Unit 4: Trigonometry

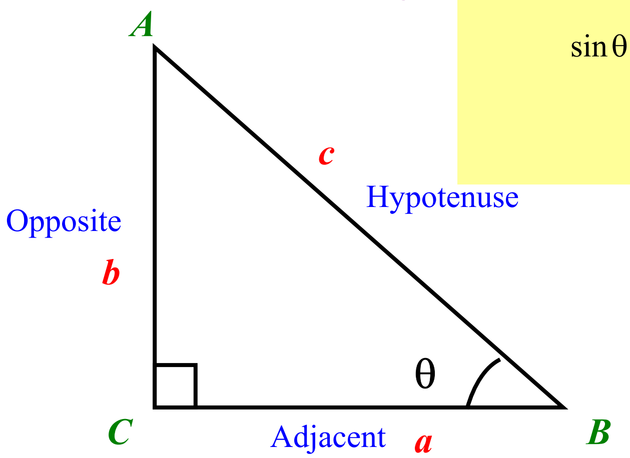
7-4: Reviewing Trigonometric Ratios

θ - "theta" – variable for angle

For any **right angle triangles**, we can use the simple trigonometric ratios.

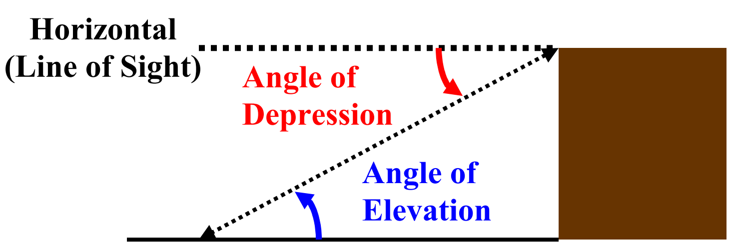
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

SOH CAH TOA

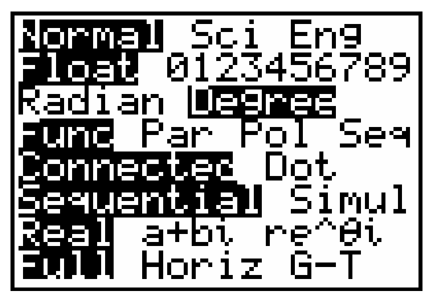


Capital letter is always used to label the angle.

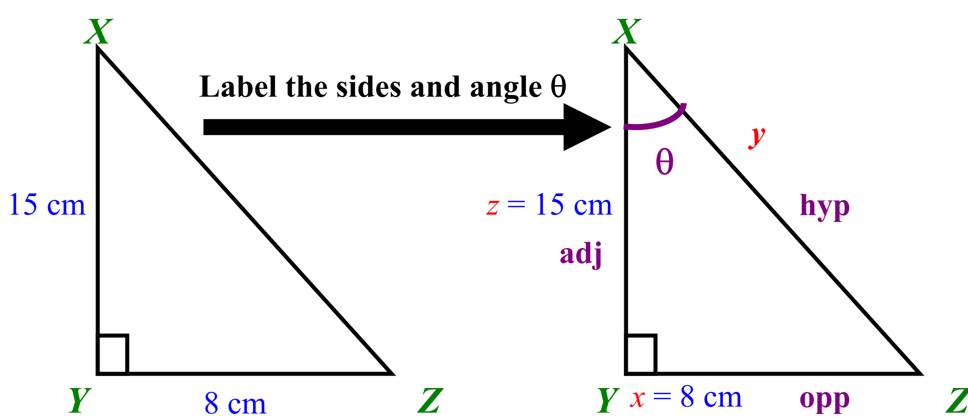
The name for the side that is opposite to the angle has the corresponding letter in small case.



Be sure that your calculator is set in DEGREE under the settings in your **MODE** menu!



Example 1: Find $\tan X$, $\angle X$ and $\overline{XZ}$.



$y^2 = 8^2 + 15^2$

$y^2 = 64 + 225$

$y^2 = 289$

$y = \sqrt{289}$

$y = 17cm$

$\tan X = \frac{\text{opp}}{\text{adj}}$

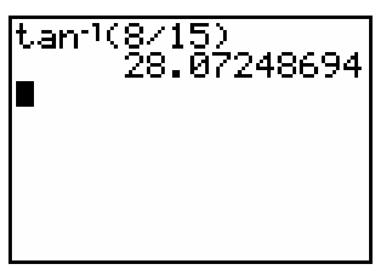
$\tan X = \frac{8}{15}$

$\tan X = \frac{8}{15}$

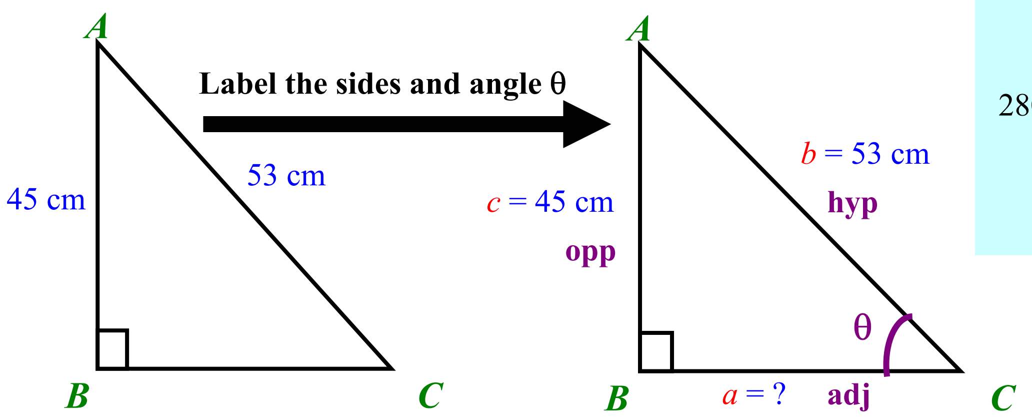
$X = 28^\circ$

2nd

TAN



Example 2: Solve the triangle (find all missing angles and sides).



$$53^2 = 45^2 + a^2$$
$$2809 = 2025 + a^2$$
$$2809 - 2025 = a^2$$
$$\sqrt{784} = a^2$$
$$a = 28cm$$

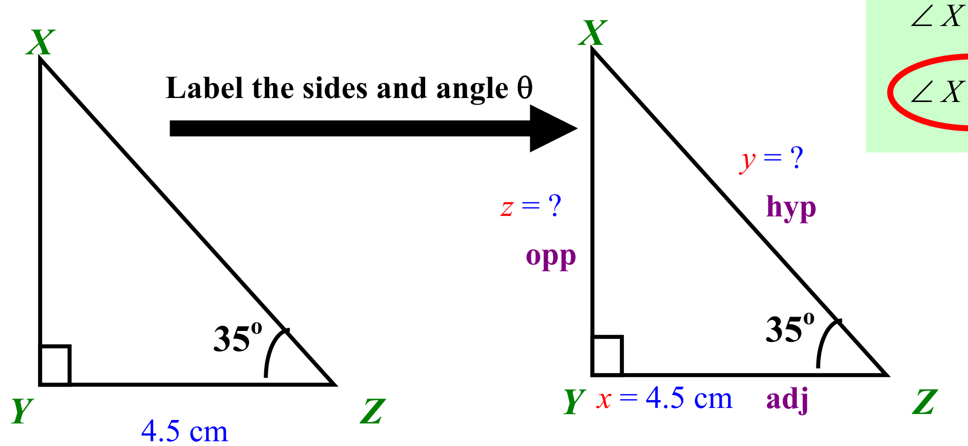
$$\sin C = \frac{\text{opp}}{\text{hyp}}$$
$$\sin C = \frac{45}{53}$$
$$\sin C = 58^\circ$$

2nd **SIN**

```
sin⁻¹(45/53)
          58.1092082
```

$$\angle A = 180^\circ - 90^\circ - 58^\circ$$
$$\angle A = 32^\circ$$

Example 3: Solve the triangle below.



$$\angle X = 180^\circ - 90^\circ - 35^\circ$$
$$\angle X = 55^\circ$$

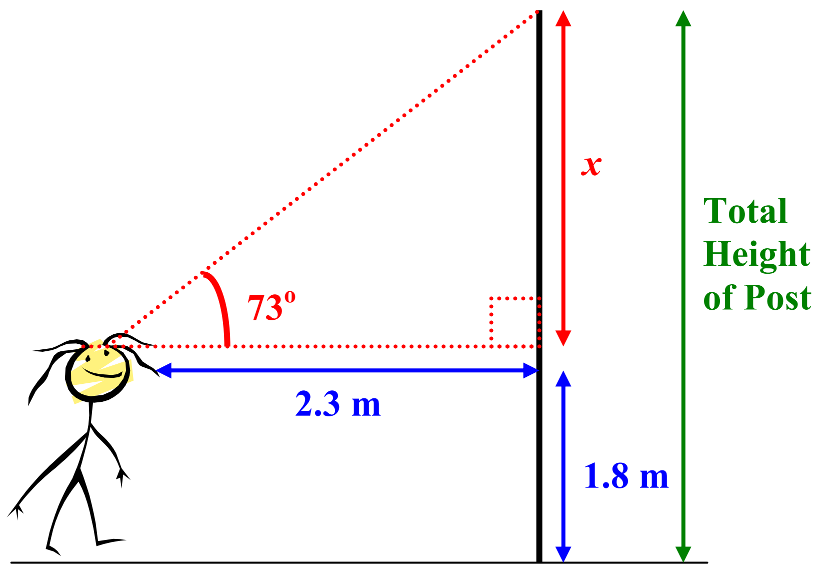
$$\tan 35^\circ = \frac{z}{4.5}$$
$$4.5\,(\tan 35^\circ) = z$$
$$z = 3.2 \text{ cm}$$

```
4.5*tan(35)
         3.150933922
```

$$\cos 35^\circ = \frac{4.5}{y}$$
$$y = \frac{4.5}{\cos 35^\circ}$$
$$y = 5.5 \text{ cm}$$

```
4.5/cos(35)
         5.493485649
```


Example 4: Adam wants to know the approximate height of the school's goal posts. He walks 2.3 m away from the base of the posts. His angle of elevation to the top of the post is 73°. What is the height of the goal post if Adam is 1.8 m tall?



