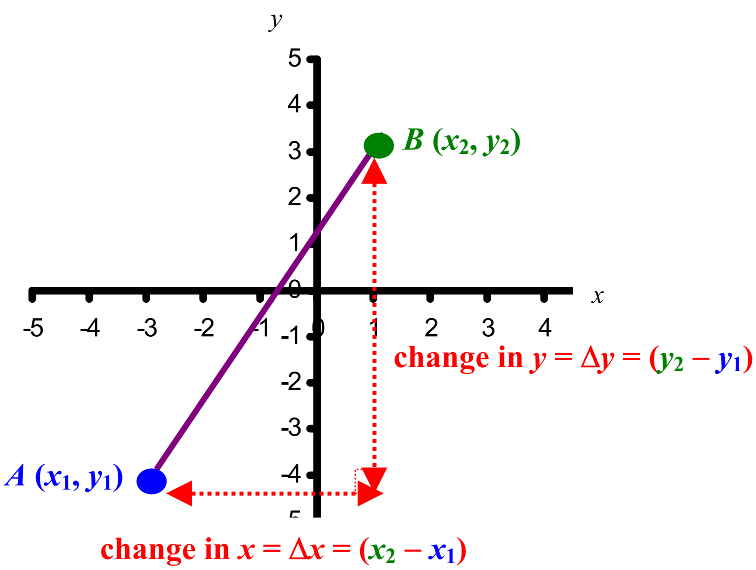
Unit 7: Coordinate Geometry

6-1: Distance Between Two Points



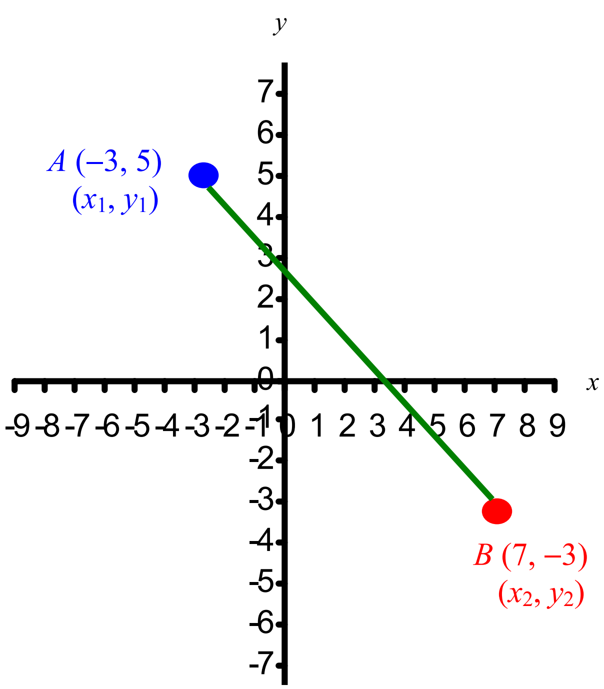
Distance Between Two Points

d^2 = (change in $x)^2$ + (change in $y)^2$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1: Find the distance (in exact value) between A (–3, 5) and B (7, –3).



$$d_{\overline{AB}} = \sqrt{(7 - -3)^2 + (-3 - 5)^2}$$
$$= \sqrt{(10)^2 + (-8)^2}$$
$$= \sqrt{100 + 64}$$
$$= \sqrt{164}$$

$$d_{\overline{AB}} = 2\sqrt{41}$$

Example 2: A circle has a diameter with endpoints (–4, –1) and (2, 6). Find the exact length of the radius.

Let $A\ (x_1, y_1) = (-4, -1)$ and $B\ (x_2, y_2) = (2, 6)$

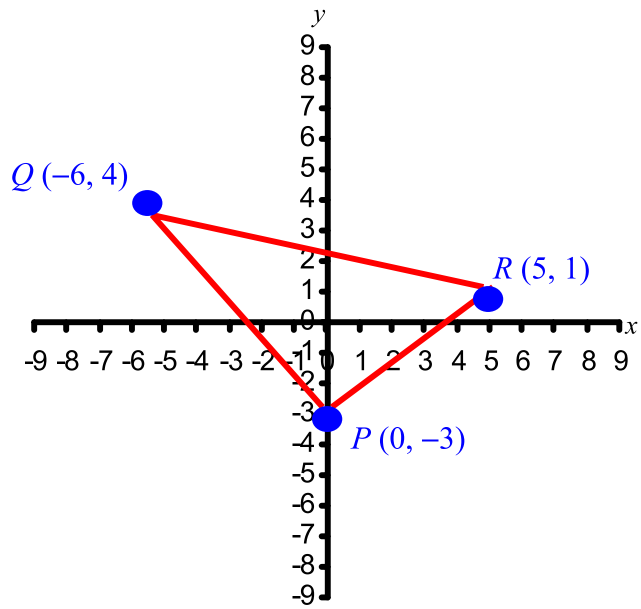
Length of Diameter $\overline{AB}$

$$d_{\overline{AB}} = \sqrt{(2 - -4)^2 + (6 - -1)^2}$$
$$= \sqrt{6^2 + 7^2} = \sqrt{36 + 49}$$
$$d_{\overline{AB}} = \sqrt{85}$$

Length of the Radius $= \dfrac{diameter\ \overline{AB}}{2}$

Length of the Radius $= \dfrac{\sqrt{85}}{2}$

Example 3: A triangle has vertices at P (0, −3), Q (−6, 4) and R (5, 1). Find the perimeter of the triangle to the nearest tenth of a unit and classify it.



For the perimeter of ΔPQR, we must find the distances of $\overline{PQ}$, $\overline{QR}$ and $\overline{RP}$.

$$d_{\overline{PQ}} = \sqrt{(-6-0)^2 + (4 - -3)^2}$$
$$= \sqrt{(-6)^2 + 7^2} = \sqrt{36+49}$$
$$d_{\overline{PQ}} = \sqrt{85}$$

$$d_{\overline{QR}} = \sqrt{(5 - -6)^2 + (1-4)^2}$$
$$= \sqrt{11^2 + (-3)^2} = \sqrt{121+9}$$
$$d_{\overline{QR}} = \sqrt{130}$$

$$d_{\overline{RP}} = \sqrt{(0-5)^2 + (-3-1)^2}$$
$$= \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16}$$
$$d_{\overline{RP}} = \sqrt{41}$$

$$Perimeter = \sqrt{85} + \sqrt{130} + \sqrt{41}$$

$$Perimeter \approx 27.0 \text{ units}$$

Since all three sides of the triangle are different in length, it is a **SCALENE TRIANGLE**.

(AP) Example 4: For points $C\ (x - 3, 2y + 1)$ and $D\ (3x + 2, y - 2)$, write an expression that represents the distance of $\overline{CD}$.

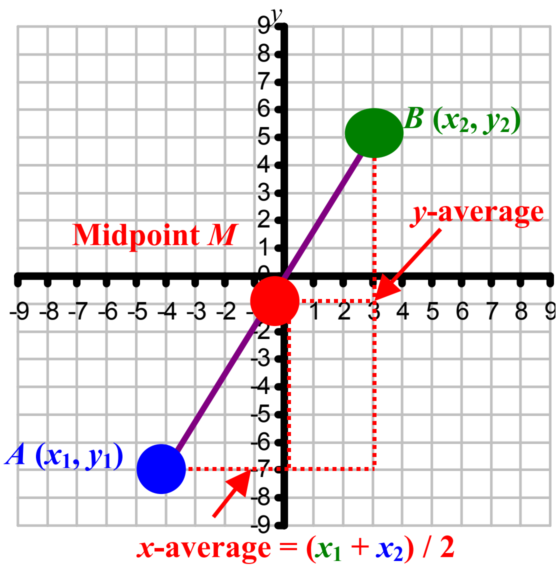
$$d_{\overline{CD}} = \sqrt{[(3x+2)-(x-3)]^2 + [(y-2)-(2y+1)]^2}$$
$$= \sqrt{[3x+2-x+3]^2 + [y-2-2y-1]^2}$$
$$= \sqrt{(2x+5)^2 + (-y-3)^2}$$
$$= \sqrt{(4x^2+20x+25) + (y^2+6y+9)}$$
$$d_{\overline{CD}} = \sqrt{4x^2 + y^2 + 20x + 6y + 34}$$

6-1 Homework Assignments

Regular: pg. 256 to 257 #1 to 11 (odd), 13 to 22, 26

AP: pg. 256 to 257 #2 to 12 (even), 13 to 23, 25 to 27

6-2: Midpoint of a Line Segment



Midpoint: - the location (coordinate) of a point in the middle of a line segment.
- the length of one side of the midpoint is equivalent to the length of the other side.

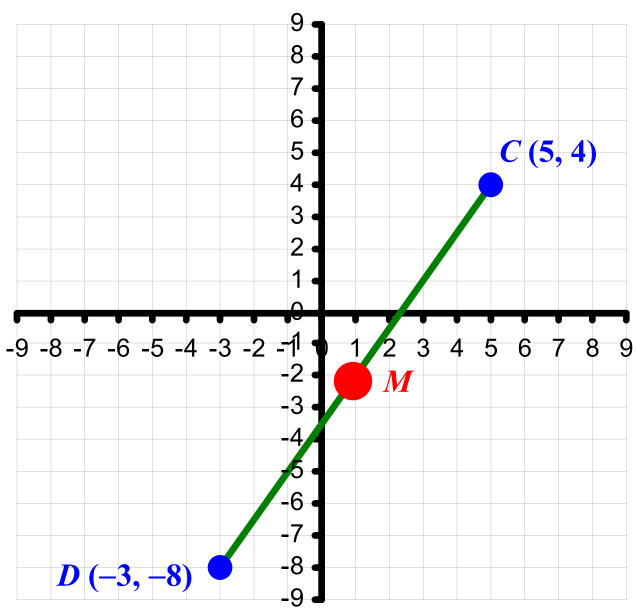
Midpoint of a Line Segment

M = (average of the x-coordinates, average of the y-coordinates)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 1: Find the midpoints of the following line segments.

a. $\overline{CD}$ where C (5, 4) and D (−3, −8)

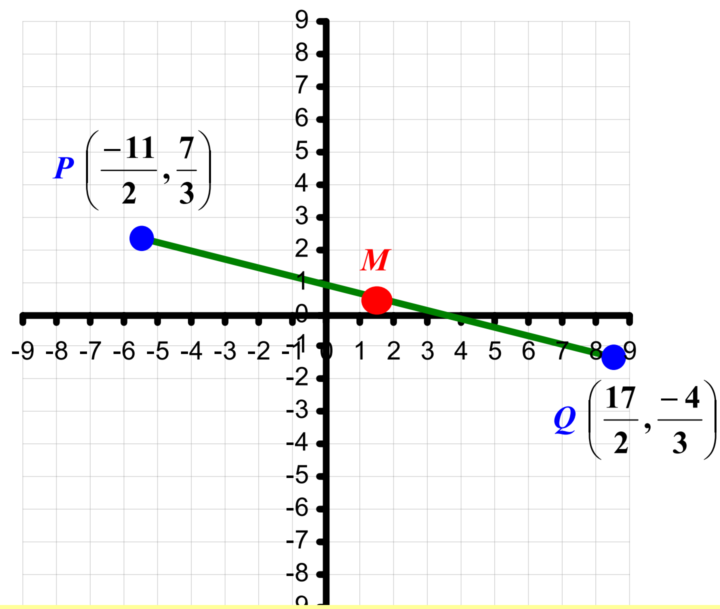


$$M_{\overline{CD}} = \left(\frac{5 + -3}{2}, \frac{4 + -8}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{-4}{2}\right)$$

$$M_{\overline{CD}} = (1, -2)$$

b. $\overline{PQ}$ where $P\left(\frac{-11}{2}, \frac{7}{3}\right)$ and $Q\left(\frac{17}{2}, \frac{-4}{3}\right)$

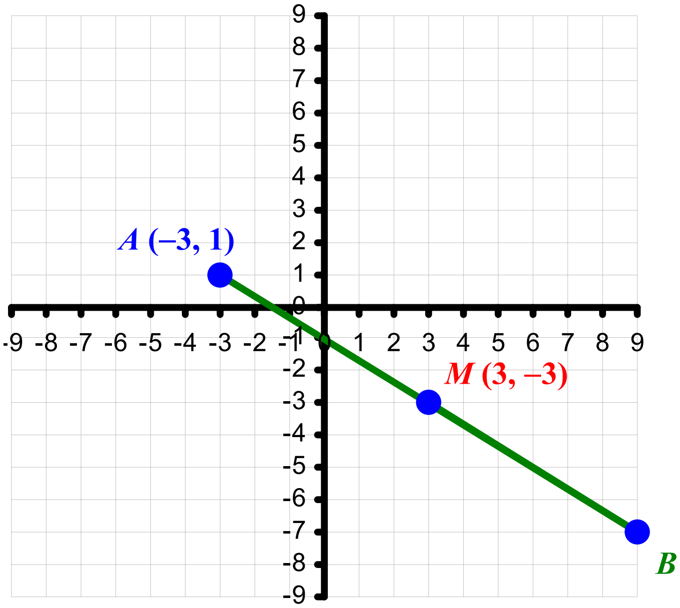


$$M_{\overline{PQ}} = \left(\frac{\left(\frac{-11}{2}\right) + \left(\frac{17}{2}\right)}{2}, \frac{\left(\frac{7}{3}\right) + \left(\frac{-4}{3}\right)}{2}\right)$$

$$= \left(\frac{\left(\frac{6}{2}\right)}{2}, \frac{\left(\frac{3}{3}\right)}{2}\right)$$

$$M_{\overline{PQ}} = \left(\frac{3}{2}, \frac{1}{2}\right)$$

Example 2: Given that the midpoint of $\overline{AB}$ is M (3, −3). If one of the endpoint of $\overline{AB}$ is A (−3, 1), find the coordinate of endpoint B.



