Unit 1: Polynomials

3-1: Reviewing Polynomials

Expressions: - mathematical sentences with no equal sign. **Example**: $3x + 2$

Equations: - mathematical sentences that are equated with an equal sign. **Example**: $3x + 2 = 5x + 8$

Terms: - are separated by an addition or subtraction sign.
- each term begins with the sign preceding the variable or coefficient.

Monomial: **-** one term expression.

Example:

Numerical Coefficient

$$5x^2 \longleftarrow \textbf{Exponent}$$

Variable

Binomial: - two terms expression. **Example**: $5x^2 + 5x$

Trinomial: - three terms expression. **Example**: $x^2 + 5x + 6$

Polynomial: - many terms (more than one) expression.

All Polynomials must have whole numbers as exponents!!

Example: $9x^{-1} + 12x^{\frac{1}{2}}$ is NOT a polynomial.

Degree: - the term of a polynomial that contains the largest sum of exponents

Example: $9x^2y^3 + \boxed{4x^5y^2} + 3x^4$ Degree 7 (5 + 2 = 7)

Example 1: Fill in the table below.

Polynomial	Number of Terms	Classification	Degree	Classified by Degree
9	1	monomial	0	constant
$4x$	1	monomial	1	linear
$9x + 2$	2	binomial	1	linear
$x^2 - 4x + 2$	3	trinomial	2	quadratic
$2x^3 - 4x^2 + x + 9$	4	polynomial	3	cubic
$4x^4 - 9x + 2$	3	trinomial	4	quartic

Like Terms: - terms that have the same variables and exponents.

Examples: $2x^2y$ and $5x^2y$ are like terms $2x^2y$ and $5xy^2$ are NOT like terms

To Add and Subtract Polynomials:

Combine like terms by adding or subtracting their numerical coefficients.

Example 2: Simplify the followings.

a. $3x^2 + 5x - x^2 + 4x - 6$

$= \underline{3x^2} \underline{+ 5x} \underline{- x^2} \underline{+ 4x} - 6$

$= \boxed{2x^2 + 9x - 6}$

b. $(9x^2y^3 + 4x^3y^2) + (3x^3y^2 - 10x^2y^3)$

$= \underline{9x^2y^3} \underline{+ 4x^3y^2} \underline{+ 3x^3y^2} \underline{-10x^2y^3}$

$= \boxed{-x^2y^3 + 7x^3y^2}$

c. $(9x^2y^3 + 4x^3y^2) - (3x^3y^2 - 10x^2y^3)$

$= \underline{9x^2y^3} \underline{+ 4x^3y^2} \boxed{-} \underline{3x^3y^2} \boxed{+} \underline{10x^2y^3}$

$= \boxed{19x^2y^3 + x^3y^2}$

(drop brackets and switch signs in the bracket that had – sign in front of it)

d. Subtract

$$\begin{array}{r} 9x^2 + 4x \\ \underline{5x^2 - 7x} \end{array}$$

This is the same as $(9x^2 + 4x) - (5x^2 - 7x)$

$= \underline{9x^2} \underline{+ 4x} \boxed{-} \underline{5x^2} \boxed{+} \underline{7x}$

$= \boxed{4x^2 + 11x}$

To Multiply and Divide Monomials:

Multiply or Divide (Reduce) Numerical Coefficients.
Add or Subtract exponents of the same variable according to basic exponential laws.

Example 3: Simplify the followings.

a. $(3x^3y^2)\,(7x^2y^4)$

$= (3)(7)\,(x^3)(x^2)\,(y^2)(y^4)$

$= \boxed{21x^5y^6}$

b. $\dfrac{24x^7y^4z^5}{6x^3yz^5}$

$= \left(\dfrac{24}{6}\right)\left(\dfrac{x^7}{x^3}\right)\left(\dfrac{y^4}{y}\right)\left(\dfrac{z^5}{z^5}\right)$

$= 4x^4y^3z^0$ $(z^0 = 1)$

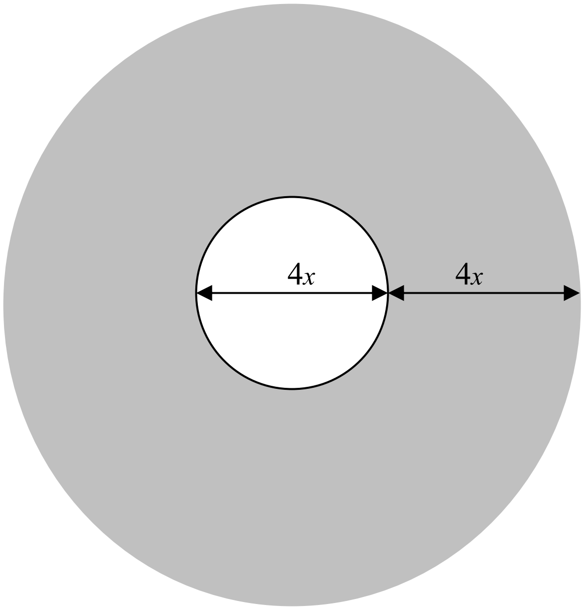
$= \boxed{4x^4y^3}$

c. $\dfrac{75a^3b^4}{25a^5b^3}$

$= \left(\dfrac{75}{25}\right)\left(\dfrac{a^3}{a^5}\right)\left(\dfrac{b^4}{b^3}\right)$

$= \boxed{3a^{-2}b \text{ or } \dfrac{3b}{a^2}}$

(AP) Example 4: Find the area of the following ring.