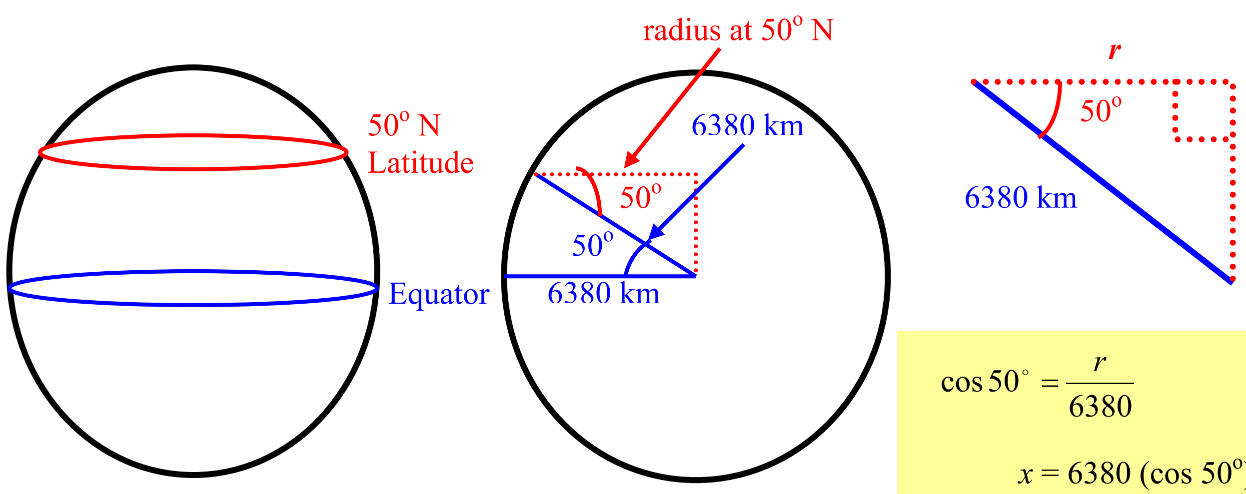
$$\tan 73^\circ = \frac{x}{2.3}$$

$x = 2.3\ (\tan 73^\circ)$
$x = 7.52$ m

Total Height = 7.52 m + 1.8 m

Total Height = 9.3 m

(AP) Example 5: Find the length of the 50° parallel of latitude, to the nearest km. Assume that the radius of the Earth is 6380 km.



$$\cos 50^\circ = \frac{r}{6380}$$

$x = 6380\ (\cos 50^\circ)$
$x = 4101$ km

Circumference of 50° N Circle
$= 2\pi r$
$= 2\pi\ (4101 \text{ km})$

Length at 50° Latitude = 25767 km

7-4 Homework Assignments

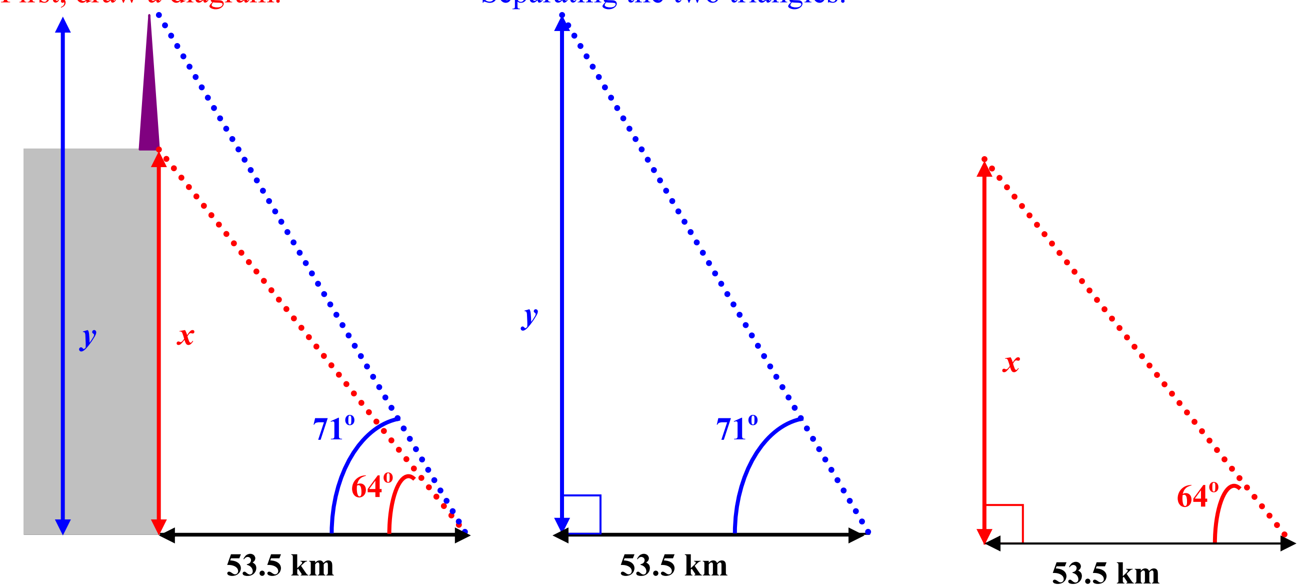
Regular: pg. 332 #1 to 13

AP: pg. 332 # 1 to 17

7-5: Problems Involving Two Right Triangles

Example 1: A TV antenna is on top of a tall building. A surveyor standing 53.5 m away measured the angle of elevation to the top of the building as 64°. She then measured the angle of elevation to the top of the antenna as 71°. What is the height of the TV antenna to the nearest tenth of a metre?

First, draw a diagram. Separating the two triangles.



$$\tan 71^\circ = \frac{y}{53.5}$$
$$y = 53.5\,(\tan 71^\circ)$$
$$\mathbf{y = 155.375\ m}$$

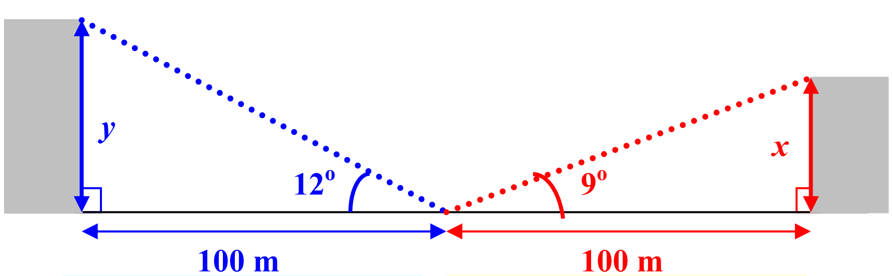
$$\tan 64^\circ = \frac{x}{53.5}$$
$$x = 53.5\,(\tan 64^\circ)$$
$$\mathbf{x = 109.691\ m}$$

Difference $= y - x$
$= 155.375\text{ m} - 109.691\text{ m}$

Diff = 45.684 m

Example 2: Two buildings are 200 m apart. From a point midway between them, the angles of elevation to the top of each building are 12° and 9° respectively. To the nearest tenth of a metre, how much taller is one building compared to the other?

First, draw a diagram.



$$\tan 12^\circ = \frac{y}{100}$$
$$y = 100\,(\tan 12^\circ)$$
$$\mathbf{y = 21.256\ m}$$

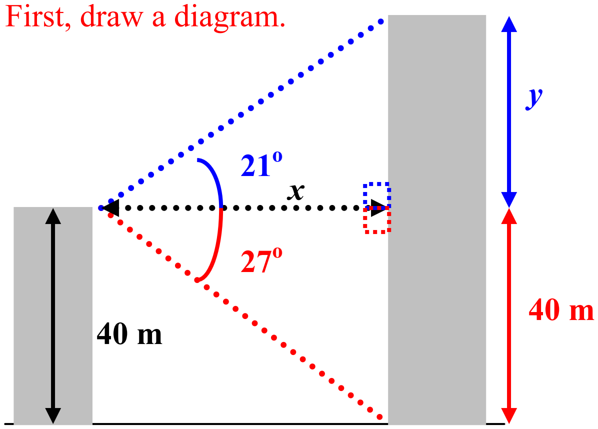
$$\tan 9^\circ = \frac{x}{100}$$
$$x = 100\,(\tan 9^\circ)$$
$$\mathbf{x = 15.838\ m}$$

Difference $= y - x$
$= 21.256\text{ m} - 15.838\text{ m}$

Diff = 5.4 m

Example 3 Two office towers are directly across the street from each other. The smaller tower is 40 m tall. The angle of elevation from the top of the smaller tower to the top of the larger tower is 21°. The angle of depression from the top of the smaller tower to the base of the larger tower is 27°. Determine the height of the larger office tower to the nearest tenth of a metre.

First, draw a diagram.



$\tan 27^\circ = \dfrac{40}{x}$

$x = \dfrac{40}{\tan 27^\circ}$

$x = 78.504$ m

$\tan 21^\circ = \dfrac{y}{x}$

$\tan 21^\circ = \dfrac{y}{78.504}$

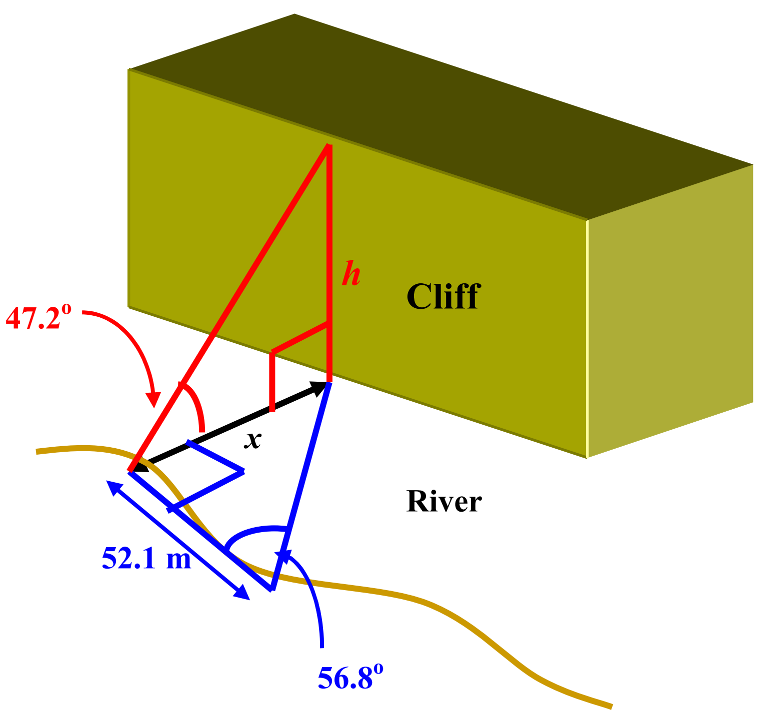
$y = 78.504\ (\tan 21^\circ)$

$y = 30.135$ m

Total Height = y + 40 m
= 30.135 m + 40 m

Total = 70.1 m

Example 4: Using the diagram below, find the height of the cliff.



$\tan 56.8^\circ = \dfrac{x}{52.1}$

$x = 52.1\ (\tan 56.8^\circ)$

$x = 79.617$ m

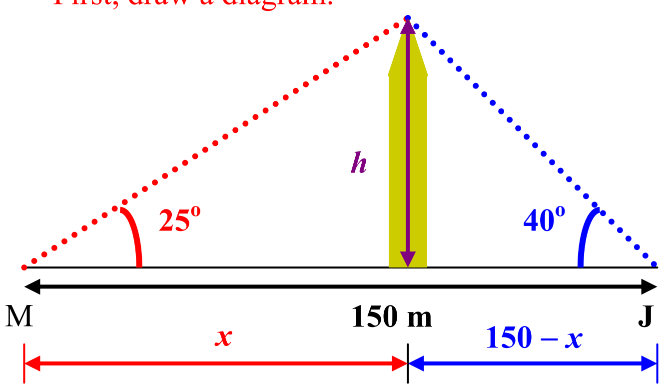
$\tan 47.2^\circ = \dfrac{h}{79.617}$

$h = 79.617\ (\tan 47.2^\circ)$

$h = 86.0$ m

Example 5: Mary stood on the east side of the church. She observed an angle of elevation of 25° to the top of the church tower. Joseph stood on the west side of the church. He measured an angle of elevation of 40° to the top of the church tower. Suppose the total distance between Mary and Joseph is 150 m, how tall is the church tower to the nearest tenth of a metre?

