} + \text{Residual with GST} \\ \text{Total Cost} &= 48 \times \$365 \times 1.07 + \$1250 + \$12500 \times 1.07 \end{aligned}$$

Total Cost = \$33371.40

$$\begin{aligned} \text{Interest Paid} &= \text{Total Cost} - (\text{Final Price with GST}) \\ \text{Interest Paid} &= \$33371.40 - (\$25000 \times 1.07) \\ \text{Interest Paid} &= \$33371.40 - \$26750 \end{aligned}$$

Interest Paid = \$6621.40

Finance Project: Automobile Leasing and Financing

Purpose: To compare the costs and interests between leasing and financing a vehicle.

Procedure: (Use the TVM Solver for most of your calculations)

1. Collect an automobile advertisement with proper referencing. The advertisement can be found in newspaper, magazine or on a website. It must contain the following information.
 - a. MSRP (Manufactured Suggested Retail Price)
 - b. Finance Rate (must be higher than 0%)
 - c. Monthly Lease Amount and Residual Value (sometimes refer to as buyback)
 - d. The finance and leasing terms (Number of Months)
 - e. Shipping and Freights (if it is not available in the fine prints, then assume \$850)
 - f. Tire and AC Tax (if it is not available in the fine prints, then assume \$100)
 - g. Document and Process Fee (if it is not available in the fine prints, then assume \$125)

2. Assuming no down payment, calculate the monthly finance payment, total cost and interest paid for the entire finance program using the information collected from the advertisement.

MSRP	
GST (7%)	
Shipping & Freights	
Tire & AC Tax	
Document & Process Fee	
Additional Accessories	
Finance Amount	

<p>N= I%= PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN</p>

3. Assuming no down payment, calculate the annual leasing rate, total cost (including GST) and interest paid for the entire leasing program using the information collected from the advertisement.

MSRP	
Shipping and Freights	
Tire and AC Tax	
Document and Process Fee	
Additional Accessories	
Lease Amount	
Monthly Payment (before GST)	
Residual before GST	

<p>N= I%= PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN</p>

4. Assuming the down payment equals to all the taxes, fees, and any additional accessories (Finance Amount = MSRP), calculate the monthly finance payment, total cost and interest paid for the new finance program using the information collected from the advertisement.

MSRP	
GST (7%)	
Shipping & Freights	
Tire & AC Tax	
Document & Process Fee	
Additional Accessories	
Down Payment	
Finance Amount	

N=
 I%=
 PV=
 PMT=
 FV=
 P/Y=
 C/Y=
 PMT: END BEGIN

5. Now assume the down payment equals to all the taxes, fees, and any additional accessories (Leasing Amount = MSRP). Calculate the new monthly leasing payment with GST, total cost (including GST) and interest paid for the entire leasing program using the information collected from the advertisement and the leasing rate found in Step 3.

MSRP	
Shipping and Freights	
Tire and AC Tax	
Document and Process Fee	
Additional Accessories	
Down Payment	
Lease Amount	
Leasing Rate (from Step 3)	
Residual before GST	

N=
 I%=
 PV=
 PMT=
 FV=
 P/Y=
 C/Y=
 PMT: END BEGIN

Analysis:

- Overall, compare the total cost of leasing and financing the same vehicle with and without down payment. Why do auto dealers advertise using the monthly leasing payment and finance rate?
- How much do you save in total when the down payment covers the taxes, fees, and additional accessories for each program? Is it always better to put a down payment when buying a new vehicle? Explain.

Note:

- The advertisement with proper referencing must be included in the final report. (If you are using the Internet, print out of the webpage.)
- Students can work together to discuss the project. **Students who copied from each other will end up sharing the mark.** Let's say the mark was 70% and two students were involved in copying each other's work. They each get 35%.
- Copy and properly fill out the table for each section of the procedure. Final write up **MUST include tables** for each section, **all TVM Solver layouts** (numbers used), and all other work must be shown.
- Be sure to **answer the analysis questions** as well in your final write up
- Project handed in **one day late** has a penalty of **30% off the total mark**. **Project handed in two days after the due date and later will not be marked.**

Due Date: _____