General Formula for Area of a Circle $A = \pi r^2$

Inner Circle Radius $= 2x$

Outer Circle Radius $= (2x + 4x) = 6x$

Inner Circle Area:

$A = \pi\,(2x)^2$

$A = \pi\,(4x^2)$

$A = 4\pi x^2$

Outer Circle Area:

$A = \pi\,(6x)^2$

$A = \pi\,(36x^2)$

$A = 36\pi x^2$

Shaded Area $= 36\pi x^2 - 4\pi x^2$

$\boxed{\textit{Shaded Area} = 32\pi x^2}$

3-1 Homework Assignment

Regular: pg. 102-103 #1 to 51, 55, 56

AP: pg. 102-103 #1 to 51, 53-57

3-3: Multiplying Polynomials

To Multiply Monomials with Polynomials

Example 1: Simplify the followings.

a. $3\,(2x^2 - 4x + 7)$

$= 3\,(2x^2 - 4x + 7)$

$= \boxed{6x^2 - 12x + 21}$

b. $2x\,(3x^2 + 2x - 4)$

$= 2x\,(3x^2 + 2x - 4)$

$= \boxed{6x^3 + 4x^2 - 8x}$

c. $3x\,(5x + 4) - 4\,(x^2 - 3x)$

$= 3x\,(5x + 4) - 4\,(x^2 - 3x)$

(only multiply the brackets right after the monomial)

$= \underline{\underline{15x^2}}\ \underline{+ 12x}\ \underline{\underline{- 4x^2}}\ \underline{+ 12x}$

$= \boxed{11x^2 + 24x}$

d. $8\,(a^2 - 2a + 3) - 4 - (3a^2 + 7)$

$= 8\,(a^2 - 2a + 3) - 4 - (3a^2 + 7)$

$= \underline{\underline{8a^2}} - 16a\ \underline{+ 24}\ \underline{- 4}\ \underline{\underline{- 3a^2}}\ \underline{- 7}$

$= \boxed{5a^2 - 16a + 13}$

To Multiply Polynomials with Polynomials

Example 2: Simplify the followings.

a. $(3x + 2)\,(4x - 3)$

$= (3x + 2)\,(4x - 3)$

$= 12x^2\ \underline{\underline{- 9x + 8x}} - 6$

$= \boxed{12x^2 - x - 6}$

b. $(x + 3)\,(2x^2 - 5x + 3)$

$= (x + 3)\,(2x^2 - 5x + 3)$

$= 2x^3\ \underline{- 5x^2}\ \underline{\underline{+ 3x}}\ \underline{+ 6x^2}\ \underline{\underline{- 15x}} + 9$

$= \boxed{2x^3 + x^2 - 12x + 9}$

c. $3\,(x + 2)\,(2x + 3) - (2x - 1)\,(x + 3)$

$= 3\,(x + 2)\,(2x + 3) - (2x - 1)\,(x + 3)$

$= 3\,(2x^2\ \underline{- 3x + 4x} - 6) - (2x^2\ \underline{\underline{+ 6x - x}} - 3)$

$= 3\,(2x^2 + x - 6) - (2x^2 + 5x - 3)$

$= \underline{\underline{6x^2}}\ \underline{+ 3x} - 18\ \underline{\underline{- 2x^2}}\ \underline{- 5x} + 3$

$= \boxed{4x^2 - 2x - 15}$

d. $(x^2 - 2x + 1)\,(3x^2 + x - 4)$

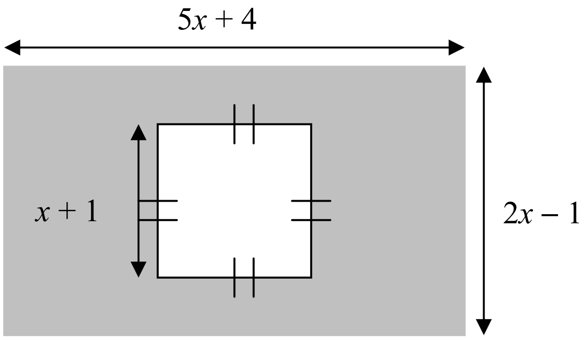
$= (x^2 - 2x + 1)\,(3x^2 + x - 4)$

$= 3x^4\ \underline{\underline{+ x^3}}\ \underline{- 4x^2}\ \underline{\underline{- 6x^3}}\ \underline{- 2x^2} + 8x\ \underline{+ 3x^2} + x - 4$

$= \boxed{3x^4 - 5x^3 - 3x^2 + 9x - 4}$

Example 3: Find the shaded area of each of the followings.

a.