First, draw a diagram.



$$\tan 25^\circ = \frac{h}{x}$$
$$x\tan 25^\circ = h$$

$$\tan 40^\circ = \frac{h}{150-x}$$
$$h = \tan 40^\circ(150-x)$$

We can equate the two expressions of *h*.

$$x\tan 25^\circ = \tan 40^\circ(150-x)$$
$$x\tan 25^\circ = 150\tan 40^\circ - x\tan 40^\circ$$
$$x\tan 25^\circ + x\tan 40^\circ = 150\tan 40^\circ$$
$$x(\tan 25^\circ + \tan 40^\circ) = 150\tan 40^\circ$$
$$x = \frac{150\tan 40^\circ}{(\tan 25^\circ + \tan 40^\circ)}$$

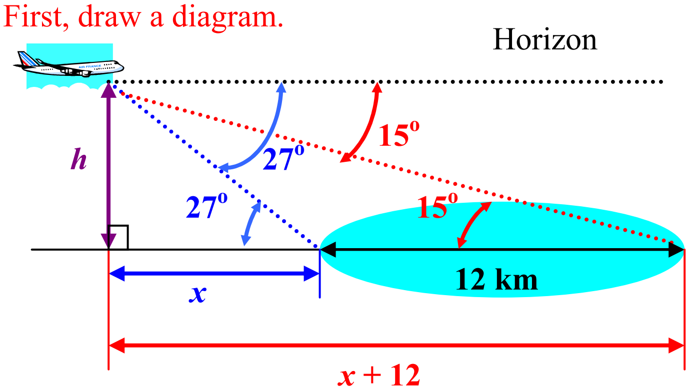
Now, we can solve for *h*.

$$\tan 25^\circ = \frac{h}{96.418}$$
$$h = 96.418(\tan 25^\circ)$$

***h* = 45.0 m**

Example 6: A plane is flying at a constant height. The pilot observed of an angle of depression of 27° to one end of the lake. At the same time her co-pilot measured an angle of depression of 15° to the opposite end of the lake. Both of them know that the lake is 2 km long. Determine the height of the plane to the nearest tenth of a kilometre.

First, draw a diagram.



$$\tan 27^\circ = \frac{h}{x}$$
$$x\tan 27^\circ = h$$

$$\tan 15^\circ = \frac{h}{x+2}$$
$$h = \tan 15^\circ(x+2)$$

We can equate the two expressions of *h*.

$$x\tan 27^\circ = \tan 15^\circ(x+2)$$
$$x\tan 27^\circ = x\tan 15^\circ + 2\tan 15^\circ$$
$$x\tan 27^\circ - x\tan 15^\circ = 2\tan 15^\circ$$
$$x(\tan 27^\circ - \tan 15^\circ) = 2\tan 15^\circ$$
$$x = \frac{2\tan 15^\circ}{(\tan 27^\circ - \tan 15^\circ)}$$

***x* = 2.218 km**

Now, we can solve for *h*.

$$\tan 27^\circ = \frac{h}{2.218}$$
$$h = 2.218(\tan 27^\circ)$$

***h* = 1.1 km**

7-5 Homework Assignments

Regular: pg. 335 to 337 #1 to 17 AP: pg. 335 to 337 # 1 to 19

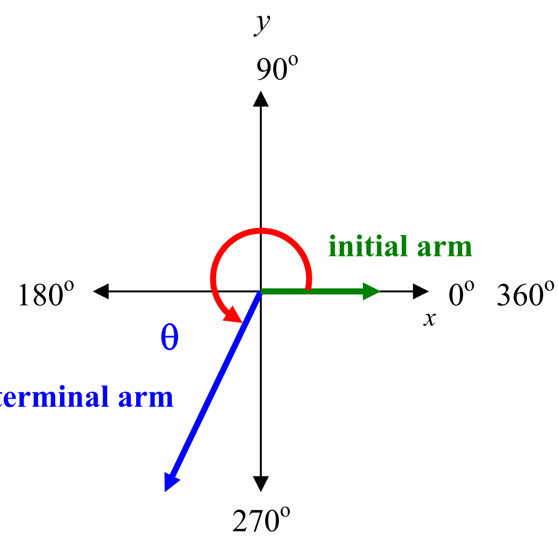
7-6: Angles in Standard Position

Standard Position Angles: - angles that can be defined on a coordinate grid.

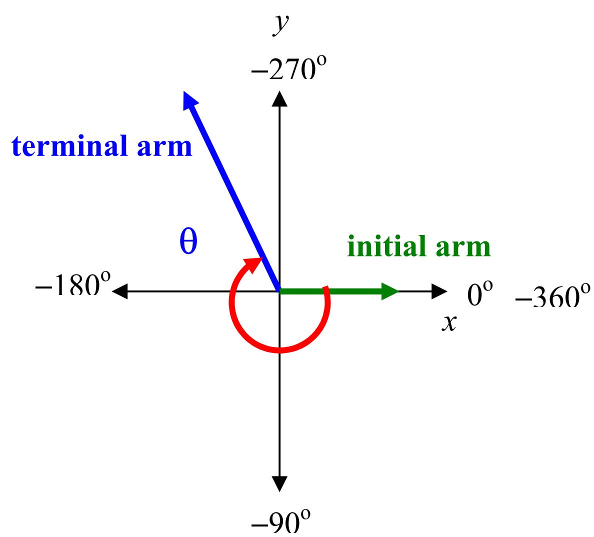
Initial Arm: - the beginning ray of the angle, which is fixed on the positive *x*-axis.

Terminal Arm: - rotates about the origin (0,0).
- the standard angle (θ) is then measured between the initial arm and terminal arm..

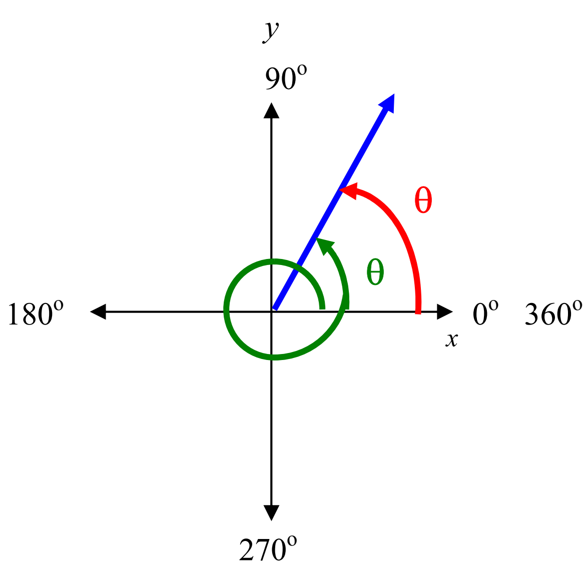
Positive Angle: - angle formed by the terminal arm rotated **counter-clockwise**.



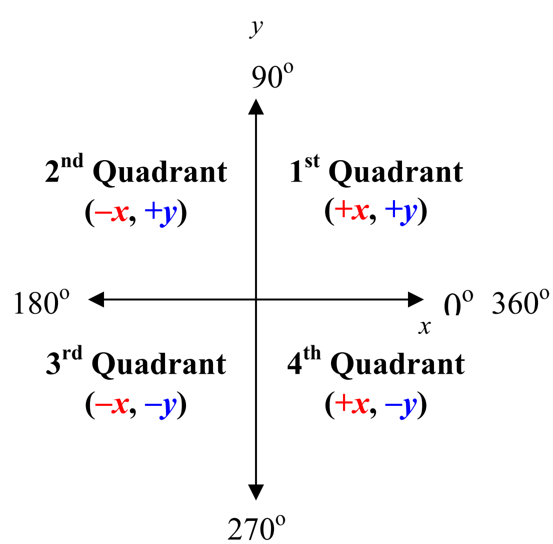
Negative Angle: - angle formed by the terminal arm rotated **clockwise**.



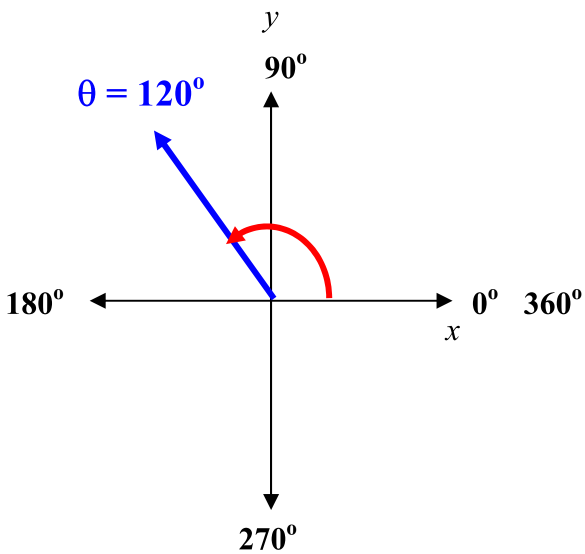
Coterminal Angles: - angles form when the terminal arms ends in the same position.

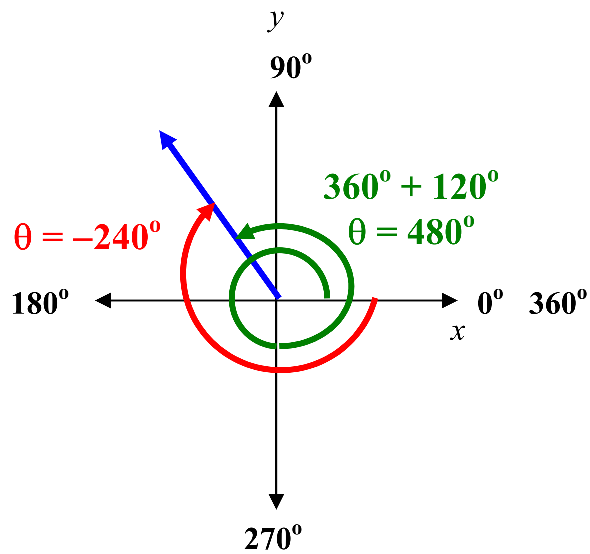


Quadrants: - the four parts of the Cartesian Coordinate Grid.



Example 1: Given θ = 120°. Draw the angle θ in standard position. Find and draw diagrams for two other angles which are coterminal to θ.