$$M_{\overline{AB}} = \left(\frac{x_2 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

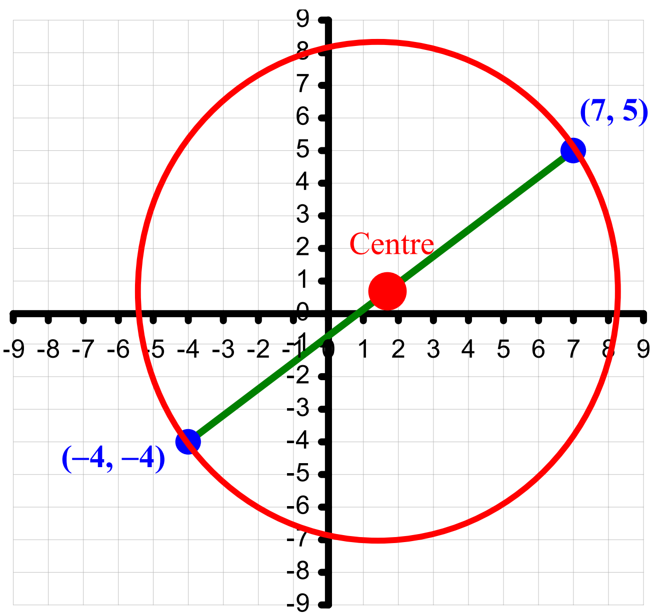
$$(3, -3) = \left(\frac{-3 + x_2}{2}, \frac{1 + y_2}{2}\right)$$

$$3 = \frac{-3 + x_2}{2}$$
$$3 \times 2 = -3 + x_2$$
$$6 = -3 + x_2$$
$$6 + 3 = x_2$$
$$x_2 = 9$$

$$-3 = \frac{1 + y_2}{2}$$
$$-3 \times 2 = 1 + y_2$$
$$-6 = 1 + y_2$$
$$-6 - 1 = y_2$$
$$y_2 = -7$$

***B* = (9, −7)**

Example 3: A diameter of a circle has endpoints (7, 5) and (−4, −4). Find the coordinate of the centre.



$$M = \left(\frac{x_2 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{7 + -4}{2}, \frac{5 + -4}{2}\right)$$

$$Centre = \left(\frac{3}{2}, \frac{1}{2}\right)$$

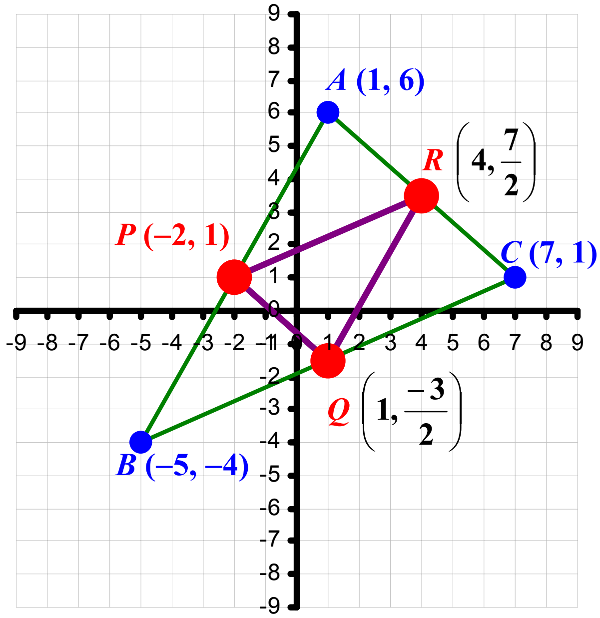
(AP) Example 4: For points C (x −3, 2y +1) and D (3x +2, y −2), write an expression that represents the midpoint of $\overline{CD}$.

$$M_{\overline{CD}} = \left(\frac{x_2 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{(x-3)+(3x+2)}{2}, \frac{(2y+1)+(y-2)}{2}\right)$$

$$M_{\overline{CD}} = \left(\frac{4x-1}{2}, \frac{3y-1}{2}\right)$$

Example 5: ΔPQR is inscribed in ΔABC where A (1, 6), B (−5, −4), and C (7, 1). If vertices P, Q and R are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively, find the perimeter of ΔPQR.



$$P = M_{\overline{AB}} = \left(\frac{1+-5}{2}, \frac{6+-4}{2}\right)$$
$$P = (-2, 1)$$

$$Q = M_{\overline{BC}} = \left(\frac{-5+7}{2}, \frac{-4+1}{2}\right)$$
$$Q = \left(1, \frac{-3}{2}\right)$$

$$R = M_{\overline{AC}} = \left(\frac{1+7}{2}, \frac{6+1}{2}\right)$$
$$R = \left(4, \frac{7}{2}\right)$$

Perimeter of $\Delta PQR = d_{\overline{PQ}} + d_{\overline{QR}} + d_{\overline{PR}}$

$$d_{\overline{PQ}} = \sqrt{(1--2)^2 + \left(\frac{-3}{2}-1\right)^2}$$
$$= \sqrt{3^2 + \left(\frac{-5}{2}\right)^2}$$
$$= \sqrt{9 + \frac{25}{4}}$$
$$= \sqrt{\frac{36}{4} + \frac{25}{4}}$$
$$= \sqrt{\frac{61}{4}}$$
$$d_{\overline{PQ}} = \frac{\sqrt{61}}{2}$$

$$d_{\overline{QR}} = \sqrt{(4-1)^2 + \left(\frac{7}{2}-\frac{-3}{2}\right)^2}$$
$$= \sqrt{3^2 + \left(\frac{10}{2}\right)^2}$$
$$= \sqrt{9+25}$$
$$d_{\overline{QR}} = \sqrt{34}$$

$$d_{\overline{PR}} = \sqrt{(4--2)^2 + \left(\frac{7}{2}-1\right)^2}$$
$$= \sqrt{6^2 + \left(\frac{5}{2}\right)^2}$$
$$= \sqrt{36 + \frac{25}{4}}$$
$$= \sqrt{\frac{144}{4} + \frac{25}{4}}$$
$$= \sqrt{\frac{169}{4}}$$
$$d_{\overline{PR}} = \frac{13}{2}$$

$$P_{\Delta PQR} = \frac{\sqrt{61}}{2} + \sqrt{34} + \frac{13}{2}$$

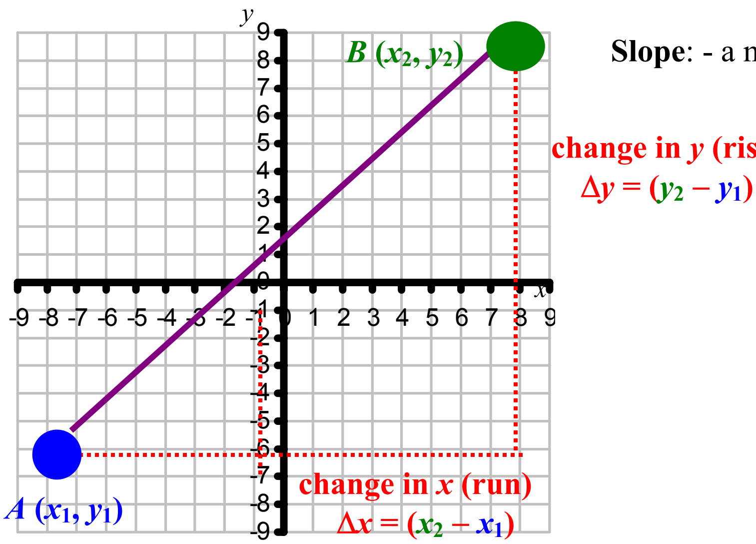
$P_{\Delta PQR} \approx$ 16.2 units

6-2 Homework Assignments

Regular: pg. 261 to 262 #1 to 17 (odd), 19 to 24, 27, 28 to 30, and 31a to 31c.

AP: pg. 261 to 262 #2 to 18 (even), 19 to 30, 31a to 31c, 32, 33 and 36.

6-3: Slope



Slope: - a measure on the steepness of the line segment.

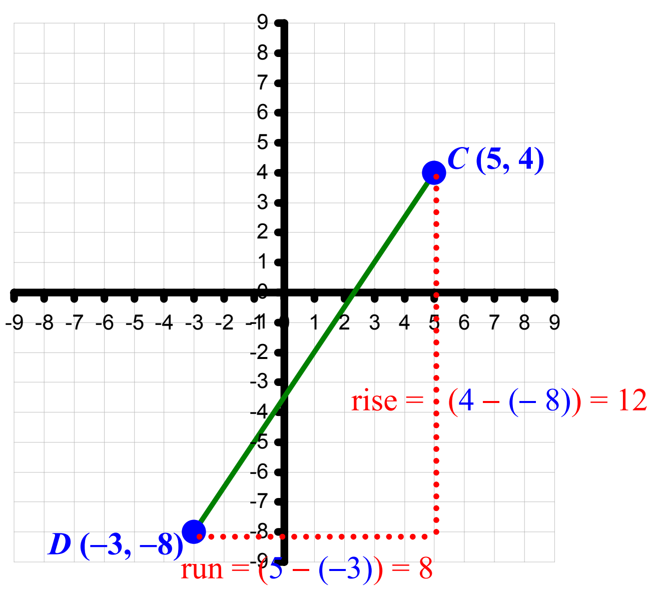
Slope of a Line Segment

$$\boldsymbol{m} = \frac{rise}{run} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Find the slope of the following line segments.

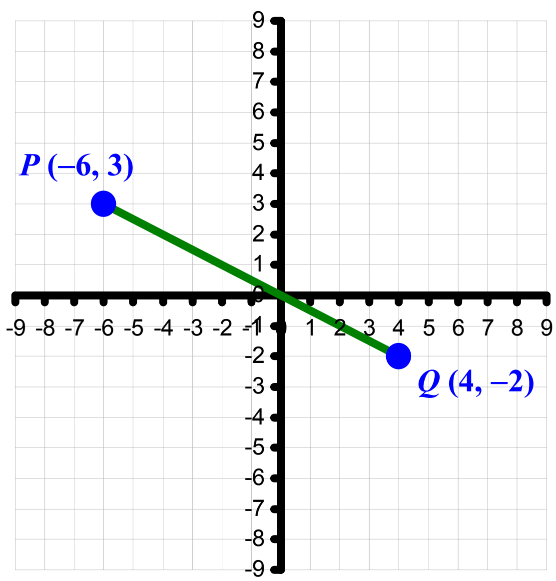
a. $\overline{CD}$ where C (5, 4) and D (−3, −8)



$$m_{\overline{CD}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-8)}{5-(-3)} = \frac{12}{8}$$

$$\boldsymbol{m_{\overline{CD}} = \frac{4}{3}}$$

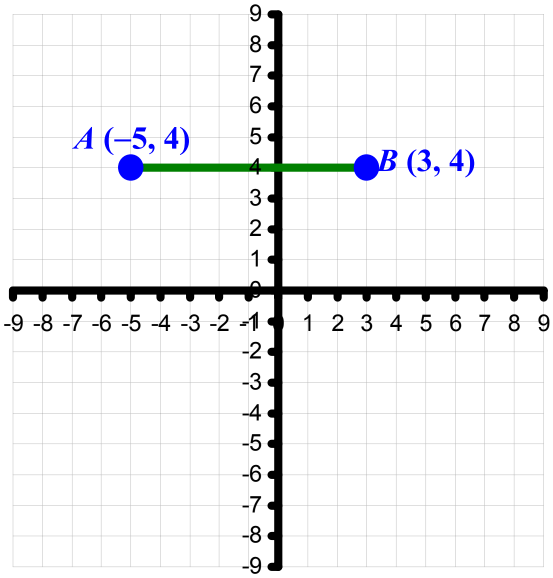
b. $\overline{PQ}$ where P (−6, 3) and Q (4, −2)



$$m_{\overline{PQ}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{4-(-6)} = \frac{-5}{10}$$

$$\boldsymbol{m_{\overline{PQ}} = \frac{-1}{2}}$$

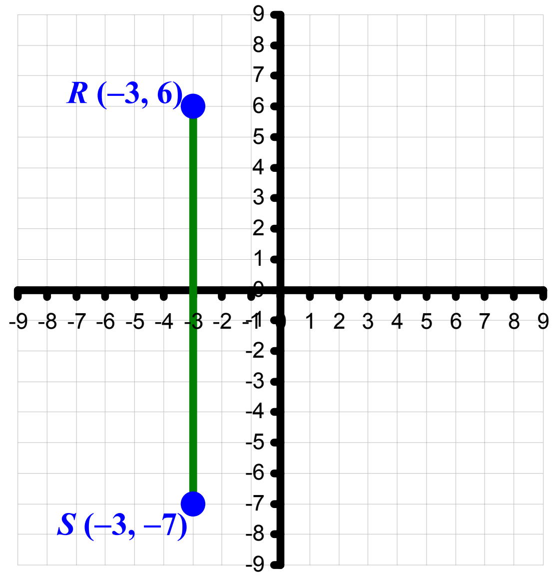
c. $\overline{AB}$ where A (–5, 4) and B (3, 4)



$$m_{\overline{AB}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-4}{3-(-5)} = \frac{0}{8}$$

$$m_{\overline{AB}} = 0$$

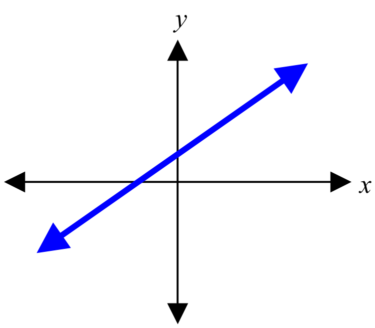
d. $\overline{RS}$ where R (–3, 6) and S (–3, –7)



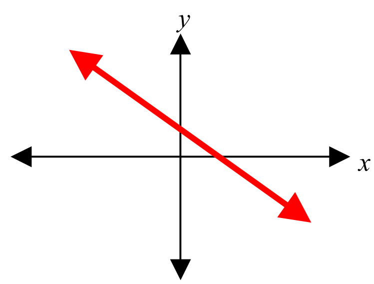
$$m_{\overline{RS}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-6}{-3-(-3)} = \frac{-13}{0}$$

$$m_{\overline{RS}} = \text{undefined}$$

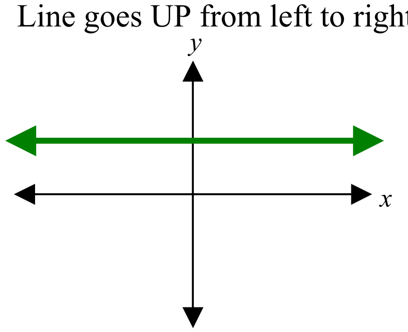
In general, slopes can be classified as follows:



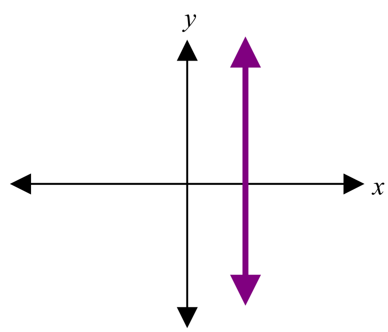
Positive Slope ($m > 0$)
Line goes UP from left to right.