Shaded Area = Big Rectangle – Small Square

$= (5x + 4)(2x - 1) - (x + 1)(x + 1)$

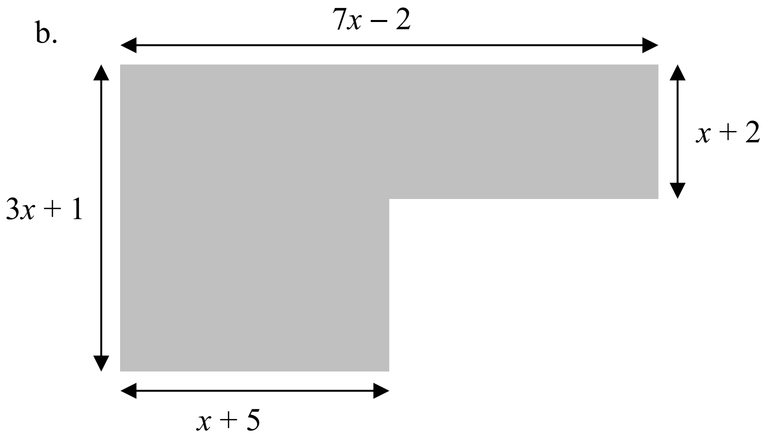
$= (10x^2 \underline{\underline{- 5x}} \underline{\underline{+ 8x}} - 4) - (x^2 \underline{+ x} \underline{+ x} + 1)$

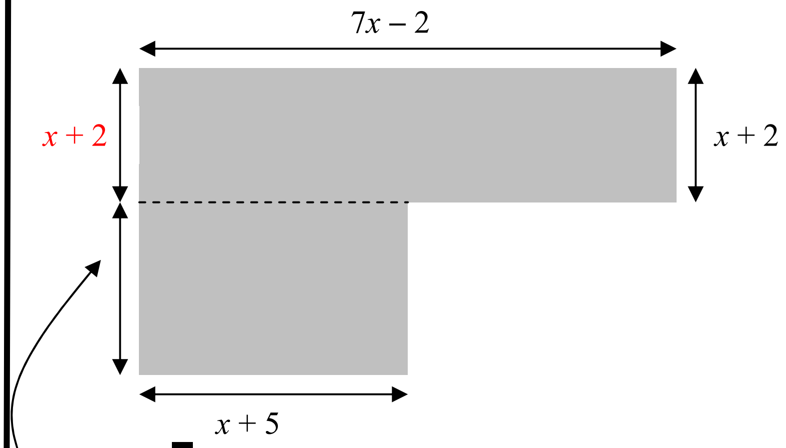
$= (10x^2 + 3x - 4) - (x^2 + 2x + 1)$

$= \underline{\underline{10x^2}} \underline{+ 3x} \underline{- 4} - \underline{\underline{x^2}} - \underline{2x} - \underline{1}$

$\boxed{\textit{Shaded Area} = 9x^2 + x - 5}$

b.





$(3x + 1) - (x + 2)$
$= 2x - 1$

Total Area = Top Rectangle + Bottom Rectangle

$= (7x - 2)(x + 2) + (2x - 1)(x + 5)$

$= (7x^2 \underline{\underline{+ 14x}} \underline{\underline{- 2x}} - 4) + (2x^2 \underline{\underline{+ 10x}} \underline{\underline{- x}} - 5)$

$= (7x^2 + 12x - 4) + (2x^2 + 9x - 5)$

$= \underline{\underline{7x^2}} \underline{+ 12x} \underline{- 4} \underline{\underline{+ 2x^2}} \underline{+ 9x} \underline{- 5}$

$\boxed{\textit{Total Area} = 9x^2 + 21x - 9}$

3-3 Homework Assignment

Regular: pg. 107-109 #1 to 77 (odd), 87, 88

AP: pg. 107-109 #2 to 84 (even) , 85, 87, 88, 91

3-4: Special Products

$$\mathbf{(x+y)^2 = (x+y)(x+y)}$$
$$\mathbf{(x+y)^3 = (x+y)(x+y)(x+y)}$$

$\mathbf{(x+y)^2}$ **is NOT** $\mathbf{x^2+y^2}$

Example 1: Simplify the followings.

a. $(2x+3)^2$

$= (2x+3)(2x+3)$

$= 4x^2 + 6x + 6x + 9$

$= \boxed{4x^2 + 12x + 9}$

b. $(x-2)^3$

$= (x-2)(x-2)(x-2)$

$= (x-2)(x^2 - 2x - 2x + 4)$

$= (x-2)(x^2 - 4x + 4)$

$= x^3 - 4x^2 + 4x - 2x^2 + 8x - 8$

$= \boxed{x^3 - 6x^2 + 12x - 8}$

c. $(3x+2)^2 - (2x-1)^2$

$= (3x+2)(3x+2) - (2x-1)(2x-1)$

$= (9x^2 + 6x + 6x + 4) - (4x^2 - 2x - 2x + 1)$

$= (9x^2 + 12x + 4) - (4x^2 - 4x + 1)$

$= 9x^2 + 12x + 4 - 4x^2 + 4x - 1$

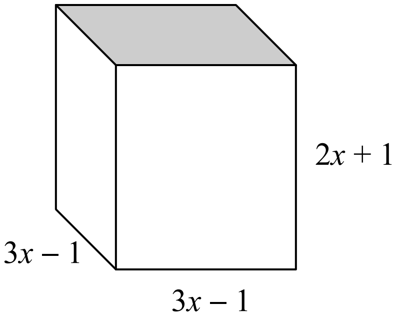
$= \boxed{5x^2 + 16x + 3}$

3-4 Homework Assignment

Regular: pg.112-113 #1 to 47 (odd), 49, 51, 54 (a, c, e, g), 55 (a, c, e, g), 56

AP: pg.112-113 #2 to 48 (even), 49 to 53, 54 (b, d, f, h), 55 (b, d, f, h), 56, 57

Example 2: Find the volume of the box below.