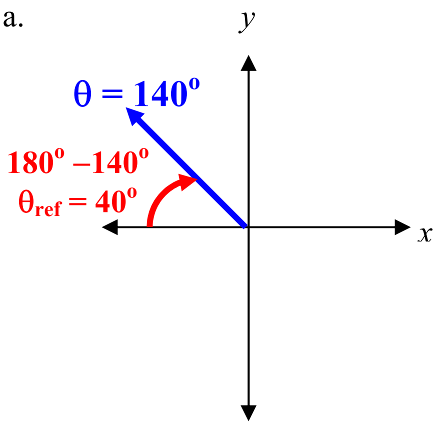




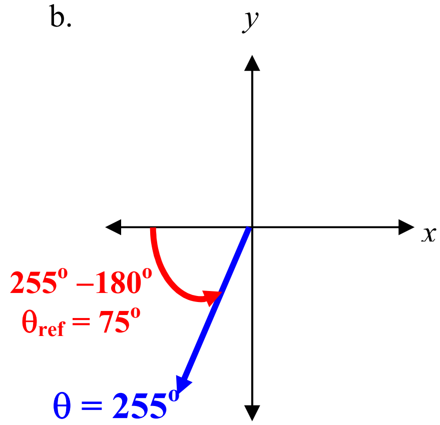
Reference Angle: - the acute angle between the terminal arm and the *x*-axis for any standard position angle.

Example 2: Find the reference angle for the following angles in standard position.

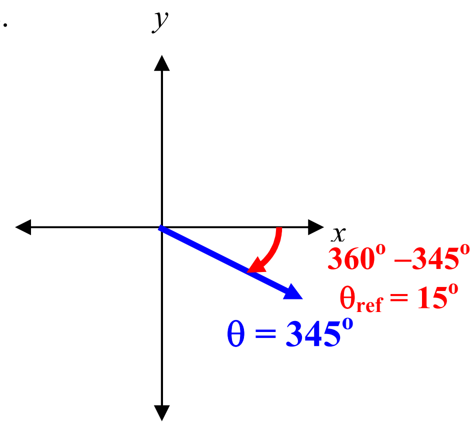
a.



b.

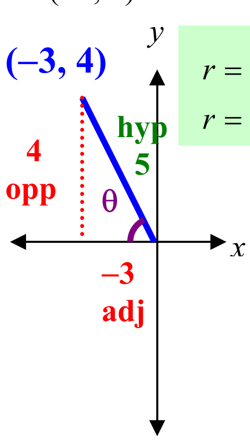


c.



Example 3: Each of the following points is on the terminal arm of angle θ. Use diagrams to find the exact values of all three trigonometric ratios of these angles.

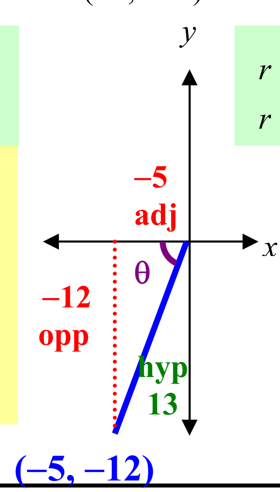
a. (–3, 4)



$$r = \sqrt{(-3)^2 + (4)^2}$$
$$r = 5$$

$$\sin\theta = \frac{4}{5}$$
$$\cos\theta = \frac{-3}{5}$$
$$\tan\theta = \frac{4}{-3}$$

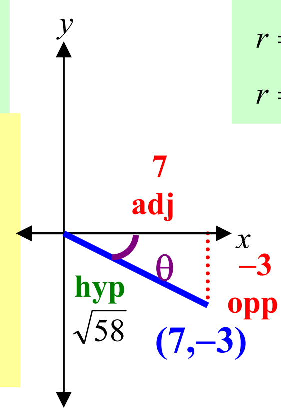
b. (–5, –12)



$$r = \sqrt{(-5)^2 + (-12)^2}$$
$$r = 13$$

$$\sin\theta = \frac{-12}{13}$$
$$\cos\theta = \frac{-5}{13}$$
$$\tan\theta = \frac{-12}{-5} = \frac{12}{5}$$

c. (7, –3)

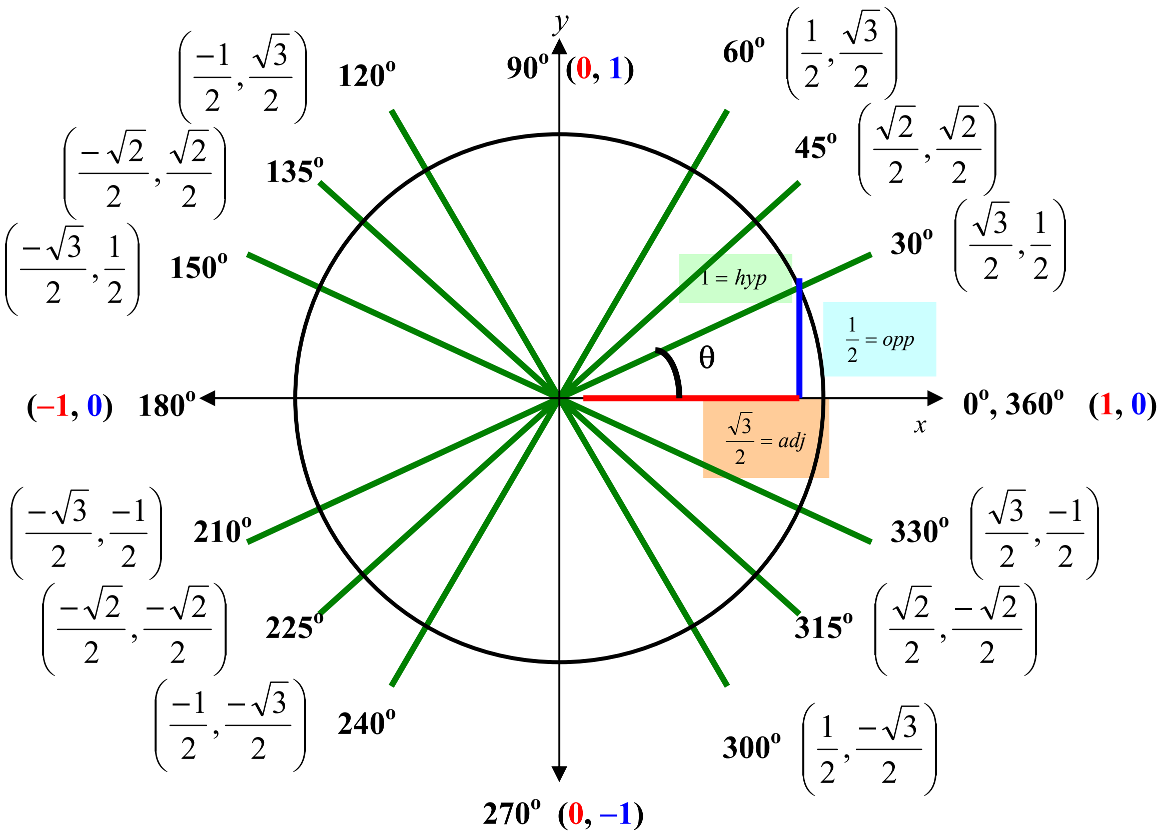


$$r = \sqrt{(7)^2 + (-3)^2}$$
$$r = \sqrt{58}$$

$$\sin\theta = \frac{-3}{\sqrt{58}}$$
$$\cos\theta = \frac{7}{\sqrt{58}}$$
$$\tan\theta = \frac{-3}{7}$$

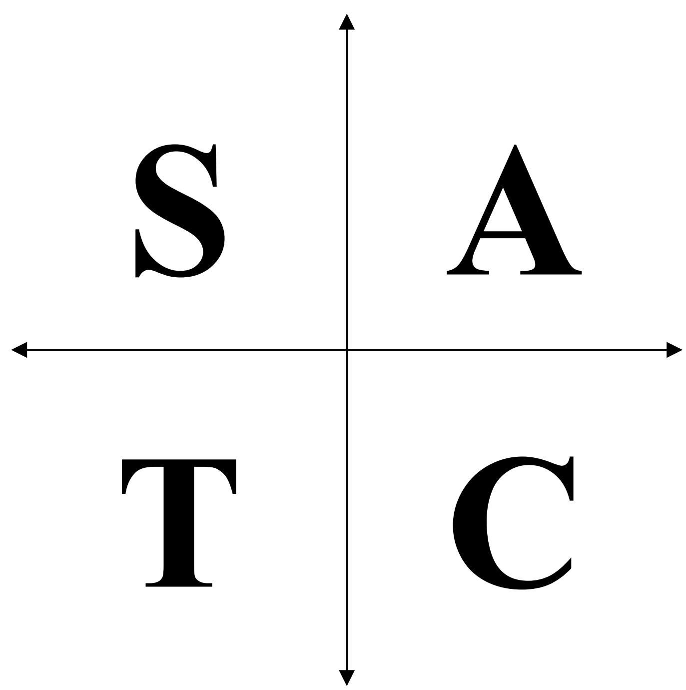
Unit Circle and the CAST Rule

(AP) If we draw a circle with a radius of 1 unit on the Cartesian coordinate gird, and overlay on it some angles in standard position, we will find the following diagram.



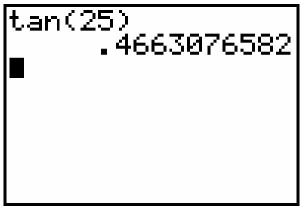
The coordinates (x, y) are the same as (cos θ, sin θ) of any angle θ in standard positions.

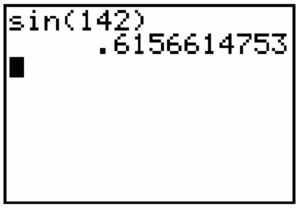
2nd Quadrant	**1st Quadrant**
Sin	**All**
$\sin\theta = +$ $\cos\theta = -$ $\tan\theta = -$	$\sin\theta = +$ $\cos\theta = +$ $\tan\theta = +$
Tan	**Cos**
$\sin\theta = -$ $\cos\theta = -$ $\tan\theta = +$	$\sin\theta = -$ $\cos\theta = +$ $\tan\theta = -$
3rd Quadrant	**4th Quadrant**

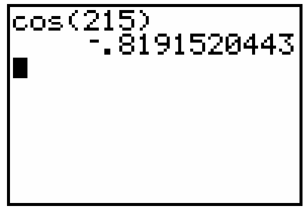


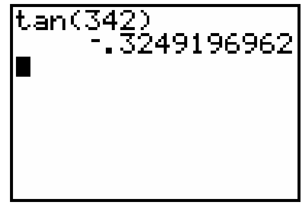
Example 4: Evaluate the followings to four decimal places.

a. $\tan 25^{\circ}$ b. $\sin 142^{\circ}$ c. $\cos 215^{\circ}$ d. $\tan 342^{\circ}$









$\tan 25^{\circ} = 0.4663$ **$\sin 142^{\circ} = 0.6157$** **$\cos 215^{\circ} = -0.8192$** **$\tan 342^{\circ} = -0.3249$**

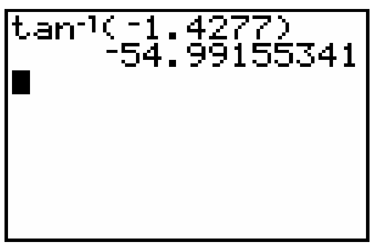
(AP) Example 5: Using the unit circle, evaluate the exact values of the followings.