Negative Slope ($m < 0$)
Line goes DOWN from left to right.



Zero Slope ($m = 0$)
Horizontal (Flat) Line [Rise = 0]



Undefined Slope
Vertical Line [Run = 0]

Example 2: If the slope of a line is $\frac{-2}{3}$, and it passes through (4, 5) and (–8, *p*), find the value of *p*.

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$\frac{-2}{3}=\frac{p-5}{-8-4}$$
$$\frac{-2}{3}=\frac{(p-5)}{-12}$$

$$-2(-12)=3(p-5)$$
$$24=3p-15$$
$$24+15=3p$$
$$39=3p$$
$$\frac{39}{3}=p$$

$$p=13$$

Example 3: If the slope of a line is $\frac{3}{4}$, and it passes through *A* (2*t*, *t* – 2) and *B* (2, –3), find the value of *t* and point *A*.

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$\frac{3}{4}=\frac{-3-(t-2)}{2-2t}$$
$$\frac{3}{4}=\frac{-t-1}{2-2t}$$

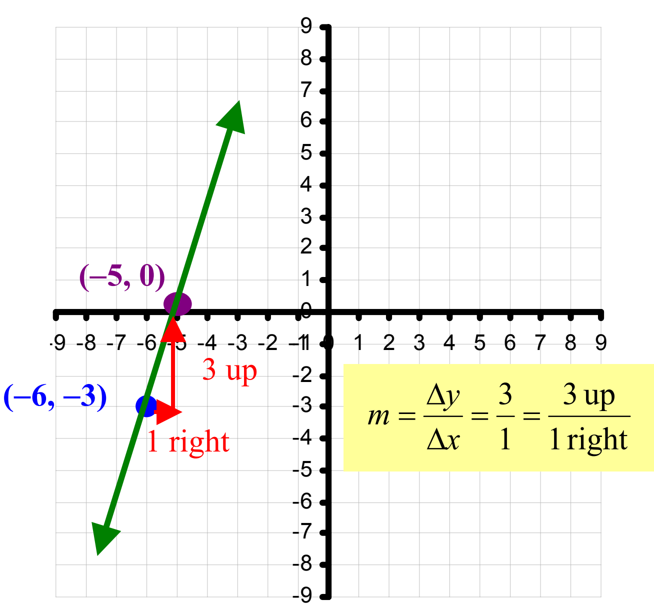
$$4(-t-1)=3(2-2t)$$
$$-4t-4=6-6t$$
$$-4t+6t=6+4$$
$$2t=10$$
$$t=\frac{10}{2}$$

$$t=5$$

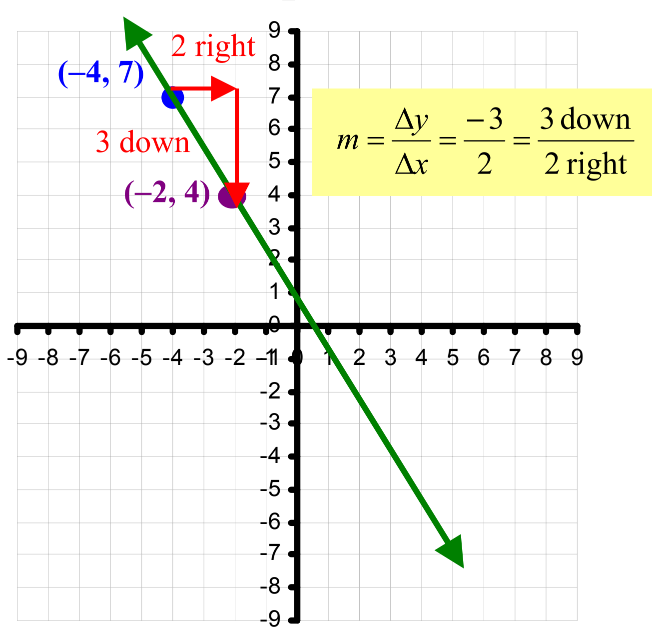
A (2(5), (5) – 2) = *A* (10, 3)

Example 4: Sketch the graph of a line given a point and a slope below.

a. *E* (–6, –3) and *m* = 3



b. *F* (–4, 7) and $m=\frac{-3}{2}$



Example 5: Sketch the graph of the following equations. Find the slope of the equation by selecting two points on the line.

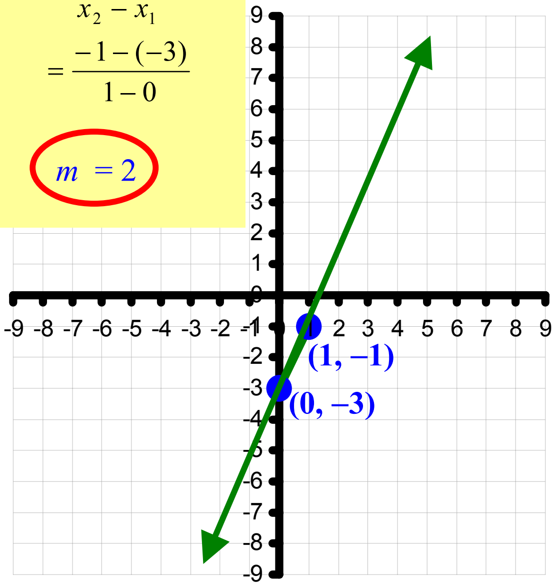
a. $2x - y = 3$

$$2x - y = 3$$
$$2x - y - 3 = 0$$
$$2x - 3 = y$$

$y = 2x - 3$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-3)}{1-0}$$

$m = 2$



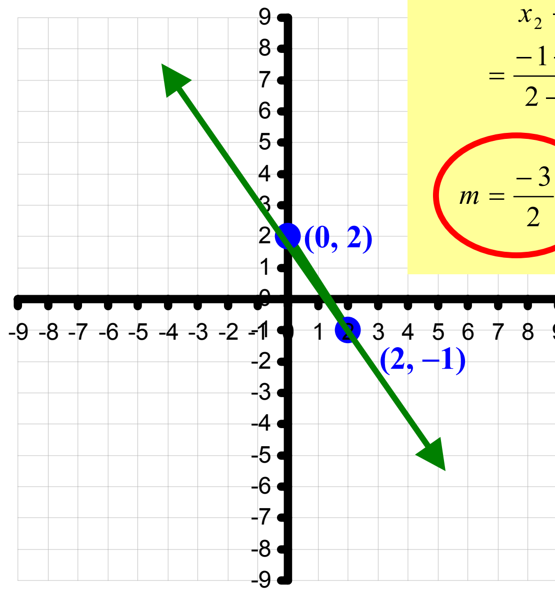
b. $3x + 2y - 4 = 0$

$$3x + 2y - 4 = 0$$
$$2y = -3x + 4$$
$$y = \frac{-3x+4}{2}$$

$y = \frac{-3x}{2} + 2$

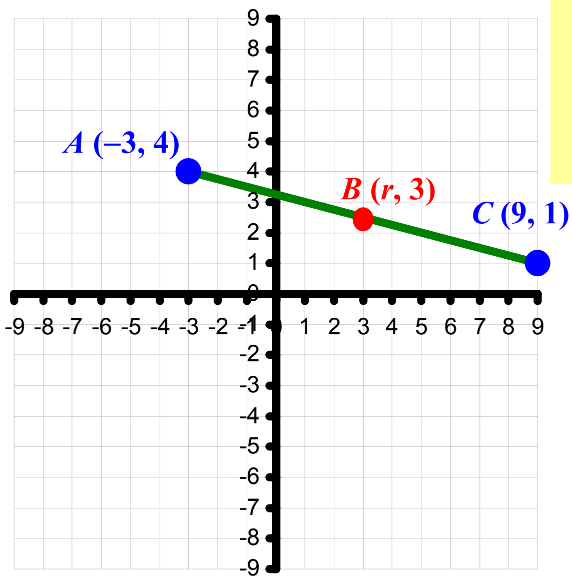
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{2-0}$$

$m = \frac{-3}{2}$



Collinear: - three or more points that lie on the same straight line.

Example 6: If the points A (–3, 4), B (r, 3), and C (9, 1) are collinear, find the value of r.



$$m_{\overline{AC}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{9-(-3)} = \frac{-3}{12}$$

$$m_{\overline{AC}} = \frac{-1}{4}$$

For collinear points: $m_{\overline{AB}} = m_{\overline{AC}}$

$$\frac{-1}{4} = \frac{3-4}{r-(-3)}$$
$$\frac{-1}{4} = \frac{-1}{r+3}$$
$$-r - 3 = -4$$
$$-r = -4 + 3$$
$$-r = -1$$

$r = 1$

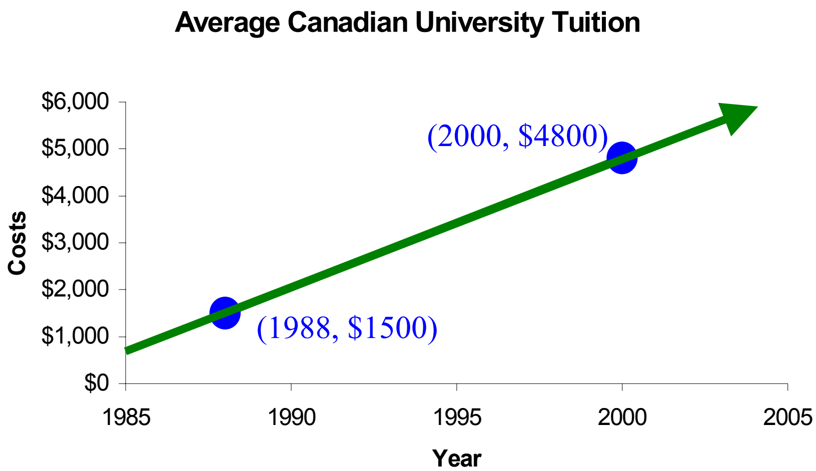
6-3 Homework Assignments

Regular: pg. 267 to 269 #1 to 6, 7 to 23 (odd), 24, 25, 30, 31e, 31g, 32a, 32c, 33 to 35, 37, 40, 41, 42a, 42b and 42d.

AP: pg. 267 to 269 #1 to 6, 8 to 22 (even), 24, 25, 30, 31e, 31g, 32a, 32c, 33 to 35, 37, 40, 41, 42a, 42b, 42d, 43 and 44.

6-4: Slope as a Rate of Change

Example 1: In 1988, the average tuition for a full time university student in Canada is $1500. In 2000, the cost is $4800. Graph the information and find the average rate of change for the full time university tuition in Canada.



Rate of Change = Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{\$4800 - \$1500}{2000 - 1988}$$
$$= \frac{\$3300}{12 \text{ years}}$$

Rate of Change = $275/year

Example 2: John was running a 10 km race. His time was 3.2 min at the 3 km mark, and 8.6 min at the 7 km mark. Find his speed to the nearest tenth of a km/min and m/s. Estimate the finishing time for the race.

Rate of Change = Speed (km/min)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7\text{ km} - 3\text{ km}}{8.6\text{ min} - 3.2\text{ min}}$$
$$= \frac{4\text{ km}}{5.4\text{ min}}$$

Speed = 0.7 km/min

To Convert km/min to m/s:

$$\frac{4\text{ km}}{5.4\text{ min}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ min}}{60\text{ sec}}$$

12.3 m/s

Assuming John is running at the same speed all the way to the finishing line,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$0.7\text{ km/min} = \frac{10\text{ km} - 7\text{ km}}{t - 8.6\text{ min}}$$
$$0.7(t - 8.6) = 3$$
$$0.7t - 6.02 = 3$$
$$0.7t = 3 + 6.02$$
$$0.7t = 9.02$$
$$t = \frac{9.02}{0.7}$$
$$t = 12.9\text{ min}$$

Example 3: Jill was driving at 30 km/h at t = 1.6 min. She accelerated to 100 km/h at t = 2.5 min. Find the acceleration of Jill's vehicle to the nearest tenth of a m/s^2.

Rate of Change = Acceleration

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{100\text{ km/h} - 30\text{ km/h}}{2.5\text{ min} - 1.6\text{ min}}$$
$$= \frac{70\text{ km/h}}{0.9\text{ min}}$$

Converting units to m/s^2:

$$\frac{70\text{ km}}{1\text{ hr}} \times \frac{1}{0.9\text{ min}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ hr}}{3600\text{ sec}} \times \frac{1\text{ min}}{60\text{ sec}}$$

0.4 m/s^2

6-4 Homework Assignments

Regular: pg. 271 to 272 #1 to 10. AP: pg. 271 to 272 #1 to 10.