$Volume = Length \times Width \times Height$

$V = (3x-1)^2(2x+1)$

$V = (9x^2 - 3x - 3x + 1)(2x+1)$

$V = (9x^2 - 6x + 1)(2x+1)$

$V = 18x^3 + 9x^2 - 12x^2 - 6x + 2x + 1$

$\boxed{\mathbf{Volume = 18x^3 - 3x^2 - 4x + 1}}$

3-6: Common Factors

Common Factors can consist of two parts:

a. Numerical GCF: - Greatest Common Factor of all numerical coefficients and constant.
b. Variable GCF: - the lowest exponent of a particular variable.

After obtaining the GCF, use it to divide each term of the polynomial for the remaining factor.

Example 1: Factor the followings

a. $3x^2 + 6x + 12$

GCF = 3

$= \boxed{3\,(x^2 + 2x + 4)}$

b. $4a^2b - 8ab^2 + 6ab$

GCF = $2ab$

$= \boxed{2ab\,(2a - 4b + 3)}$

Factor by Grouping (Common Brackets as GCF)

$$\mathbf{a\,(c + d) + b\,(c + d) = (c + d)\,(a + b)}$$

Common Brackets **Take common bracket out as GCF**

Example 2: Factor.

a. $3x\,(2x - 1) + 4\,(2x - 1)$

$= \boxed{(2x - 1)\,(3x + 4)}$

b. $2ab + 3ac + 4b^2 + 6bc$

$= (2ab + 3ac) + (4b^2 + 6bc)$

$= a\,(2b + 3c) + 2b\,(2b + 3c)$

$= \boxed{(2b + 3c)\,(a + 2b)}$

c. $3x^2 - 6y^2 + 9x - 2xy^2$

$= (3x^2 - 6y^2) + (9x - 2xy^2)$

$= 3\,(x^2 - 2y^2) + x\,(9 - 2y^2)$ **Brackets are NOT the same! We might have to first rearrange terms.**

Try again after rearranging terms! $3x^2 + 9x - 2xy^2 - 6y^2$

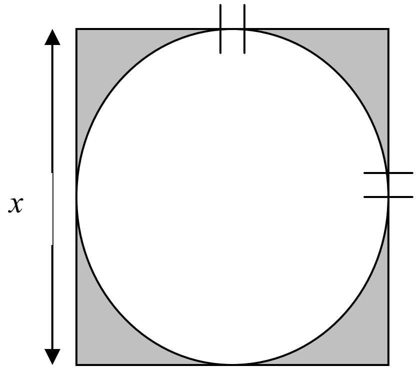
$= (3x^2 + 9x) - (2xy^2 + 6y^2)$

$= 3x\,(x + 3) - 2y^2\,(x + 3)$

$= \boxed{(x + 3)\,(3x - 2y^2)}$

Switch Sign in Second Bracket!
We have put a minus sign in front of a new bracket!

Example 3: Find the area of the shaded region in factored form and as a polynomial.



$$\text{Shaded Area} = \text{Area of Square} - \text{Area of Circle}$$

$$\text{Shaded Area} = x^2 - \pi\left(\frac{x}{2}\right)^2$$

$$\text{Shaded Area} = x^2 - \frac{\pi x^2}{4} \quad \text{(Polynomial Form)}$$

$$\text{Shaded Area} = x^2\left(1 - \frac{\pi}{4}\right) \quad \text{(Factored Form)}$$

$$\text{Area of a Circle} \quad A = \pi r^2$$

$$\text{Radius of Circle} = \frac{x}{2}$$

3-6 Homework Assignments

Regular: pg. 120 #1 to 35 (odd), 38 to 43

AP: pg.120 #2 to 36 (even), 37 to 44

3-8: Factoring $x^2 + bx + c$ (Leading Coefficient is 1)

$$x^2 + bx + c$$

What two numbers multiply to give *c*, but add up to be *b*?

Example 1: Completely factor the followings.

a. $x^2 + 5x + 6$

$= (x + 2)(x + 3)$

Product of 6			
1	6	−1	−6
2	3	−2	−3

sum of 5

b. $x^2 - 3x - 10$

$= (x + 2)(x - 5)$

Product of −10			
−1	10	1	−10
−2	5	2	−5

sum of −3

c. $a^2 - 8a + 15$

$= \boxed{(a - 3)(a - 5)}$

Product of 15			
1	15	−1	−15
3	5	−3	−5

sum of −8

d. $x^2 - 7xy + 12y^2$

$= \boxed{(x - 3y)(x - 4y)}$

Product of 12			
1	12	−1	−12
2	6	−2	−6
3	4	−3	−4

sum of −7

e. $x^2y^2 - 6xy - 16$

$= \boxed{(xy + 2)(xy - 8)}$

Product of −16			
−1	16	1	−16
−2	8	2	−8
−4	4	4	−4

sum of −6

f. $14 - 5w - w^2$

$= -w^2 - 5w + 14$

Rearrange in Descending Degree.

$= -(w^2 + 5w - 14)$

Take out −1 as common factor.

$= \boxed{-(w + 7)(w - 2)}$

$(+7)(-2) = -14$
$(+7) + (-2) = 5$

g. $3ab^2 - 3ab - 60a$

$= 3a(b^2 - b - 20)$

Take out GCF

$(+4)(-5) = -20$
$(+4) + (-5) = -1$

$= \boxed{3a(b + 4)(b - 5)}$

Example 2: List all values of k such that the trinomial $x^2 + kx - 24$ can be factored.