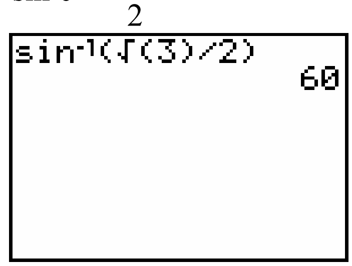
a. $\cos 150^{\circ}$ b. $\sin 270^{\circ}$ c. $\sin 135^{\circ}$ d. $\cos 300^{\circ}$

$\cos 150^{\circ} = \dfrac{-\sqrt{3}}{2}$ $\sin 270^{\circ} = -1$ $\sin 135^{\circ} = \dfrac{\sqrt{2}}{2}$ $\cos 300^{\circ} = \dfrac{1}{2}$

Example 6: Find the angles from the followings trigonometric ratios to the nearest degree if $0^{\circ} \le \theta \le 180^{\circ}$

a. $\cos\theta = -0.5467$ b. $\tan\theta = -1.4277$ c. $\sin\theta = \dfrac{\sqrt{3}}{2}$

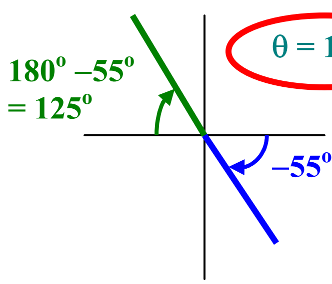




$\theta = 123^{\circ}$

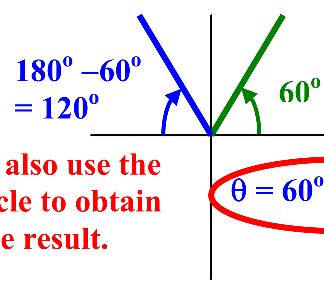
We know that for θ between 0° to 180°, when cos θ is negative, the angle must be in the second quadrant.

We know that for θ between 0° to 180°, when tan θ is negative, the angle must be in the 2nd quadrant. However, the answer on the calculator indicates that it is at the 4th quadrant. We will have to use the reference angle of 55°.



$\theta = 125^{\circ}$

We know that for θ between 0° to 180°, when sin θ is positive, the angle can be in the 1st or 2nd quadrant. However, the answer on the calculator indicates that it is at the 1st quadrant only. We will have to use the reference angle of 55° to figure out the next possible angle.



We can also use the unit circle to obtain the same result.

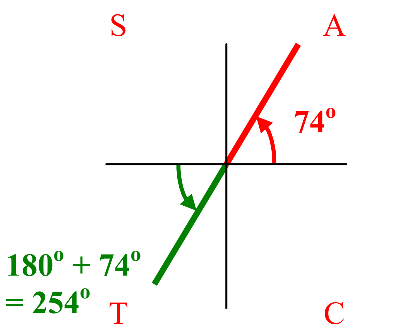
$\theta = 60^{\circ}$ and 120°

Example 7: Find the angles from the followings trigonometric ratios to the nearest degree if $0^{\circ} \leq A \leq 360^{\circ}$

Step 1: Ignore negative sign when using the 2nd sin, cos, or tan on the calculator to find the reference angle.
Step 2: Draw the *x*-*y* gird and label the possible quadrants for the angles based on whether the trig ratio is positive or negative.
Step 3: Find the actual angles between 0° and 360° using the reference angle.

a. $\tan A = 3.425$

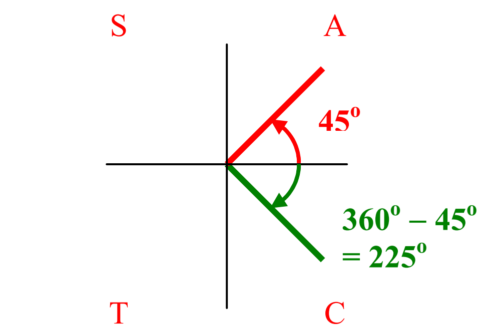
$\tan^{-1}(3.425) = 74^{\circ}$



$A = 74^{\circ}$ and 254°

b. $\cos A = \dfrac{\sqrt{2}}{2}$

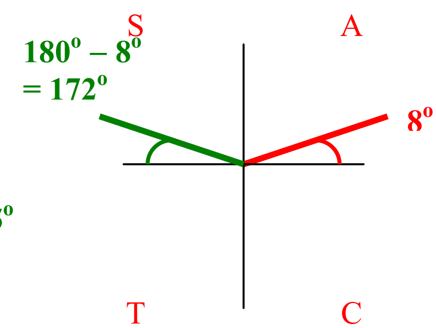
$\cos^{-1}\left(\sqrt{2}/2\right) = 45^{\circ}$



$A = 45^{\circ}$ and 225°

c. $\sin A = 0.1465$

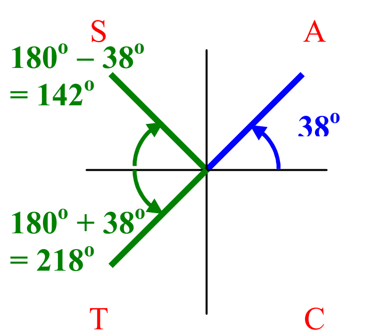
$\sin^{-1}(0.1465) = 8^{\circ}$



$A = 8^{\circ}$ and 172°

d. $\cos A = -0.7894$

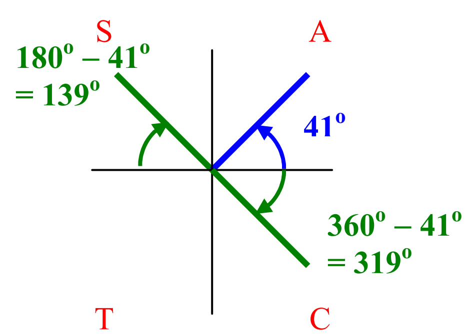
$\cos^{-1}(0.7894) = 38^{\circ}$



$A = 142^{\circ}$ and 218°

e. $\tan A = -0.8549$

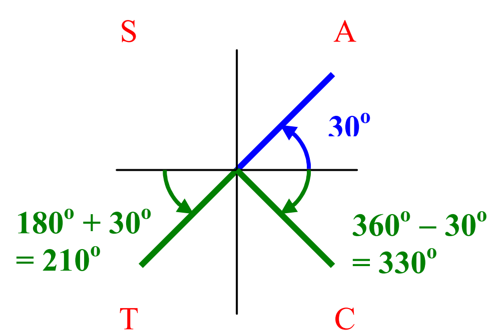
$\tan^{-1}(0.8549) = 41^{\circ}$



$A = 139^{\circ}$ and 319°

f. $\sin A = -\dfrac{1}{2}$

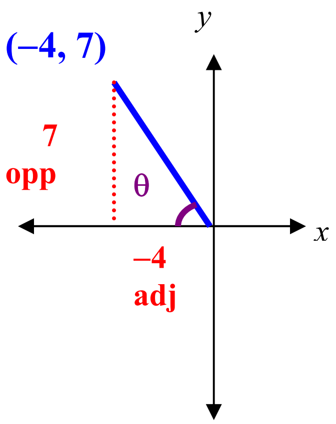
$\sin^{-1}(1/2) = 30^{\circ}$



$A = 210^{\circ}$ and 330°

Example 8: Each of the following coordinates is on the terminal arm of angle θ in standard positions. Find the angles to the nearest degree.

a. (–4, 7)



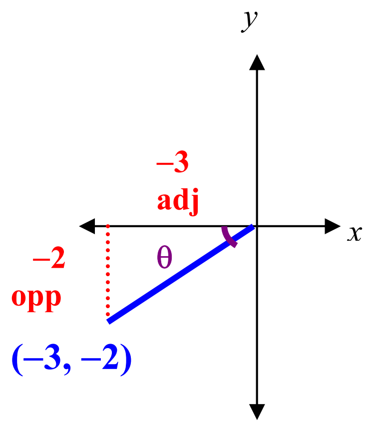
Use $\tan\theta = \frac{7}{4}$, find the reference angle θ.

$\tan^{-1}(7/4) = 60^{\circ}$

2^{nd} quadrant: $180^{\circ} - 60^{\circ}$

$\theta = 120^{\circ}$

b. (–3, –2)



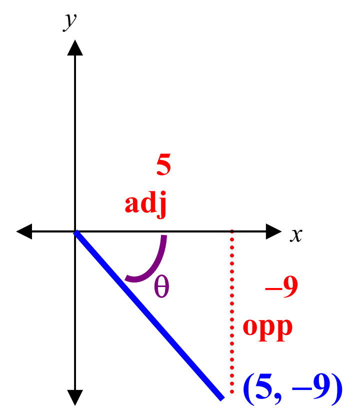
Use $\tan\theta = \frac{2}{3}$, find the reference angle θ.

$\tan^{-1}(2/3) = 34^{\circ}$

3^{rd} quadrant: $180^{\circ} + 34^{\circ}$

$\theta = 214^{\circ}$

c. (5, –9)



Use $\tan\theta = \frac{9}{5}$, find the reference angle θ.

$\tan^{-1}(9/5) = 61^{\circ}$

4^{th} quadrant: $360^{\circ} - 61^{\circ}$

$\theta = 299^{\circ}$

7-6 Homework Assignments

Regular: pg. 341-342 #1 to 26 & Worksheet 7-6: Angles in Standard Positions

AP: pg. 341-342 # 1 to 29 & Worksheet 7-6: Angles in Standard Positions

7-6 Worksheet: Angles in Standard Positions

1. Draw each angle in standard position and find its reference angle.

a) $\theta = 50^\circ$ b) $\theta = 120^\circ$ c) $\theta = 165^\circ$ d) $\theta = 240^\circ$
e) $\theta = 90^\circ$ f) $\theta = -180^\circ$ g) $\theta = 45^\circ$ h) $\theta = 270^\circ$
i) $\theta = 400^\circ$ j) $\theta = 750^\circ$ k) $\theta = -270^\circ$ l) $\theta = -60^\circ$

2. Find two other angles, which are coterminal with θ.

a) $\theta = 60^\circ$ b) $\theta = -210^\circ$ c) $\theta = 225^\circ$ d) $\theta = -90^\circ$
e) $\theta = 180^\circ$ f) $\theta = 90^\circ$ g) $\theta = -60^\circ$ h) $\theta = -360^\circ$

3. *P* is a point on the terminal arm of an angle θ in standard position. For the following angles, find

- the number of complete rotations.
- the quadrant where *P* is located.
- draw a diagram to show the position of *P*

a) $\theta = 480^\circ$ b) $\theta = 660^\circ$ c) $\theta = 870^\circ$ d) $\theta = 1000^\circ$
e) $\theta = 180^\circ$ f) $\theta = 270^\circ$ g) $\theta = 360^\circ$ h) $\theta = 450^\circ$

4. Each point *P* is on the terminal arm of an angle θ. Use a diagram to calculate the exact values of sin θ, cos θ, and tan θ.

a) *P* (12, −5) b) *P* (−4, −2) c) *P* (−3, 1) d) *P* (−3, −4)
e) *P* (6, −2) f) *P* (2, 9) (AP) g) *P* (0, 4) (AP) h) *P* (−5, 0)

5. Find the values of the following to 4 decimal places.

a) $\sin 130^\circ$ b) $\cos 145^\circ$ c) $\tan 130^\circ$ d) $\sin 200^\circ$
e) $\cos 260^\circ$ f) $\sin 325^\circ$ g) $\tan 347^\circ$ h) $\cos 534^\circ$
i) $\sin 23^\circ$ j) $\cos 34^\circ$ k) $\tan 72^\circ$ l) $\sin 103^\circ$
m) $\cos 172^\circ$ n) $\tan 238^\circ$ o) $\sin 309^\circ$ p) $\cos 501^\circ$

6. The angle θ is in the first quadrant, and $\tan\theta = \frac{2}{3}$,

a) Draw a diagram showing the angle in standard position and a point *P* on its terminal arm.
b) Determine possible coordinates for *P*.
c) Find the other two trigonometric ratios for θ.