6-5: Linear Equations: Point-Slope Form

When given a slope (m) and a point (x_1, y_1) on the line, we can find the equation of the line using the **point-slope form**:

$$\frac{y - y_1}{x - x_1} = m \quad \text{(slope formula)} \qquad \boldsymbol{y - y_1 = m\,(x - x_1)} \quad \textbf{(Point-Slope form)}$$

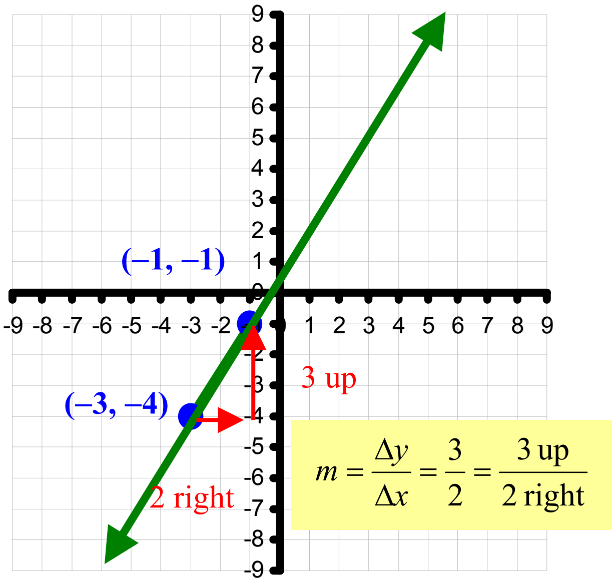
If we rearrange the equations so that all terms are on one side, it will be in **standard (general) form**:

$$\boldsymbol{Ax + By + C = 0} \qquad \textbf{(Standard or General form)}$$

($A \geq 0$, the leading coefficient for the x term must be positive)

Example 1: Find the equation in point-slope form and standard form given the followings.

a. (−3, −4) and $m = \dfrac{3}{2}$



For Slope-Point form:

$y - y_1 = m(x - x_1)$

$y - (-3) = \frac{3}{2}(x - (-4))$

$y + 3 = \frac{3}{2}(x + 4)$

For Standard form:

$y + 3 = \frac{3}{2}(x + 4)$

$2(y + 3) = 3(x + 4)$

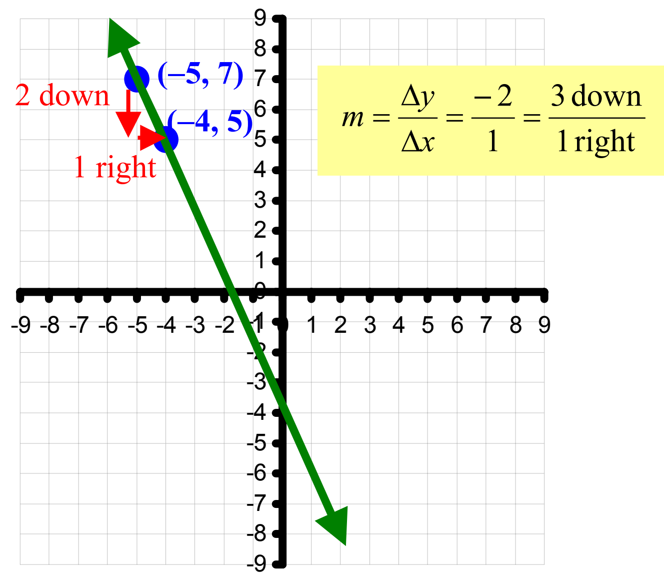
$2y + 6 = 3x + 12$

$0 = 3x - 2y + 12 - 6$

$\boldsymbol{0 = 3x - 2y - 6}$

Bringing all the terms to the <u>right-hand side of equation</u> will ensure a <u>positive coefficient for the *x* term</u>.

b. (−5, 7) and $m = -2$



For Slope-Point form:

$y - y_1 = m(x - x_1)$

$y - 7 = -2(x - (-5))$

$y - 7 = -2(x + 5)$

For Standard form:

$y - 7 = -2(x + 5)$

$y - 7 = -2x - 10$

$2x + y - 7 + 10 = 0$

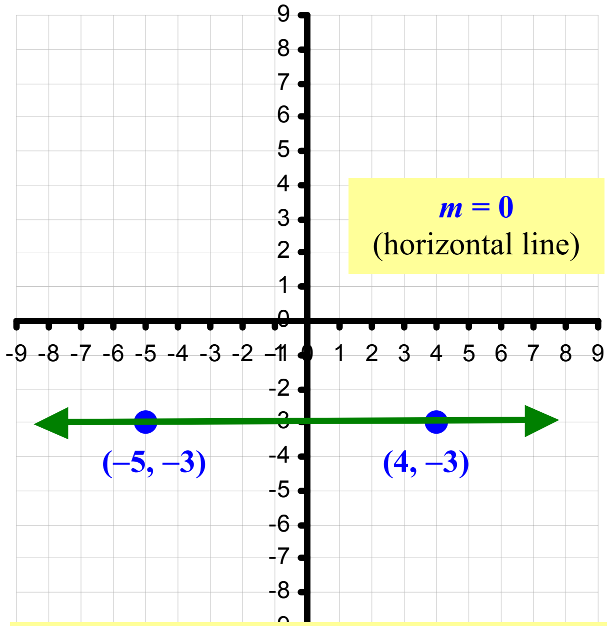
$\boldsymbol{2x + y + 3 = 0}$

Bringing all the terms to the <u>left-hand side of equation</u> will ensure a <u>positive coefficient for the *x* term</u>.

Example 2: Find the equation in point-slope form and standard form given the following points.

a. (−5, −3) and (4, −3) b. (−2, 5) and (7, −1)

a.



$m = 0$
(horizontal line)

For Slope-Point form:

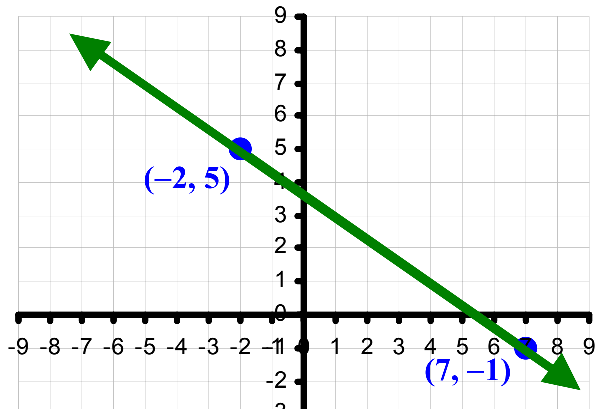
$y - y_1 = m(x - x_1)$

$y - (-3) = 0(x - (-5))$

$y + 3 = 0$

It is in **standard form**, also.
(Everything is already on one side.)

b.



$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-5}{7-(-2)} = \dfrac{-6}{9}$

$m = \dfrac{-2}{3}$

For Slope-Point form:

$y - y_1 = m(x - x_1)$

$y - 5 = \dfrac{-2}{3}(x - (-2))$

$y - 5 = \dfrac{-2}{3}(x + 2)$

For Standard form:

$y - 5 = \dfrac{-2}{3}(x + 2)$

$3(y - 5) = -2(x + 2)$

$3y - 15 = -2x - 4$

$2x + 3y - 15 + 4 = 0$

$\mathbf{2x + 3y - 11 = 0}$

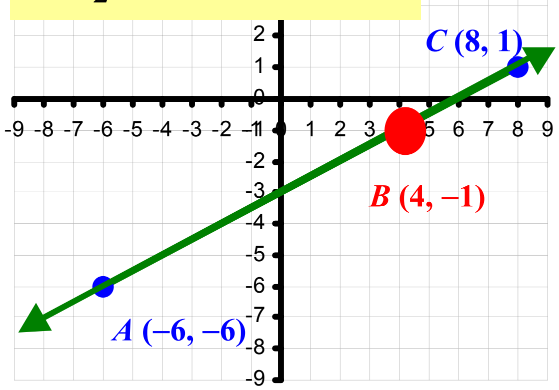
Example 3: A (−6, −6) and C (8, 1) satisfy a linear equation.

a) Express the equation in point-slope form and general form.

b) Prove that B (4, −1) is collinear with points A and C.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6-1}{-6-8} = \dfrac{-7}{-14}$

$m = \dfrac{1}{2}$



For Slope-Point form:

$y - y_1 = m(x - x_1)$

$y - 1 = \dfrac{1}{2}(x - 8)$

$y - 1 = \dfrac{1}{2}(x - 8)$

For Standard form:

$y - 1 = \dfrac{1}{2}(x - 8)$

$2(y - 1) = 1(x - 8)$

$2y - 2 = x - 8$

$0 = x - 2y - 8 + 2$

$\mathbf{0 = x - 2y - 6}$

To prove B (4, −1) is collinear to $\overline{AC}$, substitute B into the standard equation.

0 = (4) − 2 (−1) −6

0 = 4 + 2 − 6 **0 = 0** **∴ B is collinear with $\overline{AC}$.**

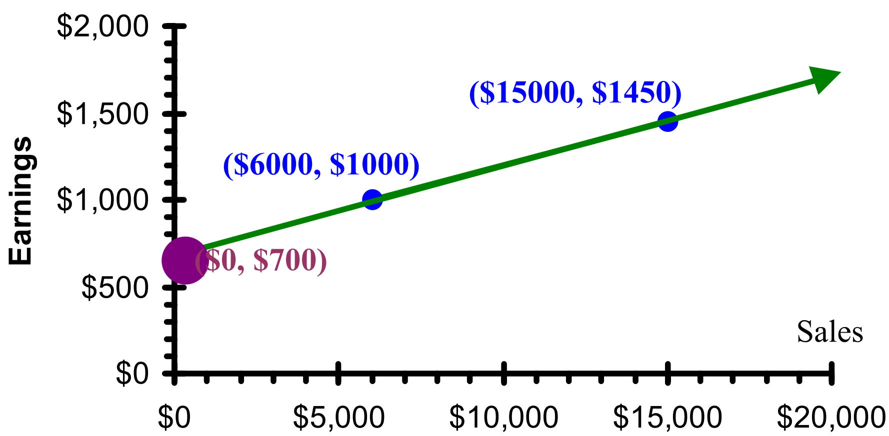
Example 4: A line with the equation, $kx + 6y - 2 = 0$, passes through (6, –3). Find the value of k.

(6, –3) means that $x = 6$ and $y = -3$. Substituting them into the equation gives:

$$k(6) + 6(-3) - 2 = 0$$
$$6k - 18 - 2 = 0$$
$$6k - 20 = 0$$
$$6k = 20$$
$$k = \frac{20}{6}$$
$$k = \frac{10}{3}$$

Example 5: A salesperson earned \$1000 when his sales totalled up to \$6000 in one month. In another month, he earned \$1450 when his total sales were \$15000.

a) Sketch a earning versus total sales graph.
b) Calculate his commission rate.
c) Find the equation of the line in standard form.
d) What is his monthly base salary?



Commission Rate = Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\$1450 - \$1000}{\$15000 - \$6000} = \frac{\$450}{\$900}$$

Commission Rate = 0.05 = 5%

For Standard form:

$$y - y_1 = m(x - x_1)$$
$$y - 1000 = 0.05(x - 6000)$$
$$y - 1000 = 0.05x - 300$$
$$0 = 0.05x - y - 300 + 1000$$

$$\mathbf{0 = 0.05x - y + 700}$$

For Base Salary, x = \$0 in sales:

$0 = 0.05(0) - y + 700$

$0 = -y + 700$

y = \$700 (Base Salary)

(AP) Example 6: For the linear equation $3x - 4y + 10 = 0$, find the coordinates of a point when

a. the y-coordinate is twice the x-coordinate.
b. the x-coordinate is 6 less than the y-coordinate.

a) y-coordinate is twice the x-coordinate means $P(x, 2x)$.

$$3(x) - 4(2x) + 10 = 0$$
$$3x - 8x + 10 = 0$$
$$-5x + 10 = 0$$
$$10 = 5x$$

$x = 2$
$P(2, 4)$

b) x-coordinate is 6 less than the y-coordinate means $P(y - 6, y)$.

$$3(y - 6) - 4(y) + 10 = 0$$
$$3y - 18 - 4y + 10 = 0$$
$$-y - 8 = 0$$
$$-y = 8$$

$y = -8$
$P(-14, -8)$

6-5 Homework Assignments

Regular: pg. 282 to 283 #1 to 37 (odd), 38 to 41, 43 to 50.

AP: pg. 282 to 283 #2 to 36 (even), 38 to 53.