Product of – 24

−1 24	1 −24
−2 12	2 −12
−3 8	3 −8
−4 6	4 −6

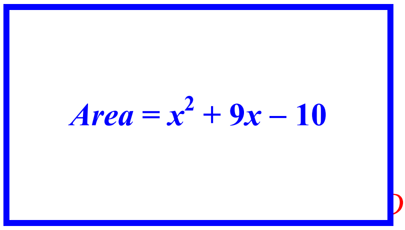
Possible Sums for k

23	−23
10	−10
5	−5
2	−2

Example 3: A rectangular has an area of $x^2 + 9x - 10$.

a. What are the dimensions of the rectangle?

b. If $x = 5$ cm, what are the actual dimensions?



Factor Area for dimensions

Area = *length* × *width*

$Area = x^2 + 9x - 10$

$Dimensions = (x + 10)(x - 1)$

if $x = 5$ cm

$Dimensions = (5 + 10)(5 - 1)$

$Dimensions$ = 15 cm × 4 cm

(AP) Example 4: Factor the followings.

a. $x^4 + 14x^2 - 32$

$= (x^2 + 16)(x^2 - 2)$

Assume $x^4 + bx^2 + c$ as the same as $x^2 + bx + c$ and factor. The answer will be $(x^2 \quad)(x^2 \quad)$.

b. $(x + 3)^2 + 6(x + 3) + 8$

$= y^2 + 6y + 8$

Do NOT Expand!!
Let $y = (x + 3)$

$= (y + 4)(y + 2)$

$= (x + 3 + 4)(x + 3 + 2)$

$= (x + 7)(x + 5)$

3-8 Homework Assignments

Regular: pg. 127 #19 to 59 (odd), 61, 65, 66

AP: pg. 127 #20 to 60 (even), 61, 65-68

3-9: Factoring $ax^2 + bx + c$ (Leading Coefficient is not 1, $a \neq 1$)

For factoring trinomial with the form $ax^2 + bx + c$, we will have to factor by grouping.

Example 1: Factor $6x^2 + 11x + 4$

First, we look for GCF. But there is no GCF!

$6x^2 + 11x + 4$

Multiply *a* and *c*.

Product of 24

1	24	−1	−24
2	12	−2	−12
3	8	−3	−8
4	6	−4	−6

sum of 11

Split the *bx* term into two separate terms.

$= 6x^2 + 3x + 8x + 4$

$= (6x^2 + 3x) + (8x + 4)$ Group by brackets

$= 3x\,(2x + 1) + 4\,(2x + 1)$ Take out GCF for each bracket.

$= \boxed{(3x + 4)\,(2x + 1)}$ Factor by Common Bracket!

Example 2: Completely factor the followings.

a. $2y^2 - 3y - 9$

(2) (−9) = −18

−1	18	1	−18
−2	9	2	−9
−3	6	3	−6

sum of −3

$= 2y^2 + 3y - 6y - 9$

$= (2y^2 + 3y)\,\boxed{-}\,(6y\,\boxed{+}\,9)$

$= y\,(2y + 3) - 3\,(2y + 3)$

$= \boxed{(2y + 3)\,(y - 3)}$

switch sign! (– sign in front of brackets)

b. $8d^2 - 2d - 3$

(8) (−3) = −24

−1	24	1	−24
−2	12	2	−12
−3	8	3	−8
−4	6	4	−6

sum of −2

$= 8d^2 + 4d - 6d - 3$

$= (8d^2 + 4d)\,\boxed{-}\,(6d\,\boxed{+}\,3)$

$= 4d\,(2d + 1) - 3\,(2d + 1)$

$= \boxed{(2d + 1)\,(4d - 3)}$

switch sign! (– sign in front of brackets)

c. $6x^3 - 14x^2 + 4x$ GCF = $2x$

$= 2x\,(3x^2 - 7x + 2)$

(2) (3) = 6

1	6	−1	−6
2	3	−2	−3

sum of −7

$= 2x\,(3x^2 - x - 6x + 2)$

$= 2x\,[\,(3x^2 - x)\,\boxed{-}\,(6x\,\boxed{-}\,2)\,]$

$= 2x\,[\,x\,(3x - 1) - 2\,(3x - 1)\,]$

$= \boxed{2x\,(3x - 1)\,(x - 2)}$

switch sign! (– sign in front of brackets)

d. $8m^2 - 6mn - 9n^2$

$8 \times -9 = -72$

$(6)\,(-12) = -72$

$(6) + (-12) = -6$

$= 8m^2 + 6mn - 12mn - 9n^2$

$= (8m^2 + 6mn)\,\boxed{-}\,(12mn\,\boxed{+}\,9n^2)$

$= 2m\,(4m + 3n) - 3n\,(4m + 3n)$

$= \boxed{(4m + 3n)\,(2m - 3n)}$

switch sign! (– sign in front of brackets)

Example 3: List all possible values for k in $5x^2 + kx - 4$ so it could be factored.