7. Repeat **Question 6** if θ is in the second quadrant, and $\tan\theta = -\frac{5}{2}$.

8. Repeat **Question 6** if θ is in the second quadrant, and $\sin\theta = \frac{2}{\sqrt{5}}$.

9. Solve for θ to the nearest degree for $0^\circ \le \theta \le 90^\circ$.

a) $\sin\theta = 0.35$ b) $\cos\theta = 0.112$ c) $\tan\theta = 0.485$
d) $\cos\theta = \frac{4}{5}$ e) $\sin\theta = \frac{9}{10}$ f) $\tan\theta = 2$

10. Solve for θ to the nearest degree for $0^\circ \le \theta \le 180^\circ$.

a) $\sin\theta = 0.82$ b) $\cos\theta = 0.75$ c) $\tan\theta = -0.685$
d) $\cos\theta = -\frac{1}{9}$ e) $\sin\theta = \frac{1}{4}$ f) $\tan\theta = \frac{16}{5}$

11. Solve for θ to the nearest degree for $0^\circ \le \theta < 360^\circ$.

a) $\sin\theta = 0.75$ b) $\cos\theta = 0.0965$ c) $\tan\theta = 0.1392$ d) $\cos\theta = 0.3558$
e) $\sin\theta = 0.6666$ f) $\tan\theta = 2.671$ g) $\sin\theta = -0.6855$ h) $\cos\theta = -0.1881$
i) $\tan\theta = -0.2550$ j) $\cos\theta = -0.8245$ k) $\tan\theta = -3.1067$ l) $\sin\theta = -0.8040$

12. Solve for θ to the nearest degree for $0^\circ \le \theta < 360^\circ$.

a) $\tan\theta = 1$ b) $\cos\theta = \frac{1}{2}$ c) $\cos\theta = \frac{\sqrt{2}}{2}$
d) $\sin\theta = \frac{\sqrt{3}}{2}$ e) $\tan\theta = \frac{1}{\sqrt{3}}$ f) $\cos\theta = -\frac{\sqrt{3}}{2}$
g) $\tan\theta = -\sqrt{3}$ h) $\cos\theta = -\frac{1}{2}$ i) $\sin\theta = -\frac{\sqrt{2}}{2}$

13. The point given is on the terminal arm of an angle θ in standard position. Find a value of θ to the nearest degree.

a) $P(-1, -4)$ b) $Q(3, -4)$ c) $R(2, -3)$ d) $S(-1, 2)$

(AP) **14.** State the exact value of each ratio.

a) $\sin 120^\circ$ b) $\cos 150^\circ$ c) $\sin 90^\circ$ d) $\cos 270^\circ$
e) $\sin 240^\circ$ f) $\cos 315^\circ$ g) $\sin 360^\circ$ h) $\cos 90^\circ$
i) $\sin 300^\circ$ j) $\cos 480^\circ$ k) $\sin(-150^\circ)$ l) $\cos(-60^\circ)$

(AP) **15.** Using the unit circle, solve for θ to the nearest degree for $0^\circ \le \theta < 360^\circ$.

a) $\sin\theta = 0$ b) $\cos\theta = -1$ c) $\sin\theta = 1$ d) $\cos\theta = 0$
e) $\sin\theta = -1$ f) $\cos\theta = 1$ f) $\tan\theta = 0$ g) $\tan\theta$ = undefined

ANSWERS

1a) 50° **b)** 60° **c)** 15° **d)** 60° **e)** 90° **f)** 0° **g)** 45° **h)** 90° **i)** 40° **j)** 30° **k)** 90° **l)** 60° **2a)** 420°, –300° **b)** 150°, 510° **c)** –135°, 585° **d)** 270°, 630° **e)** –180°, 540° **f)** –270°, 450° **g)** 300°, 660° **h)** 0°, 360° **3a)** 1 rotation, 2nd quadrant **b)** 1 rotation, 4th quadrant **c)** 2 rotations, 2nd quadrant **d)** 2 rotations, 3rd quadrant **e)** 0 rotation, $-x$ axis **f)** 0 rotation, $-y$ axis **g)** 1 rotation, $+x$ axis **h)** 0 rotation, $+y$ axis **4a)** $\sin\theta = -\frac{5}{13}$ $\cos\theta = \frac{12}{13}$ $\tan\theta = -\frac{5}{12}$ **b)** $\sin\theta = -\frac{2}{\sqrt{20}}$ $\cos\theta = -\frac{4}{\sqrt{20}}$ $\tan\theta = \frac{1}{2}$ **c)** $\sin\theta = \frac{1}{\sqrt{10}}$ $\cos\theta = -\frac{3}{\sqrt{10}}$ $\tan\theta = -\frac{1}{3}$ **d)** $\sin\theta = -\frac{4}{5}$ $\cos\theta = -\frac{3}{5}$ $\tan\theta = \frac{4}{3}$ **e)** $\sin\theta = -\frac{2}{\sqrt{40}}$ $\cos\theta = \frac{6}{\sqrt{40}}$ $\tan\theta = -\frac{1}{3}$ **f)** $\sin\theta = \frac{9}{\sqrt{85}}$ $\cos\theta = \frac{2}{\sqrt{85}}$ $\tan\theta = \frac{9}{2}$ **g)** $\sin\theta = 1$ $\cos\theta = 0$ $\tan\theta$ = undefined **h)** $\sin\theta = -1$ $\cos\theta = 1$ $\tan\theta = 0$

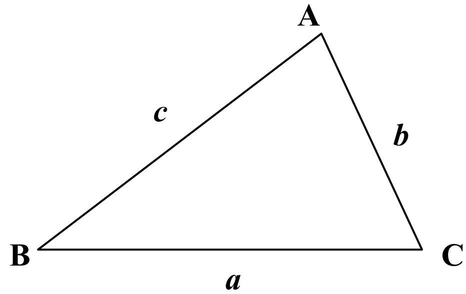
5a) 0.7660 **b)** –0.8192 **c)** –1.1918 **d)** –0.3420 **e)** –0.1736 **f)** –0.5736 **g)** –0.2309 **h)** –0.9945 **i)** 0.3907 **j)** 0.8290 **k)** 3.0777 **l)** 0.9744 **m)** –0.9903 **n)** 1.6003 **o)** –0.7771 **p)** –0.7771 **6b)** (3, 2) **c)** $\sin\theta = \frac{2}{\sqrt{13}}$ $\cos\theta = \frac{3}{\sqrt{13}}$

7b) (–2, 5) **c)** $\sin\theta = \frac{5}{\sqrt{29}}$ $\cos\theta = -\frac{2}{\sqrt{29}}$ **8b)** (–1, 2) **c)** $\cos\theta = -\frac{1}{\sqrt{5}}$ $\tan\theta = -2$ **9a)** 20° **b)** 84° **c)** 26° **d)** 37° **e)** 64° **f)** 63° **10a)** 55° and 125° **b)** 41° **c)** 146° **d)** 96° **e)** 14° and 166° **f)** 73° **11a)** 49° and 131° **b)** 84° and 276° **c)** 8° and 188° **d)** 69° and 291° **e)** 42° and 138° **f)** 69° and 249° **g)** 223° and 317° **h)** 101° and 259° **i)** 166° and 346° **j)** 146° and 214° **k)** 108° and 288° **l)** 234° and 306° **12a)** 45° and 225° **b)** 60° and 300° **c)** 45° and 315° **d)** 60° and 120° **e)** 30° and 210° **f)** 150° and 210° **g)** 120° and 300° **h)** 120° and 240° **i)** 225° and 315° **13a)** 256° **b)** 307° **c)** 304° **d)** 117° **14a)** $\frac{\sqrt{3}}{2}$ **b)** $-\frac{\sqrt{3}}{2}$ **c)** 1 **d)** 0 **e)** $-\frac{\sqrt{3}}{2}$ **f)** $\frac{\sqrt{2}}{2}$ **g)** 0 **h)** 0 **i)** $-\frac{\sqrt{3}}{2}$ **j)** $-\frac{1}{2}$ **k)** $-\frac{1}{2}$ **l)** $\frac{1}{2}$

15a) 0° and 180° **b)** 180° **c)** 90° **d)** 90° and 270° **e)** 270° **f)** 0° and 180° **g)** 90° and 270°

7-7: The Law of Sines

For any triangle, the **Law of Sines** allows us to solve the rest of the triangle if we <u>know</u> the measure of <u>an angle</u> and the <u>length of its opposite side</u>, plus <u>one other angle or side</u>.

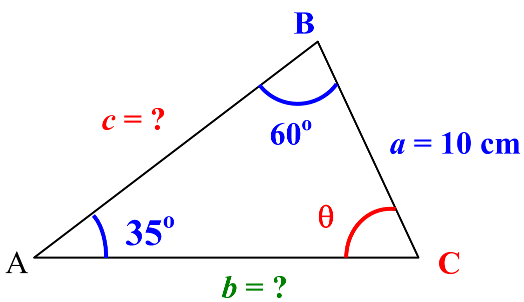


$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

When using the Sine Law, we only use a ratio of two fractions at one time.

Example 1: In ΔABC, ∠A = 35°, ∠B = 60°, and a = 10 cm. Solve the triangle to the nearest degree and to the nearest centimetre.

First, draw ΔABC and label appropriately.



$\angle C = 180^\circ - 35^\circ - 60^\circ$

∠C = 85°

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 35^\circ}{10} = \frac{\sin 60^\circ}{b}$$
$$b = \frac{10(\sin 60^\circ)}{\sin 35^\circ}$$

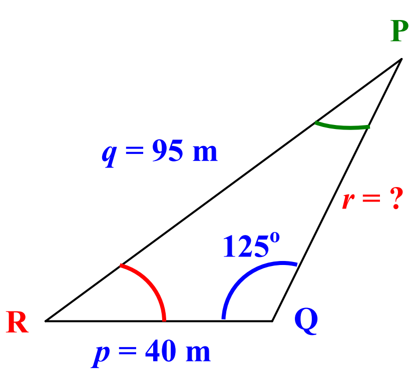
***b* = 15 cm**

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 35^\circ}{10} = \frac{\sin 85^\circ}{c}$$
$$c = \frac{10(\sin 85^\circ)}{\sin 35^\circ}$$

***c* = 17 cm**

Example 2: In ΔPQR, ∠Q = 125°, p = 40 m, and q = 95 m. Solve the triangle to the nearest degree and to the nearest metre.