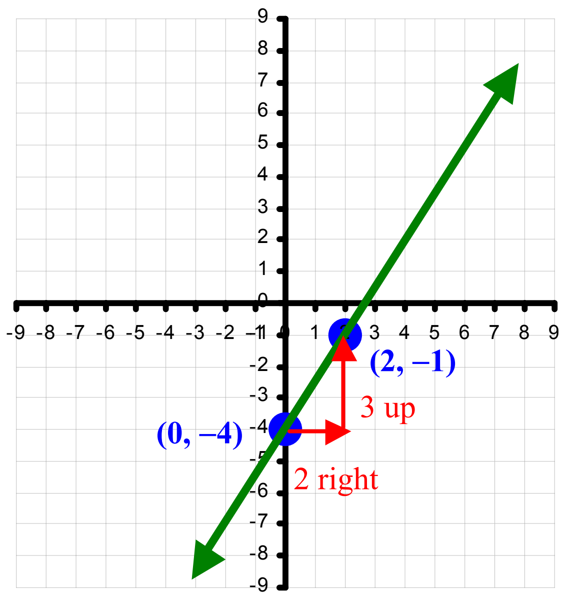
6-6: Linear Equations: Slope and *y*-intercept Form

When given a slope (m) and the y-intercept $(0, b)$ of the line, we can find the equation of the line using the **slope and *y*-intercept form**:

$$y = mx + b$$ where m = **slope** and b = ***y*-intercept**

Example 1: Given the y-intercept and slope, write the equation of the line in slope and y-intercept form, and standard form. Sketch a graph of the resulting equation.

a. $(0, -4)$ and $m = \frac{3}{2}$



$$m = \frac{\Delta y}{\Delta x} = \frac{3}{2} = \frac{3 \text{ up}}{2 \text{ right}}$$

y-intercept $= b = -4$

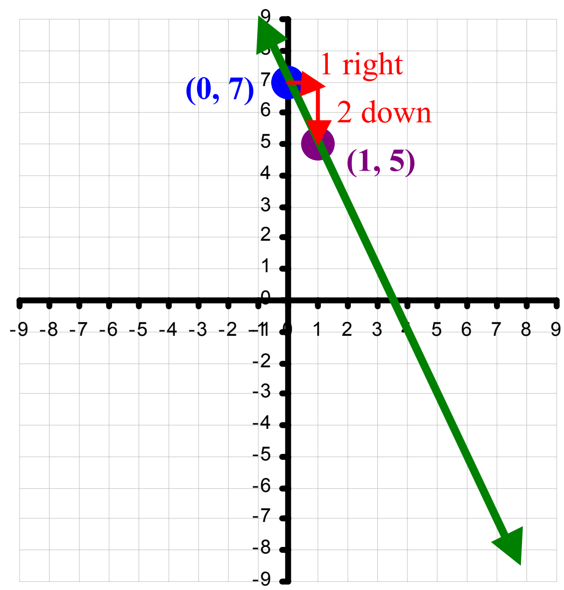
$$y = \frac{3}{2}x - 4$$

Multiply both sides by 2.

$$2y = 3x - 8$$

$$0 = 3x - 2y - 8$$

b. $b = 7$ and $m = -2$



$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = \frac{2 \text{ down}}{1 \text{ right}}$$

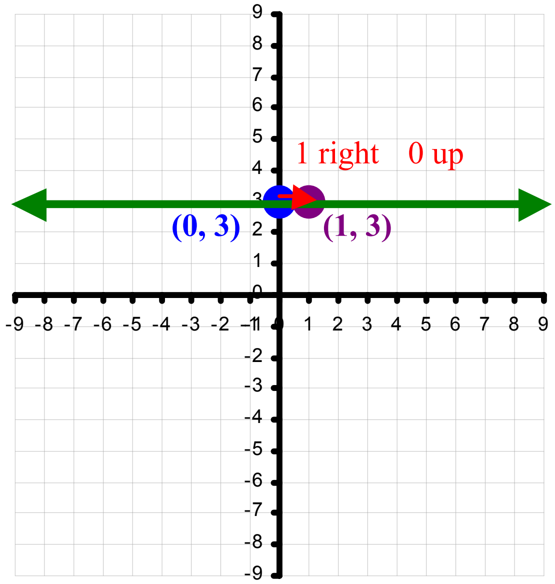
y-intercept $= b = 7$

$$y = -2x + 7$$

Rearrange to the left side.

$$2x + y - 7 = 0$$

c. $b = 3$ and $m = 0$



$$m = \frac{\Delta y}{\Delta x} = \frac{0}{1} = \frac{0\text{ up}}{1\text{ right}}$$

y-intercept = b = 3

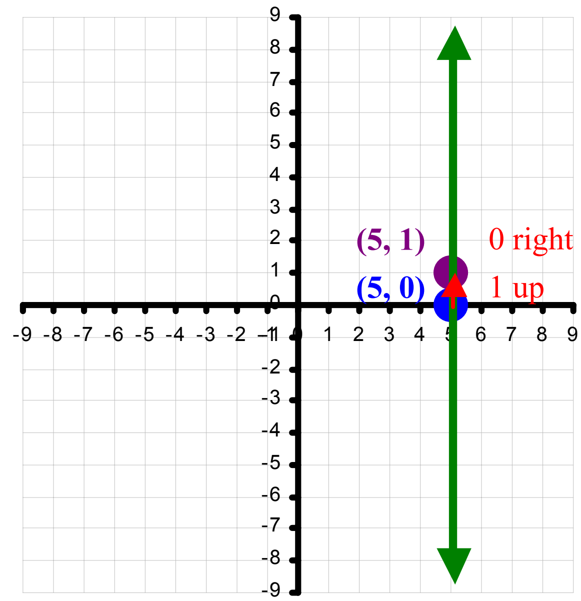
$y = 0x + 3$

$y = 3$

Rearrange to the left side.

$y - 3 = 0$

d. x-intercept = 5 and m = undefined



x-intercept = 5 and undefined slope (Vertical Line).

$x = 5$

Rearrange to the left side.

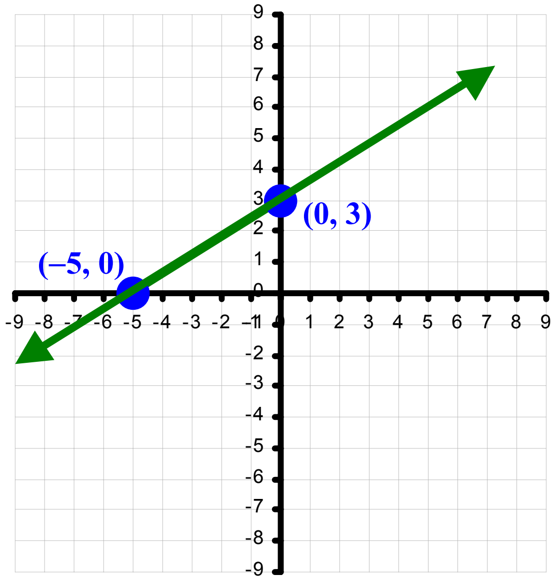
$x - 5 = 0$

Vertical Line – NO x term.

Horizontal Line – NO y term.

Example 2: Given the information below, write the equation of the line in slope and y-intercept form, and standard form. Sketch a graph of the resulting equation.

a. (−5, 0) and (0, 3)



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{0-(-5)} = \frac{3}{5}$$

y–intercept = b = 3

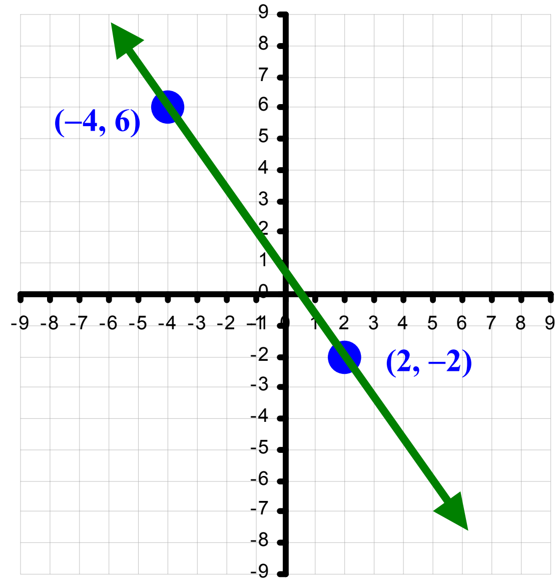
$$y = \frac{3}{5}x + 3$$

Multiply both sides by 5.

$$5y = 3x + 15$$

$$\mathbf{0 = 3x - 5y + 15}$$

b. (−4, 6) and (2, −2)



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-6}{2-(-4)} = \frac{-8}{6} = \frac{-4}{3}$$

Solve for b by using a point on the line (2, −2) and slope by using $y = mx + b$.

$$-2 = \frac{-4}{3}(2) + b$$

$$-2 = \frac{-8}{3} + b$$

$$-2 + \frac{8}{3} = b$$

$$b = \frac{2}{3}$$

$$y = \frac{-4}{3}x + \frac{2}{3}$$

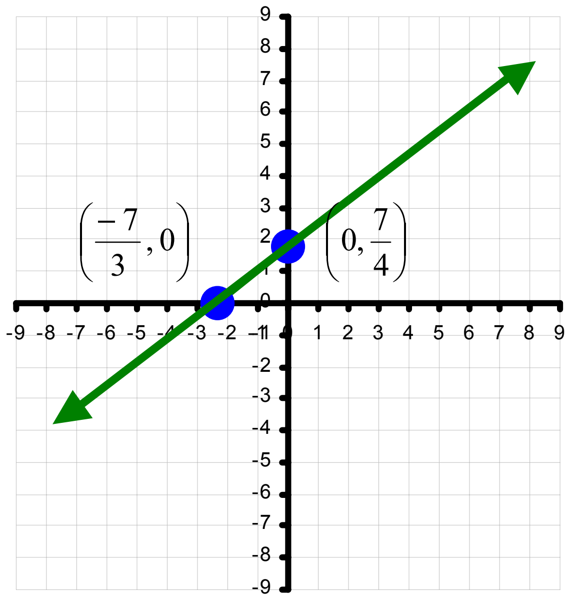
Multiply both sides by 3.

$$3y = -4x + 2$$

$$\mathbf{4x + 3y - 2 = 0}$$

Example 3: Find the slope, x and y-intercepts of the lines below. Sketch the graphs of the equations.

a. $3x - 4y + 7 = 0$



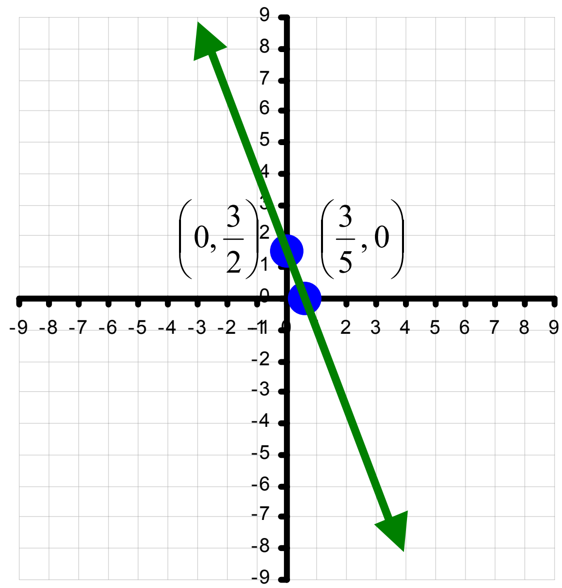
To find y-intercept, let $x = 0$.

$$3(0) - 4y + 7 = 0$$
$$-4y = -7$$
$$y = \frac{7}{4}$$

To find x-intercept, let $y = 0$.

$$3x - 4(0) + 7 = 0$$
$$3x = -7$$
$$x = \frac{-7}{3}$$

b. $0 = 5x + 2y - 3$



To find y-intercept, let $x = 0$.

$$0 = 5(0) + 2y - 3$$
$$3 = 2y$$
$$y = \frac{3}{2}$$

To find x-intercept, let $y = 0$.

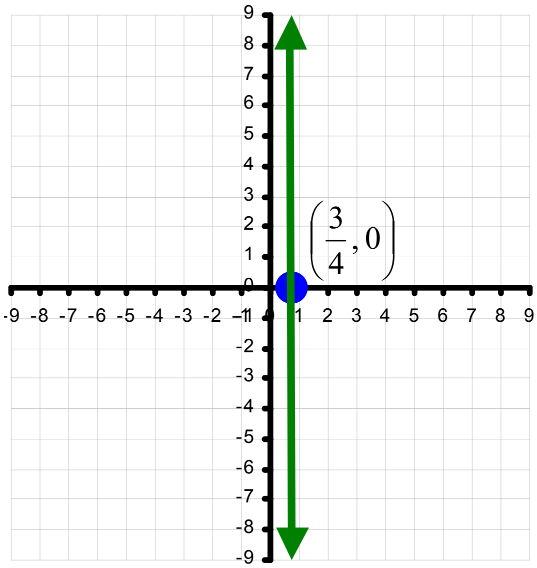
$$0 = 5x + 2(0) - 3$$
$$3 = 5x$$
$$x = \frac{3}{5}$$

c. $4x - 3 = 0$

NO *y* term. (Vertical Line)

$$4x = 3$$

$$x = \frac{3}{4}$$

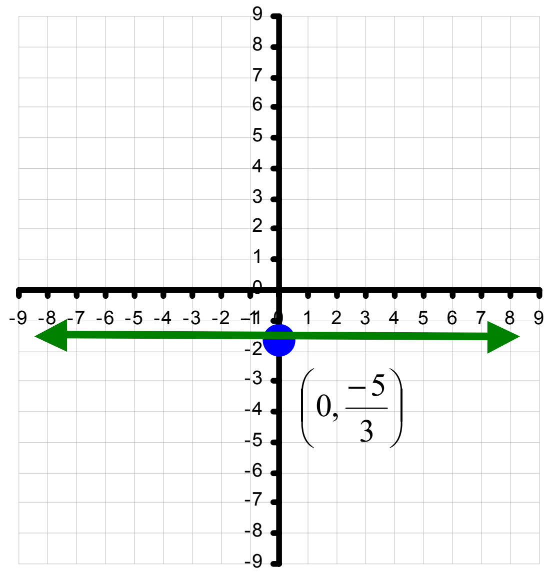


d. $0 = 3y + 5$

NO *x* term. (Horizontal Line)

$$-3y = 5$$

$$y = \frac{-5}{3}$$



Family of Lines: - when lines are parallel (having the same slope) **or** have the same *y*-intercept.

Example 4: Given the following equations, graph them on the same grid and determine which lines belong to a family.