Product of (5) (–4) = –20		Possible Sums for k	
–1 20	1 –20	19	–19
–2 10	2 –10	8	–8
–4 5	4 –5	1	–1

(AP) Example 4: Factor the followings.

a. $4x^4 + 13x^2 + 9$

$\underline{4 \times 9 = 36}$

$(4)\,(9) = 36$

$(4) + (9) = 13$

$$= 4x^4 + 4x^2 + 9x^2 + 9$$

$$= (4x^4 + 4x^2) + (9x^2 + 9)$$

$$= x^2\,(x^2 + 1) + 9\,(x^2 + 1)$$

$$= \boxed{(x^2 + 1)\,(x^2 + 9)}$$

b. $18x^4 - 27x^2y + 4y^2$

$\underline{18 \times 4 = 72}$

$(-3)\,(-24) = 72$

$(-3) + (-24) = 72$

$$= 18x^4 - 3x^2y - 24x^2y + 4y^2$$

$$= (18x^4 - 3x^2y) - (24x^2y - 4y^2)$$

switch sign!
(– sign in front
of brackets)

$$= 3x^2\,(6x^2 - y) - 4y\,(6x^2 - y)$$

$$= \boxed{(6x^2 - y)\,(3x^2 - 4y)}$$

3-9 Homework Assignments

Regular: pg. 130 - 131 #7 to 49 (odd), 51, 54, 55

AP: pg. 130 - 131 #8 to 50 (even), 51, 54-56

3-10: Factoring Special Quadratics

Difference of Squares (Square – Square) $x^2 - y^2 = (x - y)(x + y)$

Example 1: Completely factor the followings.

a. $x^2 - 25$

$= (x - 5)(x + 5)$

b. $x^2 + 9$

(NOT Factorable – Sum of Squares)

c. $3x^2 - 300$

$= 3(x^2 - 100)$

$= 3(x - 10)(x + 10)$

d. $x^4 - 81$

$= (x^2 - 9)(x^2 + 9)$

$= (x - 3)(x + 3)(x^2 + 9)$

e. $9x^2 - 64y^2$

$= (3x - 8y)(3x + 8y)$

(AP) Example 2: Completely factor the followings.

a. $(x - 4)^2 - 49$ Look at $(x - 4)$ as a single item!

$= [(x - 4) - 7][(x - 4) + 7]$

$= (x - 11)(x + 3)$

b. $(2x + 3)^2 - (3x - 1)^2$ Look at $(x - 2)$ and $(3x + 1)$ as individual items!

$= [(2x + 3) - (3x + 1)][(2x + 3) + (3x + 1)]$

Watch Out! Subtracting a bracket!

$= [-x + 4][5x + 2]$

$= -(x - 4)(5x + 2)$ Take out negative sign from the first bracket!

Perfect Trinomial Square $ax^2 + bx + c = \left(\sqrt{a}\,x + \sqrt{c}\right)^2$

where a, c are square numbers, and $b = 2\left(\sqrt{a}\right)\left(\sqrt{c}\right)$

Example 3: Expand $(3x + 2)^2$.

$= (3x + 2)(3x + 2)$

$= 9x^2 + 6x + 6x + 4$

$= 9x^2 + 12x + 4$

$\boxed{\sqrt{9} = 3}$ $\boxed{2\left(\sqrt{9}\right)\left(\sqrt{4}\right) = 12}$ $\boxed{\sqrt{4} = 2}$

Example 4: Completely factor the followings.

a. $9x^2 + 30x + 25$

$$\begin{array}{ccc} 9x^2 & +\ 30x & +\ 25 \\ \uparrow & \uparrow & \uparrow \\ \boxed{\sqrt{9}=3} & \boxed{2\left(\sqrt{9}\right)\left(\sqrt{25}\right)=30} & \boxed{\sqrt{25}=5} \end{array}$$

$$= \boxed{(3x+5)^2}$$

b. $4x^2 - 28x + 49$

$$\begin{array}{ccc} 4x^2 & \text{[redacted]}\ 28x & +\ 49 \\ \uparrow & \uparrow & \uparrow \\ \boxed{\sqrt{4}=2} & \boxed{2\left(\sqrt{4}\right)\left(\sqrt{49}\right)=28} & \boxed{\sqrt{49}=7} \end{array}$$

$$= \boxed{(2x\ \text{[redacted]}\ 7)^2}$$

(AP) Example 5: Factor $x^6 - 20x^3 + 100$.

Assumes $x^6 + bx^3 + c$ is the same as $x^2 + bx + c$.
But the answer will be in the form of $(x^3 + \quad)(x^3 + \quad)$.

$$\begin{array}{ccc} x^6 & \text{[redacted]}\ 20x^3 & +\ 100 \\ \uparrow & \uparrow & \uparrow \\ \boxed{\sqrt{x^6}=x^3} & \boxed{2\left(\sqrt{x^6}\right)\left(\sqrt{100}\right)=20x^3} & \boxed{\sqrt{100}=10} \end{array}$$

$$= \boxed{(x^3\ \text{[redacted]}\ 10)^2}$$

Example 6: List all possible values for k that can make the following polynomials as perfect squares.

a. $x^2 + kx + 64$

$$\begin{array}{ccc} x^2 & +\ kx & +\ 64 \\ \uparrow & \uparrow & \uparrow \\ \boxed{\sqrt{1}=1} & \boxed{k=2(1)(8)} & \boxed{\sqrt{64}=8} \end{array}$$

$$\boxed{k = 16 \text{ and } -16}$$

The middle term of any perfect trinomial squares can have a positive or a negative numerical coefficient!