First, draw ΔPQR and label appropriately.



$$\frac{\sin Q}{q} = \frac{\sin P}{p}$$
$$\frac{\sin 125^\circ}{95} = \frac{\sin P}{40}$$
$$\sin P = \frac{40(\sin 125^\circ)}{95}$$
$$\sin P = 0.3449061239$$

∠P = 20°

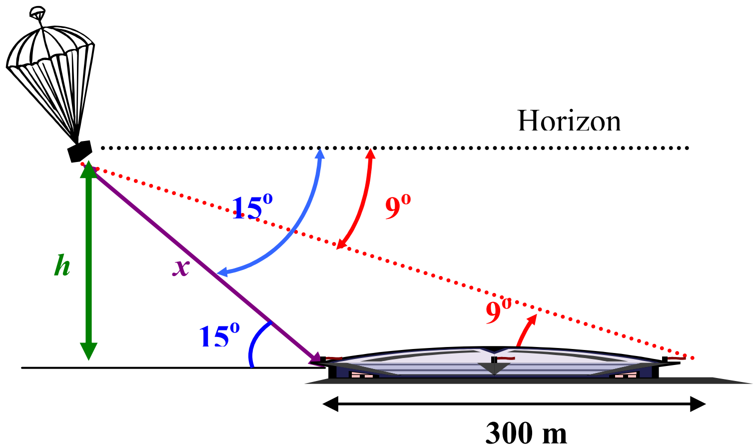
$\angle R = 180^\circ - 125^\circ - 20^\circ$

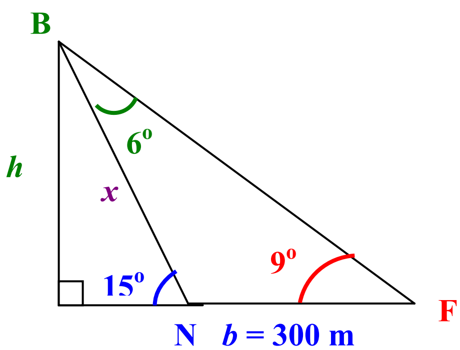
∠R = 35°

$$\frac{\sin Q}{q} = \frac{\sin R}{r}$$
$$\frac{\sin 125^\circ}{95} = \frac{\sin 35^\circ}{r}$$
$$r = \frac{95(\sin 35^\circ)}{\sin 125^\circ}$$

***b* = 67 m**

Example 3: A hot air balloon is helping to produce TV coverage for the Grey Cup. The top of the stadium is 300 m wide and the angles of depression to each side of the stadium are 9^{o} and 15^{o} respectively with the balloon off at one side of the stadium. To the nearest tenth of a metre, determine the distance between the hot air balloon to the closest side of the stadium and its altitude?





$$\frac{\sin B}{b} = \frac{\sin F}{x}$$

$$\frac{\sin 6^{\circ}}{300} = \frac{\sin 9^{\circ}}{x}$$

$$x = \frac{300(\sin 9^{\circ})}{\sin 6^{\circ}}$$

$$\boldsymbol{x = 449.0 \text{ m}}$$

$$\sin 15^{\circ} = \frac{h}{x}$$

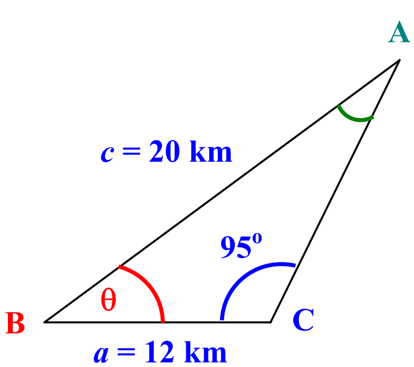
$$\sin 15^{\circ} = \frac{h}{448.97}$$

$$448.97\ (\sin 15^{\circ}) = h$$

$$\boldsymbol{h = 116.2 \text{ m}}$$

Example 4: Three watchtowers are located at positions A, B and C. Tower A and B is 20 km apart, and towers B and C are 12 km apart. The angle between the lines of sight from C to A and C to B is 95^{o}. To the nearest degree, what is the angle between the lines of sight from B to A and B to C?

Draw ΔABC and label appropriately.



Since we do not know side *b*, we need to solve for ∠A first.

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 95^{\circ}}{20} = \frac{\sin A}{12}$$

$$\sin A = \frac{12(\sin 95^{\circ})}{20}$$

$$\sin A = 0.5977168189$$

$$\angle A = 37^{o}$$

$$\angle B = 180^{o} - 95^{o} - 37^{o}$$

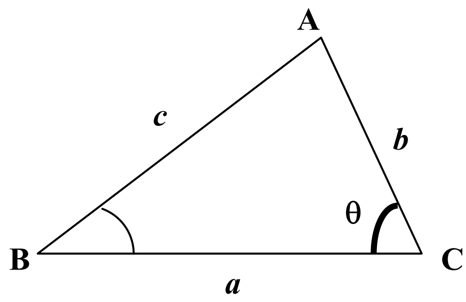
$$\boldsymbol{\angle B = 48^{o}}$$

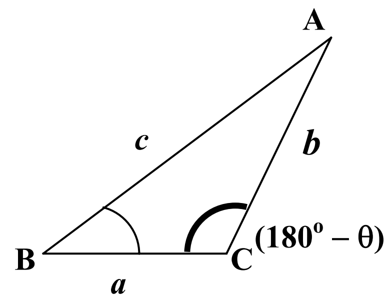
7-7 Homework Assignments

Regular: pg. 347-348 #1 to 10, 15, 16a, 18 and 19a.

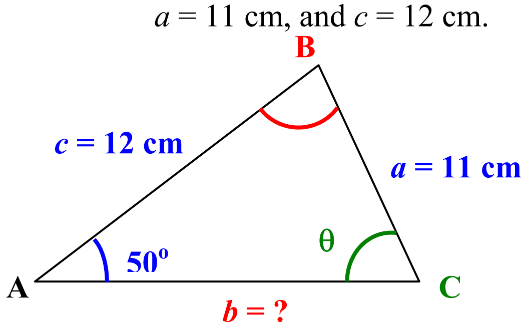
(AP) The Ambiguous Case of the Law of Sines

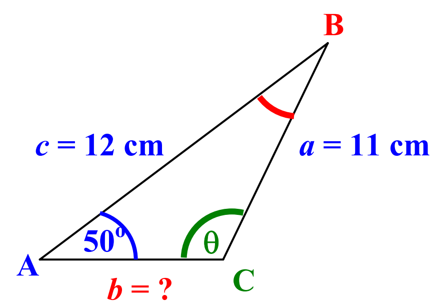
When solving for the second angle of the triangle, there exists a possibility of two solutions (doing the inverse sine, $\sin^{-1}$, between 0° and 180°). As such, both solutions must be analyzed and evaluated.





(AP) Example 5: Solve ΔABC to the nearest degree and to the nearest centimetre given $\angle A = 50^\circ$, $a = 11$ cm, and $c = 12$ cm.





Case 1

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 50^\circ}{12} = \frac{\sin C}{11}$$
$$\sin C = \frac{12(\sin 50^\circ)}{11}$$
$$\sin C = 0.835684847$$

$\angle C = 57^\circ$ (Case 1)

$$\angle B = 180^\circ - 50^\circ - 57^\circ$$

$\angle B = 73^\circ$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 50^\circ}{11} = \frac{\sin 73^\circ}{b}$$
$$b = \frac{11(\sin 73^\circ)}{\sin 50^\circ}$$

$b = 14$ cm

Case 2

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 50^\circ}{12} = \frac{\sin C}{11}$$
$$\sin C = \frac{12(\sin 50^\circ)}{11}$$
$$\sin C = 0.835684847$$

(or $180^\circ - 57^\circ$)

$\angle C = 123^\circ$ (Case 2)

$$\angle B = 180^\circ - 50^\circ - 123^\circ$$

$\angle B = 7^\circ$

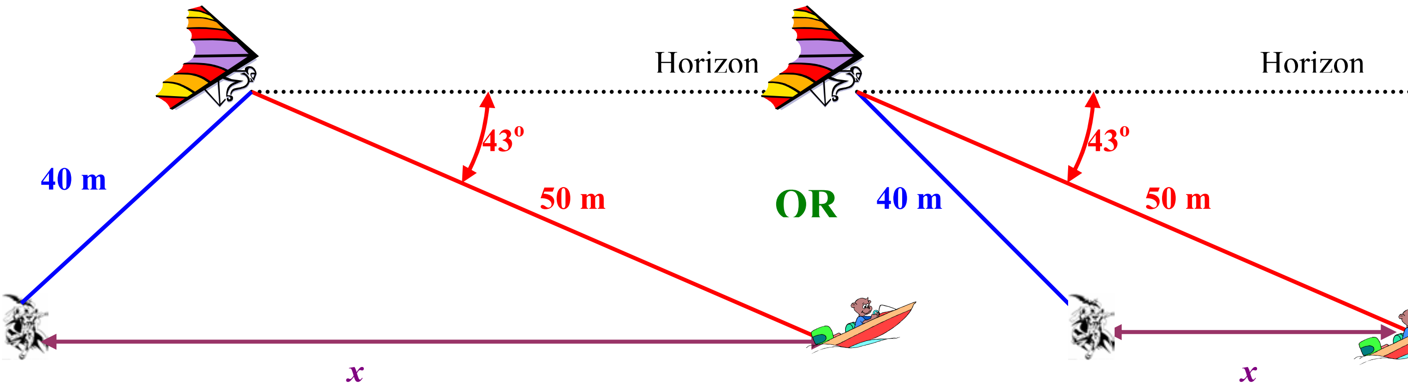
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 50^\circ}{12} = \frac{\sin 7^\circ}{b}$$
$$b = \frac{12(\sin 7^\circ)}{\sin 50^\circ}$$

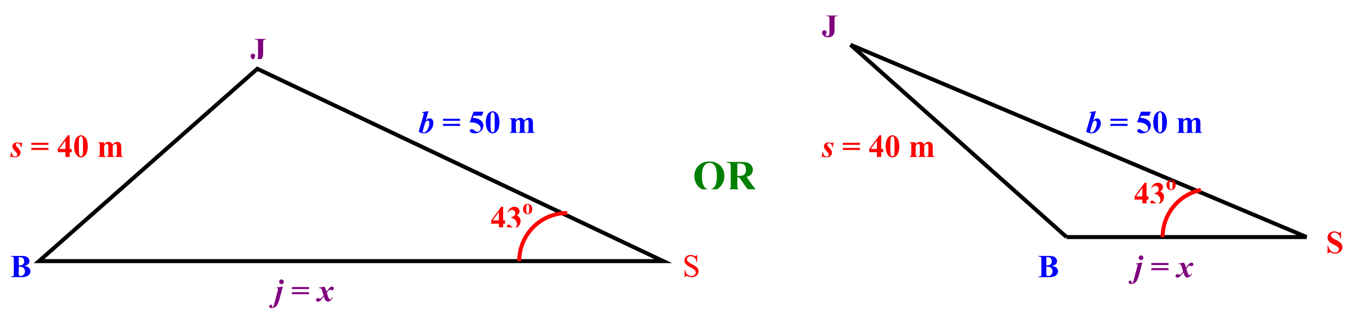
$b = 2$ cm

For Case 1: $\angle C = 57^\circ$, $\angle B = 73^\circ$ and $b = 14$ cm

For Case 2: $\angle C = 123^\circ$, $\angle B = 7^\circ$ and $b = 2$ cm

(AP) Example 6: The Joker is making an escape by hang-gliding above a speedboat with a 50 m long rope. Batman shot his hook at the Joker, which is attached to a 40 m ultra-light steel string from the shore. Suppose the Joker observed an angle of depression of 43° to the speedboat, find the distance between the speedboat and Batman to the nearest metre, and the angle of depression the Joker makes to Batman to the nearest degree.