Line 1: $y = 2x - 5$ Line 2: $6x - 3y = -12$ Line 3: $0 = 2x - 4y + 8$ Line 4: $4x - 2y + 16 = 0$

Line 1: $y = 2x - 5$

$\boldsymbol{m = 2}$ $b = -5$

Line 2:

$$6x - 3y = -12$$
$$-3y = -6x - 12$$
$$y = \frac{(-6x - 12)}{-3}$$
$$y = 2x + 4$$

$\boldsymbol{m = 2}$ $b = 4$

Line 3:

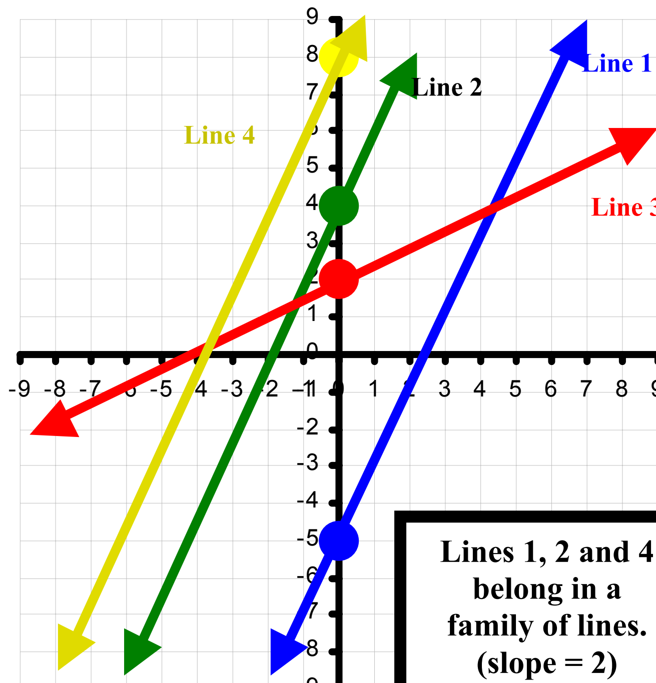
$$0 = 2x - 4y + 8$$
$$4y = 2x + 8$$
$$y = \frac{(2x + 8)}{4}$$
$$y = \frac{1}{2}x + 2$$

$\boldsymbol{m} = \frac{1}{2}$ $b = 2$

Line 4:

$$4x - 2y + 16 = 0$$
$$-2y = -4x - 16$$
$$y = \frac{(-4x - 16)}{-2}$$
$$y = 2x + 8$$

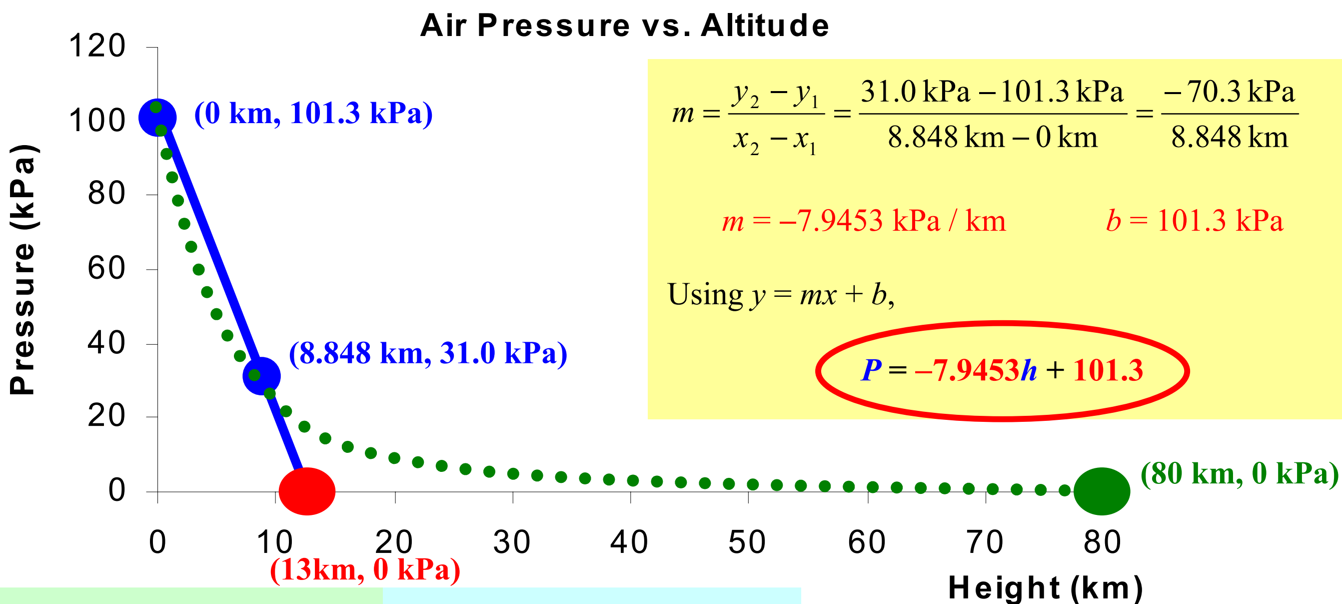
$\boldsymbol{m = 2}$ $b = 2$



Lines 1, 2 and 4 belong in a family of lines. (slope = 2)

Example 5: The atmospheric pressure at sea level measures at 101.3 kPa. On the summit of Mount Everest 8.848 km high, the atmospheric pressure measures at 31.0 kPa.

a) Sketch the graph of pressure versus height.
b) Find the equation of the graph in slope and y-intercept form, using h for height in km and P for pressure in kPa.
c) The city of Calgary has an altitude of 1532 m above sea level. What is the normal atmospheric pressure for Calgary to the nearest tenth of a kPa?
d) Assuming that the boundary between the Earth's atmosphere and space is when there is no atmospheric pressure, the thickness of the Earth's atmosphere is 80 km. Using the equation obtained above, find the thickness of the Earth's atmosphere to the nearest km. Is the answer reasonable? Explain.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{31.0\text{ kPa} - 101.3\text{ kPa}}{8.848\text{ km} - 0\text{ km}} = \frac{-70.3\text{ kPa}}{8.848\text{ km}}$$

$m = -7.9453$ kPa / km $\quad\quad$ $b = 101.3$ kPa

Using $y = mx + b$,

$$\mathbf{P = -7.9453h + 101.3}$$

For the City of Calgary,
$h = 1532$ m $= 1.532$ km, $P =$?

$$P = -7.9453(1.532) + 101.3$$

$$\mathbf{P = 89.1\text{ kPa}}$$

For the Thickness of Earth's atmosphere, $P = 0$ kPa, $h =$?

$$0 = -7.9453h + 101.3$$
$$-101.3 = -7.9453h$$
$$\frac{-101.3}{-7.9453} = h$$
$$\mathbf{h = 13\text{ km}}$$

The answer is **not reasonable** because in reality, Earth's atmosphere is 80 km thick. The linear equation model only fits the first 10 km in altitude. If one looks at the entire thickness of the atmosphere, a curve line (dotted line above) would have been a better model to describe the relation between altitude and air pressure.

(AP) Example 6: Convert standard form into the slope and y-intercept form. Find the expressions of m and b in terms of A and/or B and/or C.

$$Ax + By + C = 0$$
$$By = -Ax - C$$
$$y = \frac{-Ax - C}{B}$$
$$y = \frac{-A}{B}x - \frac{C}{B}$$

Comparing to $y = mx + b$,

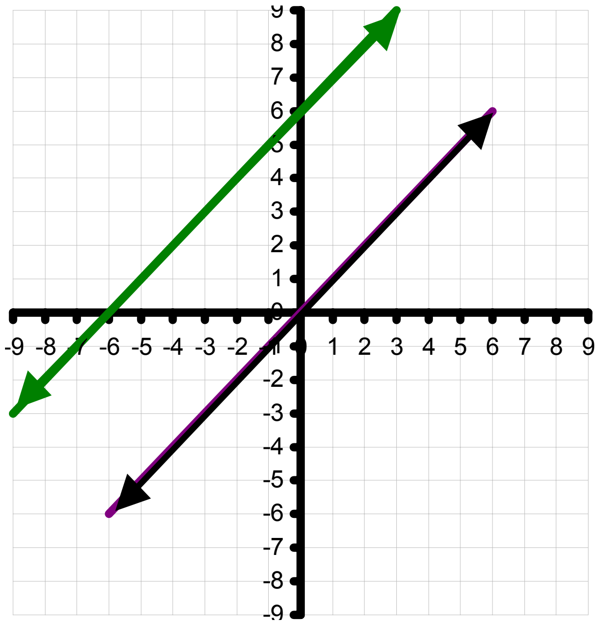
$$m = \frac{-A}{B} \text{ and } b = \frac{-C}{B}$$

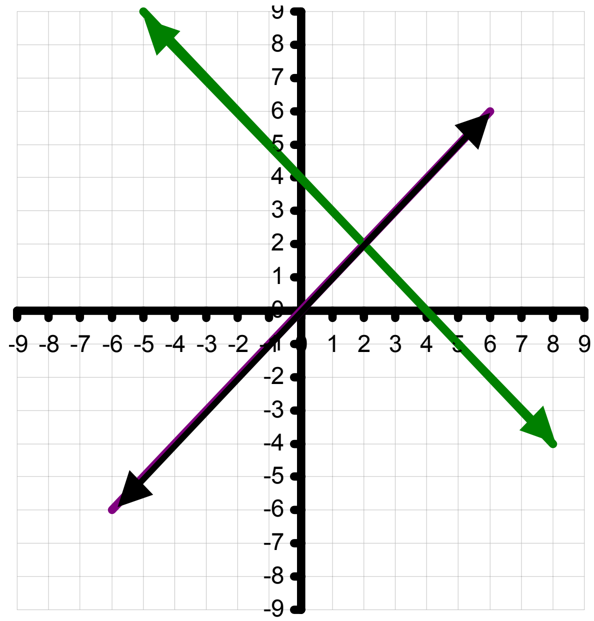
6-6 Homework Assignments

Regular: pg. 288 to 289 #1 to 31 (odd), 33 to 44.

AP: pg. 288 to 289 #2 to 32 (even), 33 to 48.

6-7: Parallel and Perpendicular Lines





Parallel Lines

slope of line 1 = slope of line 2

$$m_{l_1} = m_{l_2}$$

Perpendicular Lines

slope of line 1 = negative reciprocal slope of line 2

$$m_{l_1} = \frac{-1}{m_{l_2}}$$

Example 1: Given the slope of two lines below, determine whether the lines are parallel or perpendicular.

a. $m_1 = \frac{-3}{4}$ and $m_2 = \frac{8}{6}$

$m_1 = \frac{-3}{4}$ and $m_2 = \frac{4}{3}$

(negative reciprocal slopes)

Perpendicular Lines

b. $m_1 = \frac{4}{6}$ and $m_2 = \frac{6}{9}$

$m_1 = \frac{2}{3}$ and $m_2 = \frac{2}{3}$

(same slopes)

Parallel Lines

c. $m_1 = \frac{1}{2}$ and $m_2 = -\frac{1}{2}$

$m_1 = \frac{1}{2}$ and $m_2 = \frac{-1}{2}$

(neither the same nor negative reciprocal)

Neither Parallel nor Perpendicular Lines

Example 2: Find the slope of the line given and its perpendicular slope.

a. $y = -3x + 2$

$m = -3$

(⊥ means perpendicular)

$m_{\perp} = \frac{1}{3}$ (negative reciprocal)

b. $2x - 5y - 4 = 0$

$-5y = -2x + 4$

$y = \frac{2}{5}x - \frac{4}{5}$ $\quad m = \frac{2}{5}$

$m_{\perp} = \frac{-5}{2}$ (negative reciprocal)

c. $x - 6 = 0$

$x = 6$ (vertical line)

$m = \text{undefined} = \frac{1}{0}$

$m_{\perp} = \frac{-0}{1}$ (negative reciprocal)

$m_{\perp} = 0$

Example 3: Solve for the variables indicated below if they are parallel slopes.

a. $-5, \frac{-20}{p}$ b. $\frac{3}{8}, \frac{-9}{q}$ c. $\frac{-3}{4}, \frac{r}{9}$

Parallel Lines – Same Slopes

$$-5 = \frac{-20}{p}$$
$$p = \frac{-20}{-5}$$
$$p = 4$$

Parallel Lines – Same Slopes

$$\frac{3}{8} = \frac{-9}{q}$$
$$3q = -72$$
$$q = \frac{-72}{3}$$
$$q = -24$$

Parallel Lines – Same Slopes

$$\frac{-3}{4} = \frac{r}{9}$$
$$4r = -27$$
$$r = \frac{-27}{4}$$

Example 4: Solve for the variables indicated below if they are perpendicular slopes.

a. $-5, \frac{-2}{p}$ b. $\frac{3}{8}, \frac{q}{-6}$ c. $\frac{-3}{4}, -\frac{r}{8}$

Perpendicular Lines –
Negative Reciprocal Slopes

(Take the negative reciprocal of the first slope)

$$\frac{1}{5} = \frac{-2}{p}$$
$$p = -10$$

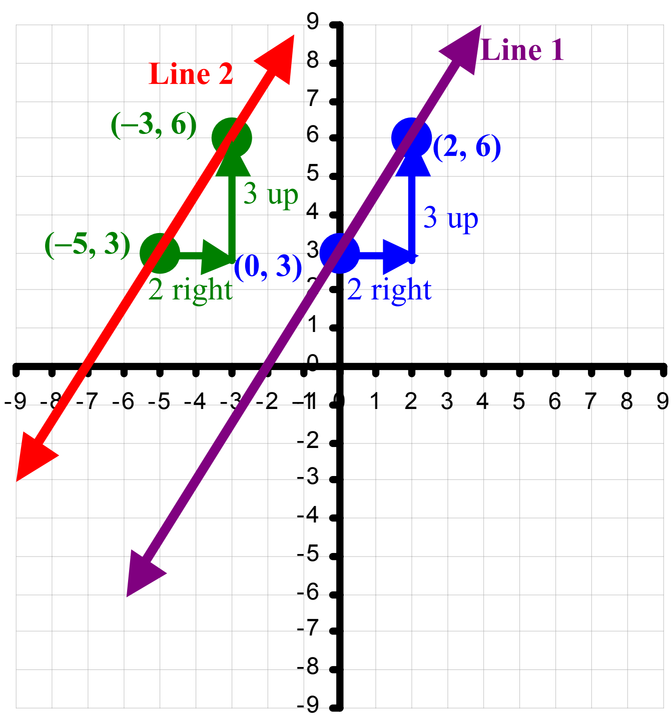
Perpendicular Lines –
Negative Reciprocal Slopes

$$\frac{-8}{3} = \frac{q}{-6}$$
$$3q = 48$$
$$q = \frac{48}{3}$$
$$q = 16$$

Perpendicular Lines –
Negative Reciprocal Slopes

$$\frac{4}{3} = \frac{-r}{8}$$
$$-3r = 32$$
$$r = \frac{32}{-3}$$
$$r = \frac{-32}{3}$$

Example 5: Find the equation of a line parallel to $3x - 2y + 6 = 0$ and passes through $(-5, 3)$.