b. $kx^2 + 20x + 25$

$$\begin{array}{ccc} kx^2 & +\ 20x & +\ 25 \\ \uparrow & \uparrow & \uparrow \\ & \boxed{\begin{array}{l} 20 = 2\left(\sqrt{k}\right)(5) \\ 20 = 10\left(\sqrt{k}\right) \\ 2 = \sqrt{k} \\ k = 4 \end{array}} & \boxed{\sqrt{25}=5} \end{array}$$

c. $49x^2 + 56xy + ky^2$

$$\begin{array}{ccc} 49x^2 & +\ 56xy & +\ ky^2 \\ \uparrow & \uparrow & \uparrow \\ \boxed{\sqrt{49}=7} & \boxed{\begin{array}{l} 56 = 2(7)\left(\sqrt{k}\right) \\ 56 = 14\left(\sqrt{k}\right) \\ 4 = \sqrt{k} \\ k = 16 \end{array}} & \end{array}$$

3-10 Homework Assignments

Regular: pg. 133 - 134 #13 to 43 (odd), 54 to 56

AP: pg. 133 - 134 #14 to 44 (even), 46 to 57, 59, 61, 63

4-1A: Dividing Polynomials

Consider 562 ÷ 3.

$$\begin{array}{r} \mathbf{187} \\ 3\overline{)\mathbf{562}} \\ \underline{\mathbf{3}} \\ \mathbf{26} \\ \underline{\mathbf{24}} \\ \mathbf{22} \\ \underline{\mathbf{21}} \\ \mathbf{R\,1} \end{array}$$

Original Number **Remainder**

We can say that $\frac{562}{3} = 187 + \frac{1}{3}$

Divisor **Quotient** **Divisor**

Divisor **Quotient**

Original Number **Remainder**

Or, we can say $562 = (3)\,(187) + 1$

Polynomial Function **Divisor Function**

In general, for $P(x) \div D(x)$, we can write

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)} \quad \textbf{or} \quad P(x) = D(x)\,Q(x) + R$$

Restriction: $D(x) \neq 0$ **Quotient Function** Remainder

Non-Permissible Value (NPV): - **restriction on what the** variable CANNOT be **equal to due to the fact that the** Denominator CANNOT be 0**.**
(You can never divide by 0!)

Division with Monomials

Example 1: Simplify the followings

a. $\frac{21x^2y^2}{3x}$

$= \boxed{7xy^2}$

$3x \neq 0$

$x \neq 0$ NPV = 0

b. $\frac{(4x^5)(6x^2)}{3x}$

$= \frac{24x^7}{3x}$

$= \boxed{8x^6}$

$x \neq 0 \Rightarrow x \neq 0$

(NPV = 0)

c. $\frac{6x^3 + 9x^2 + 15x}{3x}$

$= \frac{6x^3}{3x} + \frac{9x^2}{3x} + \frac{15x}{3x}$

$= \boxed{2x^2 + 3x + 5}$

$x \neq 0 \Rightarrow x \neq 0$

(NPV = 0)

Divide each term of the polynomial by the monomial.

Long Division to Divide Polynomials

Example 2: Divide $\dfrac{6x^3+9x^2+15x+21}{2x+1}$

$$\begin{array}{r} 3x^2 \quad +3x \quad +6 \\ (2x+1)\overline{)6x^3+9x^2+15x+21} \\ \underline{-(6x^3+3x^2)} \qquad\qquad\quad \\ 6x^2+15x \quad\;\; \\ \underline{-(6x^2+3x)} \quad\;\; \\ 12x+21 \\ \underline{-(12x+6)} \\ R=15 \end{array}$$

$$\frac{6x^3+9x^2+15x+21}{2x+1}$$
$$=\frac{6x^3}{2x+1}+\frac{9x^2}{2x+1}+\frac{15x}{2x+1}+\frac{21}{2x+1}$$

You cannot divide monomial by polynomial!

Dividing by Polynomial is only possible by Long Division!

$$\frac{6x^3+9x+15x+21}{2x+1}=(3x^2+3x+6)+\frac{15}{2x+1}$$

OR

$$6x^3+9x^2+15x+21=(2x+1)\,(3x^2+3x+6)+15$$

For NPV, we let $2x+1=0$

$2x=-1$ NPV: $x=-\dfrac{1}{2}$

Example 3: Divide $\dfrac{3x^3-4x^2+5x-8}{x-2}$

$$\begin{array}{r} 3x^2+2x+9 \\ (x-2)\overline{)3x^3-4x^2+5x-8} \\ \underline{-(3x^3-6x^2)} \qquad\qquad \\ 2x^2+5x \quad\;\; \\ \underline{-(2x^2-4x)} \quad\;\; \\ 9x-8 \\ \underline{-(9x-18)} \\ R=10 \end{array}$$

$$\frac{3x^3-4x^2+5x-8}{x-2}=(3x^2+2x+9)+\frac{10}{x-2}$$

OR

$$3x^3-4x^2+5x-8=(x-2)\,(3x^2+2x+9)+10$$

For NPV, we let $x-2=0$

$x=2$ NPV: $x=2$

Example 4: Divide $\frac{2x^3-7x+6}{x-3}$

Missing Term from Decreasing Degree!