Case 1

$$\frac{\sin S}{s} = \frac{\sin B}{b}$$

$$\frac{\sin 43^\circ}{40} = \frac{\sin B}{50}$$

$$\sin B = \frac{50(\sin 43^\circ)}{40}$$

$$\sin B = 0.8524979501$$

∠B = 58° (Case 1)

$\angle J = 180^\circ - 43^\circ - 58^\circ$

∠J = 79°

$$\frac{\sin S}{s} = \frac{\sin J}{j}$$

$$\frac{\sin 43^\circ}{40} = \frac{\sin 79^\circ}{x}$$

$$x = \frac{40(\sin 79^\circ)}{\sin 43^\circ}$$

***x* = 58 m**

Case 2

$$\frac{\sin S}{s} = \frac{\sin B}{b}$$

$$\frac{\sin 43^\circ}{40} = \frac{\sin B}{50}$$

$$\sin B = \frac{50(\sin 43^\circ)}{40}$$

$$\sin B = 0.8524979501$$

(or 180° – 58°)

∠B = 122° (Case 2)

$\angle J = 180^\circ - 43^\circ - 122^\circ$

∠J = 15°

$$\frac{\sin S}{s} = \frac{\sin J}{j}$$

$$\frac{\sin 43^\circ}{40} = \frac{\sin 15^\circ}{x}$$

$$x = \frac{40(\sin 15^\circ)}{\sin 43^\circ}$$

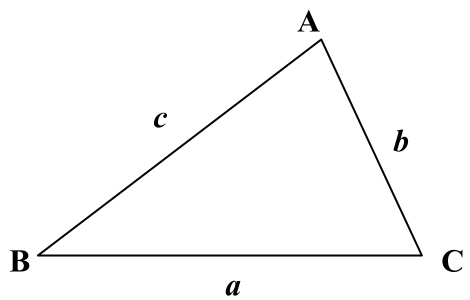
***x* = 15 m**

7-7 Homework Assignments

AP: pg. 347-348 # 1 to 10, 15, 16a, 18, 19a, 22a.

7-8: The Law of Cosines

For any triangle, the **Law of Cosines** allows us to solve the triangle if we know the measure of an angle and the length of its two adjacent sides (Case SAS), or if we know the lengths of all three sides (Case SSS).



$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

Solving for cos A:

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$2bc(\cos A) = b^2 + c^2 - a^2$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Solving for cos B:

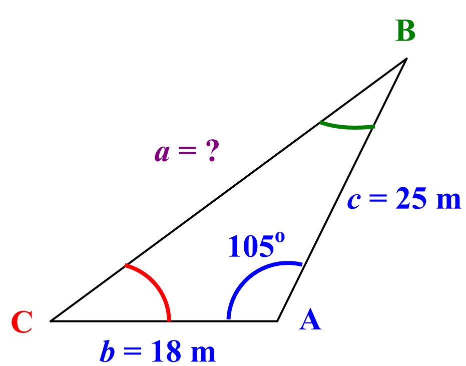
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$2ac(\cos B) = a^2 + c^2 - b^2$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

Solving for cos C:

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$
$$2ab(\cos C) = a^2 + b^2 - c^2$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

When given the length for all three sides, solve for the largest angle first!
This will eliminate any chance for an ambiguous situation.

Example 1: In ΔABC, ∠A = 105°, b = 18 m and c = 25 m. Solve the triangle to the nearest degree and to the nearest tenth of a metre.



$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$a^2 = 18^2 + 25^2 - 2(18)(25)(\cos 105)$$
$$a^2 = 1181.937141$$
$$a = \sqrt{1181.937141}$$

a = 34.4 m

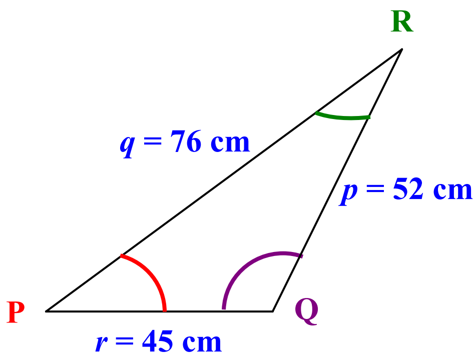
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 105^\circ}{34.4} = \frac{\sin B}{18}$$
$$\sin B = \frac{18(\sin 105^\circ)}{34.4}$$
$$\sin P = 0.5054263045$$

∠B = 30°

∠C = 180° – 105° – 30°

∠C = 45°

Example 2: In ΔPQR, p = 52 cm, q = 76 cm and r = 45 cm. Solve the triangle to the nearest degree.



Solving for the **largest angle first** will eliminate any chance for an ambiguous situation.

$$q^2 = p^2 + r^1 - 2pr(\cos Q)$$
$$2pr(\cos Q) = p^2 + r^2 - q^2$$
$$\cos Q = \frac{p^2 + r^2 - q^2}{2pr}$$
$$\cos Q = \frac{52^2 + 45^2 - 76^2}{2(52)(45)}$$
$$\cos Q = -0.2237179487$$

∠Q = 103°

Watch Out! You MUST enter Brackets!

```
(52²+45²-76²)/(2
*52*45)
     -.2237179487
cos⁻¹(Ans)
      102.9275004
```

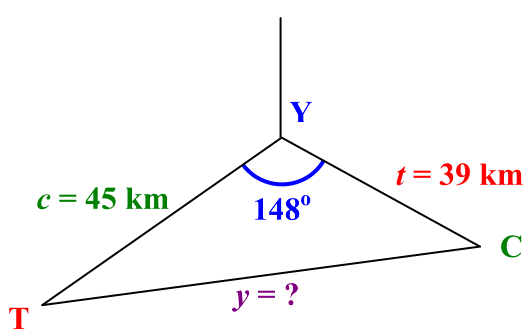
$$\frac{\sin Q}{q} = \frac{\sin P}{p}$$
$$\frac{\sin 103^\circ}{76} = \frac{\sin P}{52}$$
$$\sin P = \frac{52(\sin 103^\circ)}{76}$$
$$\sin P = 0.6666742549$$

∠P = 42°

∠R = 180° – 103° – 42°

∠C = 35°

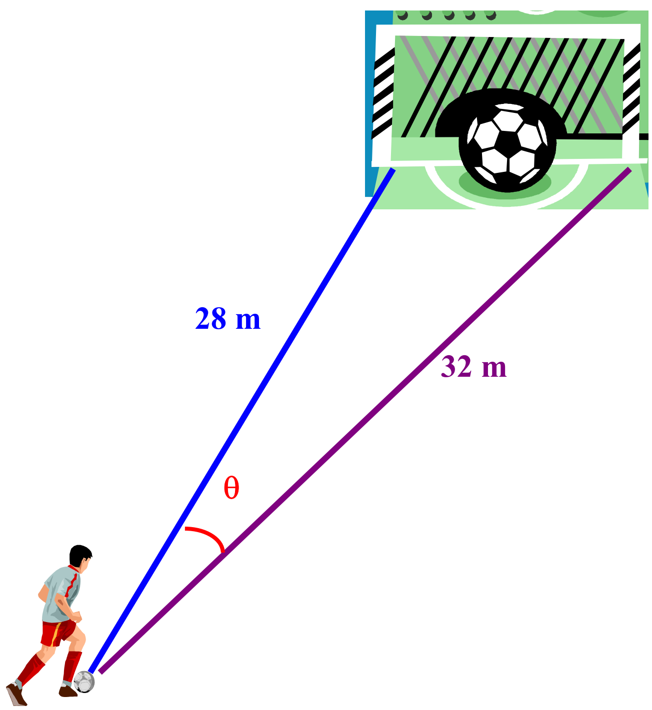
Example 3: A police and his motorcycle are at the Y intersection of two straight roads. By means of a radio scanner, he knows that a truck Is 45 km from the intersection on one road and a car is 39 km from the intersection on the other road. The roads intersect at Y at an angle of 148°. Calculate the distance between the truck and the car to the nearest tenth of a kilometre.

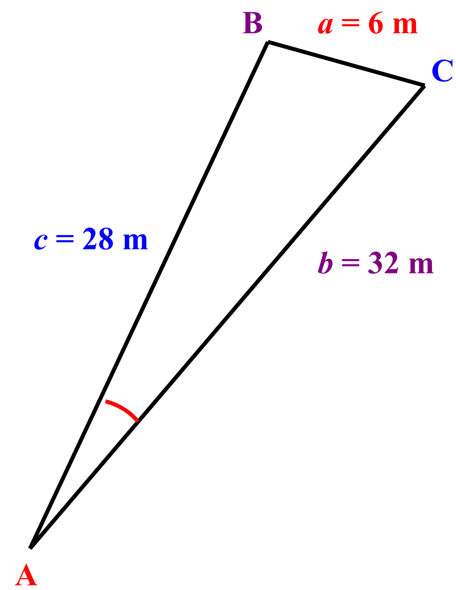


$$y^2 = c^2 + t^2 - 2ct(\cos Y)$$
$$y^2 = 45^2 + 39^2 - 2(45)(39)(\cos 148)$$
$$y^2 = 6522.648818$$
$$y = \sqrt{6522.648818}$$

***a* = 80.8 km**

Example 4: Standard soccer goal posts are 6 m apart. A player attempts to kick a ball between the posts from a point where the ball is 28 m from one end of the goal posts and 32 m from the other end of the posts. To the nearest tenth of a degree, within what angle must the player kick the ball?





$$a^2 = b^2 + c^1 - 2bc(\cos A)$$

$$2bc(\cos A) = b^2 + c^2 - a^2$$

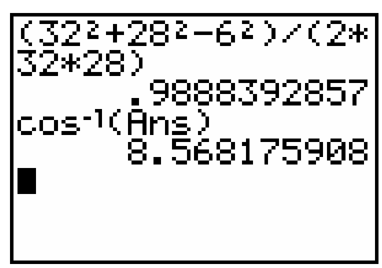
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{32^2 + 28^2 - 6^2}{2(32)(28)}$$

$$\cos A = 0.9888392857$$

$$\angle A = 8.6^{\text{o}}$$

Again Watch Out! You MUST enter Brackets!



7-8 Homework Assignments

Regular: pg. 352-353 #1 to 21

AP: pg. 352-353 #1 to 21, 23 to 28