Line 1:

$$-2y = -3x - 6$$
$$y = \frac{-3x-6}{-2}$$
$$y = \frac{3}{2}x + 3$$
$$m_1 = \frac{3}{2}$$

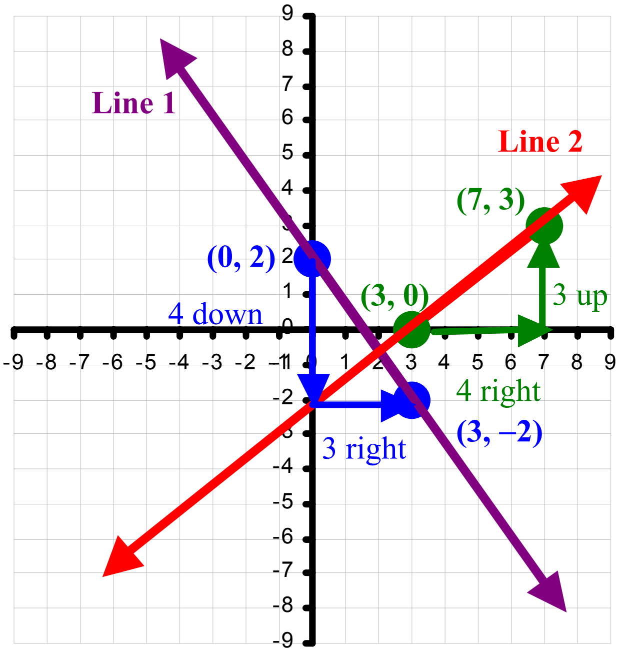
Line 2:

$m_2 = \frac{3}{2}$ (parallel lines – same slope as m_1)

Using $(-5, 3)$ as (x, y) and the form $y = mx + b$, we have:

$$(3) = \frac{3}{2}(-5) + b$$
$$3 = \frac{-15}{2} + b$$
$$3 + \frac{15}{2} = b$$
$$\frac{6}{2} + \frac{15}{2} = b$$
$$b = \frac{21}{2}$$
$$y = \frac{3}{2}x + \frac{21}{2}$$

Example 6: Find the equation of a line perpendicular to $4x + 3y - 6 = 0$ and having the same x- intercept as the line $3x - 2y - 9 = 0$.



Line 1:

$3y = -4x + 6 \qquad y = \frac{-4x+6}{3} \qquad y = \frac{-4}{3}x + 2 \quad m_1 = \frac{-4}{3}$

Line 2:

$m_2 = \frac{3}{4}$ (perpendicular lines – negative reciprocal of m_1)

To find x-intercept of $3x - 2y - 9 = 0$, we let $y = 0$.

$3x - 2(0) - 9 = 0 \qquad 3x = 9 \qquad x\text{-int} = 3$ means **(3, 0)**

Using (3, 0) as (x, y) and the form $y = mx + b$, we have:

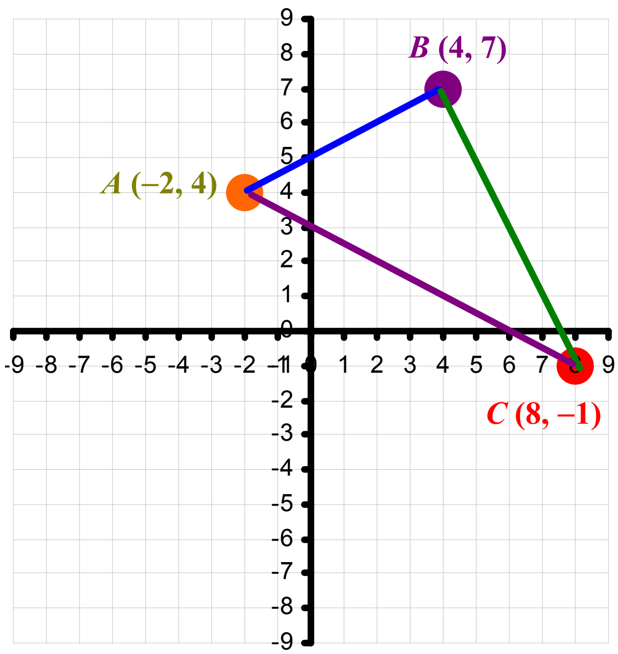
$(0) = \frac{3}{4}(3) + b$

$0 = \frac{9}{4} + b$

$b = \frac{-9}{4}$

$$y = \frac{3}{4}x - \frac{9}{4}$$

Example 7: Prove that ΔABC, where A (−2, 4), B (4, 7) and C (8, −1), is a right angle triangle.



From the diagram, it looks like $\angle B = 90°$. To prove that ΔABC is a right angle triangle at $\angle B$, we need to show that $\overline{AB} \perp \overline{BC}$.

$m_{\overline{AB}} = \frac{7-4}{4-(-2)} = \frac{3}{6} \qquad m_{\overline{AB}} = \frac{1}{2}$

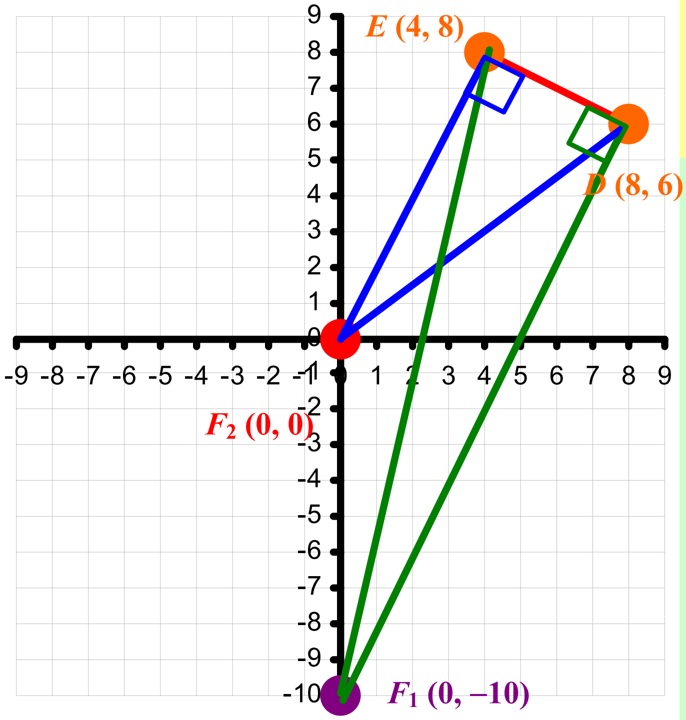
$m_{\overline{BC}} = \frac{-1-7}{8-4} = \frac{-8}{4} \qquad m_{\overline{BC}} = -2$

Since $m_{\overline{AB}} = \frac{-1}{m_{\overline{BC}}}$ (negative reciprocal slopes), we can say that $\overline{AB} \perp \overline{BC}$. Therefore, ΔABC is a right angle triangle at $\angle B$.

6-7 Homework Assignments

Regular: pg. 294 to 295 #1 to 16, 17 to 25 (odd), 27a, 27c, 27e, 27g, 28 to 60, 64.

(AP) Example 8: The line $\overline{DE}$, where *D* (8, 6) and *E* (4, 8) is the shortest side of the right Δ*DEF*. If *F* is a point on the *y*-axis, find the possible coordinates of *F*.



$$m_{\overline{DE}} = \frac{8-6}{4-8} = \frac{2}{-4} \qquad m_{\overline{DE}} = \frac{-1}{2}$$

The right angle of Δ*DEF* can be at ∠*D* or ∠*E*.

If $\angle D = 90°$, $m_{\overline{DF}} = 2$.
A point, F_1, on the *y*-axis will be $(0, b_1)$.

$$m_{\overline{DF}} = \frac{6-b_1}{8-0}$$
$$2 = \frac{6-b_1}{8}$$
$$16 = 6 - b_1$$
$$b_1 = 6 - 16$$
$$b_1 = -10$$

F_1 is at (0, −10)

If $\angle E = 90°$, $m_{\overline{EF}} = 2$.
A point, F_2, on the *y*-axis will be $(0, b_2)$.

$$m_{\overline{EF}} = \frac{8-b_2}{4-0}$$
$$2 = \frac{8-b_2}{4}$$
$$8 = 8 - b_2$$
$$b_2 = 8 - 8$$
$$b_2 = -0$$

F_2 is at (0, 0)

(AP) Example 9: Find the value of *p* if the lines $5x - py + 8 = 0$ and $px - 5y + 10 = 0$ are

a. parallel to each other. b. perpendicular to each other.

First, find the expressions of the slopes of both lines.

Line 1:
$$5x - py + 8 = 0$$
$$-py = -5x - 8$$
$$y = \frac{-5x-8}{-p}$$
$$y = \frac{5}{p}x + \frac{8}{p}$$
$$m_1 = \frac{5}{p}$$

Line 2:
$$px - 5y + 10 = 0$$
$$-5y = -px - 10$$
$$y = \frac{-px-10}{-5}$$
$$y = \frac{p}{5}x + 2$$
$$m_2 = \frac{p}{5}$$

a) Parallel lines means $m_1 = m_2$.

$$\frac{5}{p} = \frac{p}{5} \qquad p^2 = 25 \qquad p = \pm\sqrt{25}$$

$\boldsymbol{p = \pm 5}$

b) Perpendicular lines means $m_1 = \frac{-1}{m_2}$.

Equating m_2 with the negative reciprocal of m_1 gives,

$$\frac{-p}{5} = \frac{p}{5} \qquad 5p = -5p \qquad 5p + 5p = 0 \qquad 10p = 0$$

$\boldsymbol{p = 0}$

6-7 Homework Assignments

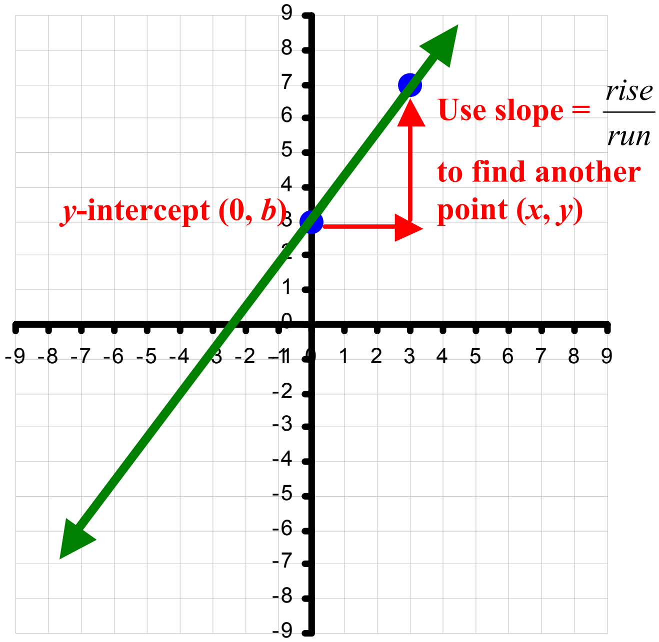
AP: pg. 294 to 295 #1 to 16, 18 to 26 (even), 27b, 27d, 27f, 27h, 28 to 64.

6-8: Graphing Linear Equations

When given the equation in **slope and *y*-intercept form**, we can graph the equation by plotting the *y*-intercept first. Then, using the slope to find another point of the equation, we can graph the line.

$$y = mx + b$$

where m = **slope** and b = ***y*-intercept**

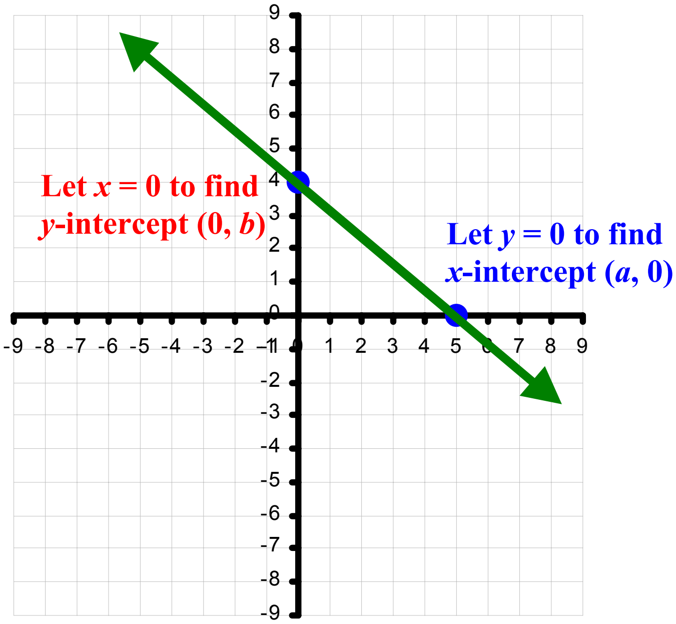


When given the equation in **standard or general form**, we can graph the equation by plotting the *x*- and *y*-intercepts.