$$\frac{2x^3-7x+6}{x-3}=\frac{2x^3+0x^2-7x+6}{x-3}$$

$$\begin{array}{r}
2x^2+6x+11 \\
(x-3)\overline{\big)2x^3+0x^2-7x+6} \\
\underline{-(2x^3-6x^2)} \\
6x^2-7x \\
\underline{-(6x^2-18x)} \\
11x+6 \\
\underline{-(11x-33)} \\
R=39
\end{array}$$

$$\frac{2x^3-7x+6}{x-3}=(2x^2+6x+11)+\frac{39}{x-3}$$

OR

$$2x^3-7x+6=(x-3)\,(2x^2+6x+11)+39$$

For NPV, we let $x-3=0$

$x=3$ NPV: $x=3$

Example 5: Divide $\frac{4x^3-8x^2+7x-1}{2x^2+3}$

$$\frac{4x^3-8x^2+7x-1}{2x^2+3}=\frac{4x^3-8x^2+7x-1}{2x^2+0x+3}$$

Missing Term from Decreasing Degree!

$$\begin{array}{r}
2x-4 \\
(2x^2+0x+3)\overline{\big)4x^3-8x^2+7x-1} \\
\underline{-(4x^3+0x^2+6x)} \\
-8x^2+x-1 \\
\underline{-(-8x^2+0x-12)} \\
R=x+11
\end{array}$$

$$\frac{4x^3-8x+7x-1}{2x^2+3}=(2x-4)+\frac{x+11}{2x^2+3}$$

OR

$$4x^3-8x^2+7x-1=(2x^2+3)\,(2x-4)+(x+11)$$

For NPV, $2x^2+3=0$ No NPV!

$2x^2=-3$ (cannot take square root

$x^2=-\frac{3}{2}$ of negative number)

4-1A Homework Assignments

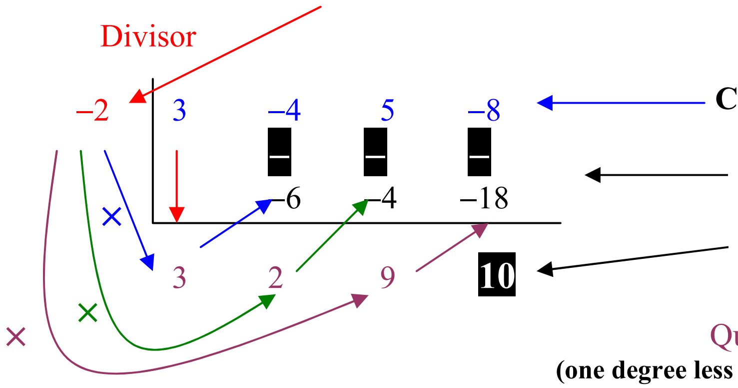
Regular: pg. 152 - 153 #1 to 63 (odd), 65 (a, c, e), 68, 70

AP: pg. 152 - 153 #2 to 64 (even), 65 (a, c, e), 68, 71, 74, 75, 76

4-1B: Synthetic Division

Only works well on divisor that is in a form of *x* + *a*, where Leading coefficient of Divisor is 1 on the divisor.

Example 1: Divide $\dfrac{3x^3 - 4x^2 + 5x - 8}{x - 2}$



Coefficients and constant of Polynomial

Subtract numbers in each column

Remainder = 10

Quotient = $3x^2 + 2x + 9$

(one degree less than the original polynomial)

Example 2: Divide $\dfrac{2x^3 - 7x + 6}{x - 3}$

$$\begin{array}{r|rrrr} -3 & 2 & 0 & -7 & 6 \\ & \downarrow & \boxed{-} & \boxed{-} & \boxed{-} \\ & & -6 & -18 & -33 \\ \hline & 2 & 6 & 11 & \boxed{39} \end{array}$$

Quotient = $2x^2 + 6x + 11$

Remainder = 39

Example 3: Divide $\dfrac{2x^3 - 3x^2 - 5x + 6}{x + 2}$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -5 & 6 \\ & & \boxed{-} & \boxed{-} & \boxed{-} \\ & & 4 & -14 & 18 \\ \hline & 2 & -7 & 9 & \boxed{-12} \end{array}$$

Quotient = $2x^2 - 7x + 9$

Remainder = −12

The Remainder Theorem

If you want to find only the remainder, you can simply substitute *a* from the Divisor, (*x* – *a*), into the original Polynomial, *P*(*x*).

In general, when $\frac{P(x)}{x-a}$, $P(a)$ = Remainder

Example 4: Find the remainder of the followings.

a. $\frac{3x^3 - 4x^2 + 5x - 8}{x - 2}$

$x - 2 = 0$
$x = 2$

$P(2) = 3(2)^3 - 4(2)^2 + 5(2) - 8$

$= 24 - 16 + 10 - 8$

$\boxed{\text{Remainder} = 10}$

b. $\frac{2x^3 - 7x + 6}{x - 3}$

$x - 3 = 0$
$x = 3$

$P(2) = 2(3)^3 - 7(3) + 6$

$= 54 - 21 + 6$

$\boxed{\text{Remainder} = 39}$

c. $\frac{2x^3 - 3x^2 - 5x + 6}{x + 2}$

$x + 2 = 0$
$x = -2$

$P(2) = 2(-2)^3 - 3(-2)^2 - 5(-2) + 6$

$= -16 - 12 + 10 + 6$

$\boxed{\text{Remainder} = -12}$

4-1B Homework Assignments

Regular: pg. 154 #1 to 17; pg. 155 Section 3 #5 (a, b, c)

AP: pg. 154 #1 to 17; pg. 155 Section 3 #5 (a, b, c)