To find the *x*-intercept, we let *y* = 0.
To find the *y*-intercept, we let *x* = 0.

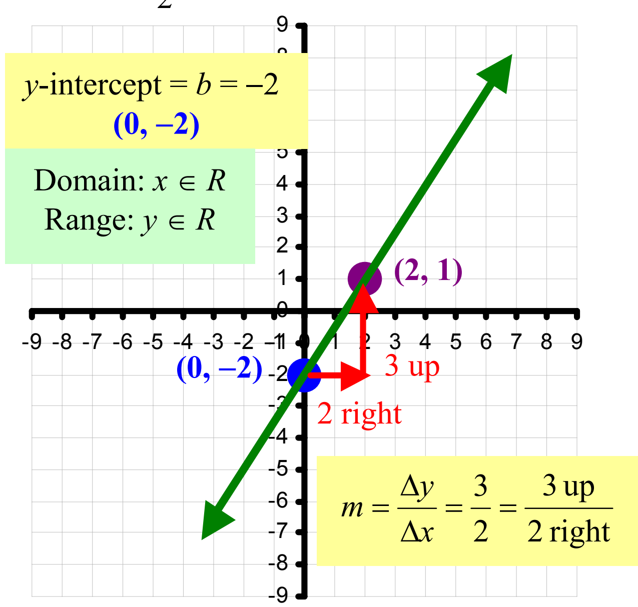
$$0 = Ax + By + C$$

If both *x*- and *y*-intercepts are = 0, then select any number for *x* and find its corresponding *y* for another point to plot.

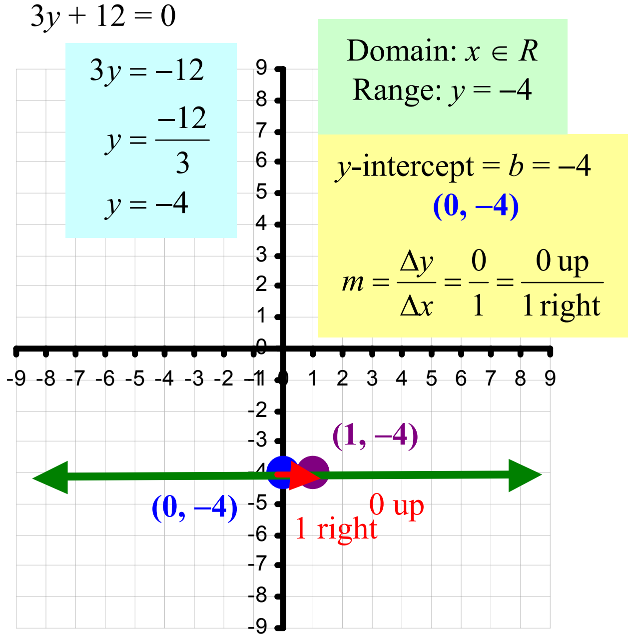


Example 1: Graph equations below using slope and *y*-intercept. State their domains and ranges.

a. $y = \frac{3}{2}x - 2$



b. $3y + 12 = 0$



Example 2: Graph the equations below using x- and y-intercepts. State their domains and ranges.

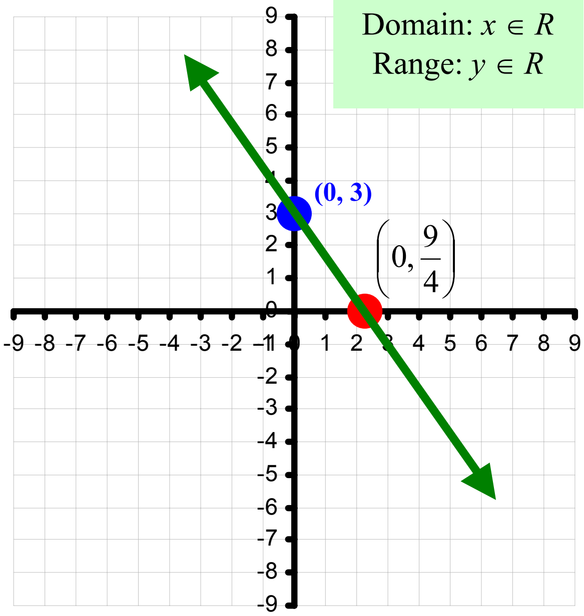
a. $4x + 3y - 9 = 0$

To find x-intercept, let $y = 0$.

$$4x + 3(0) - 9 = 0$$
$$4x = 9$$
$$x = \frac{9}{4}$$

To find y-intercept, let $x = 0$.

$$4(0) + 3y - 9 = 0$$
$$3y = 9$$
$$y = 3$$



Domain: $x \in R$
Range: $y \in R$

b. $3x - 2y = 0$

To find x-intercept, let $y = 0$.

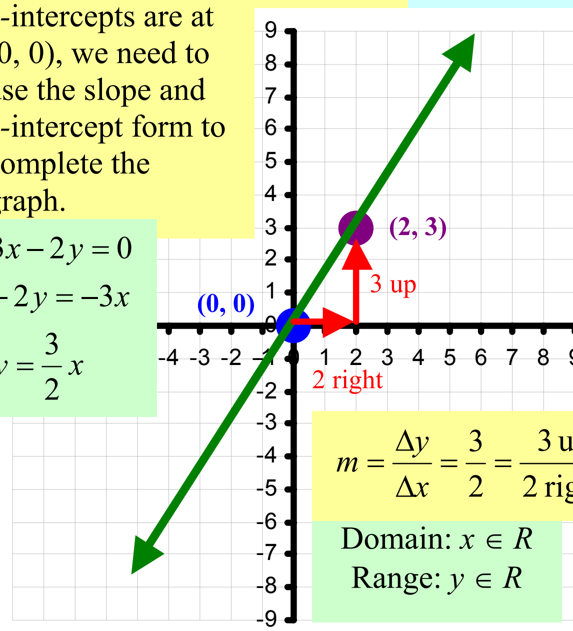
$$3x - 2(0) = 0$$
$$3x = 0$$
$$x = 0$$

To find y-intercept, let $x = 0$.

$$3(0) + 2y = 0$$
$$2y = 0$$
$$y = 0$$

Since both x- and y-intercepts are at (0, 0), we need to use the slope and y-intercept form to complete the graph.

$$3x - 2y = 0$$
$$-2y = -3x$$
$$y = \frac{3}{2}x$$

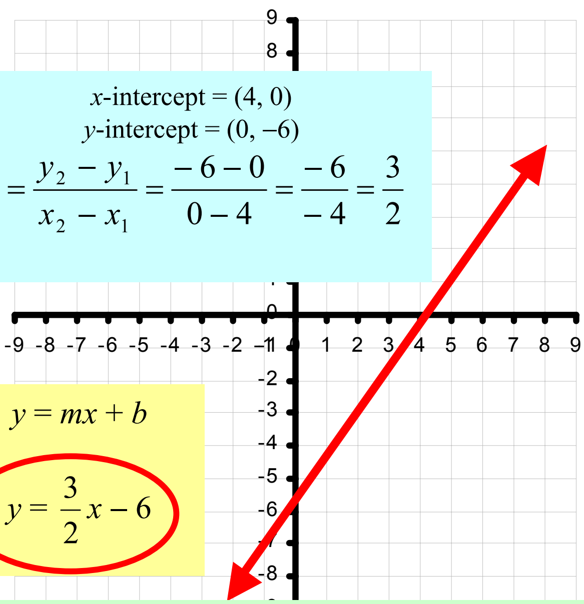


$$m = \frac{\Delta y}{\Delta x} = \frac{3}{2} = \frac{3 \text{ up}}{2 \text{ right}}$$

Domain: $x \in R$
Range: $y \in R$

Example 3: Find the equation for the lines below in both slope and *y-intercept* form and standard form.

a.



x-intercept = (4, 0)
y-intercept = (0, –6)

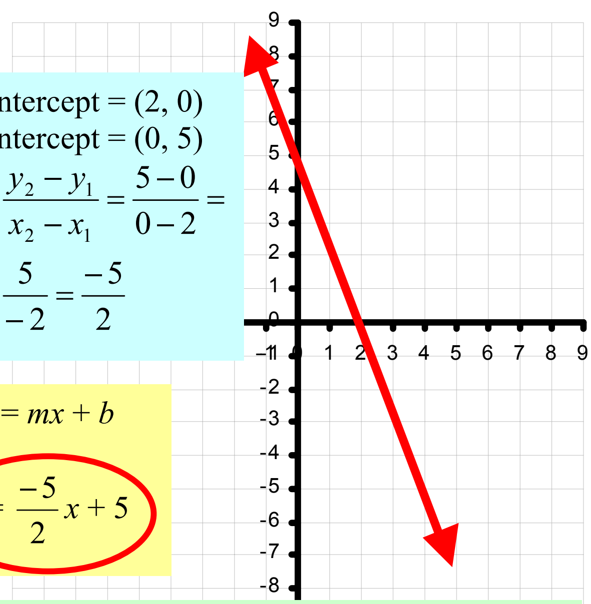
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 0}{0 - 4} = \frac{-6}{-4} = \frac{3}{2}$$

$$y = mx + b$$
$$y = \frac{3}{2}x - 6$$

Multiply 2 on both sides.

$2y = 3x - 12$ **$0 = 3x - 2y - 12$**

b.



x-intercept = (2, 0)
y-intercept = (0, 5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{0 - 2} =$$
$$m = \frac{5}{-2} = \frac{-5}{2}$$

$$y = mx + b$$
$$y = \frac{-5}{2}x + 5$$

Multiply 2 on both sides.

$2y = -5x + 10$ **$5x + 2y - 10 = 0$**

Example 4: In the world of economics, the price of an item sold is mainly depended on the supply and demand of the market place. Suppose the supply equation of a particular Star Trek model is $P = 0.001n + 40$ and the demand equation of the same model is $11n + 5000P - 1,000,000 = 0$, where P = price and n is the quantity manufactured or sold.

a. Graph the supply and demand equations of *Price* versus *Quantity* using the scales x: [0, 100,000, 10,000] and y: [0, 250, 50].
b. What do the slope and y-intercept of the supply line represent?
c. What do the x- and y-intercepts of the demand line represent?
d. What does the intersecting point of the two linear equations represent?

Supply Equation
$P = 0.001n + 40$
y-intercept = (0, \$40)

$$slope = 0.001 = \frac{1}{1000} = \frac{\$10 \text{ increase}}{10{,}000 \text{ units increase}}$$

B (0 + 10000, \$40 + \$10) = B (10000, \$50)

Demand Equation
$11n + 5000P - 1,000,000 = 0$

For n-int, let $P = 0$

$$0 = 11n + 5000(0) - 1{,}000{,}000$$
$$-11n = -1{,}000{,}000$$
$$n = \frac{-1{,}000{,}000}{-11}$$

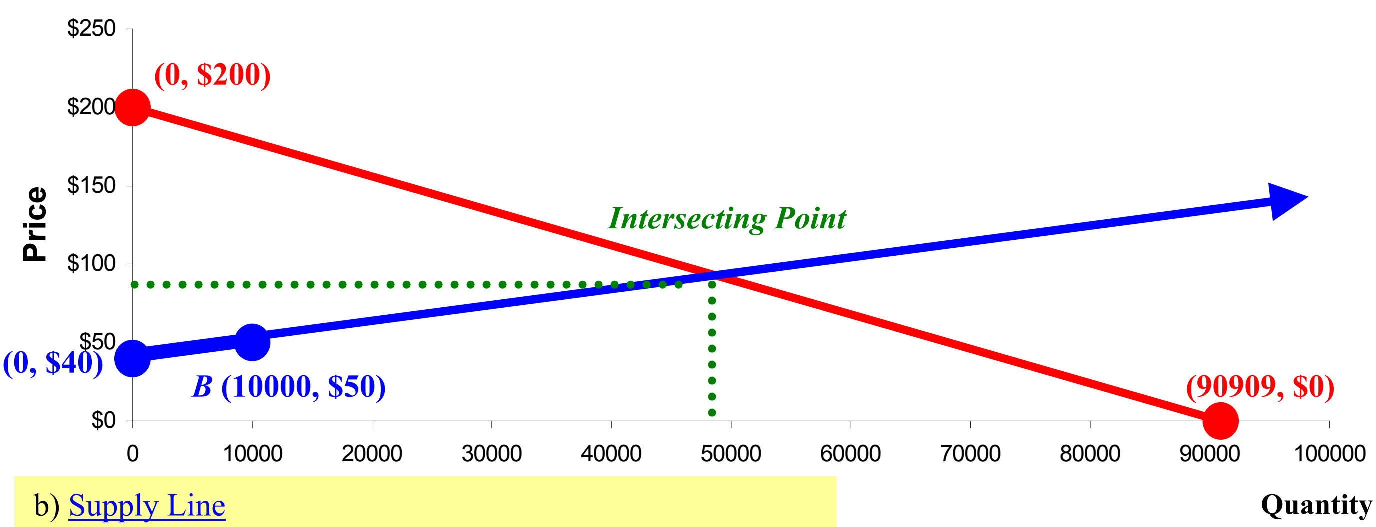
n-int = 90909 units

For P-int, let $n = 0$

$$0 = 11(0) + 5000P - 1{,}000{,}000$$
$$-5000P = -1{,}000{,}000$$
$$P = \frac{-1{,}000{,}000}{-5000}$$

P-int = \$40

Supply and Demand of a Star Trek Model



b) Supply Line
slope = Manufacturing Variable Cost (labour, material)
***y*-int = Manufacturing Fixed Cost (rent, heat, license)**

c) Demand Line
***x*-int = quantity available when item becomes worthless.**
***y*-int = price of item when it becomes absolutely rare.**

d) Intersecting Point of Supply and Demand Lines
- Optimal Price at Optimal Amount Manufactured.

6-8 Homework Assignments

Regular: pg. 298 to 299 #1 to 21 (odd), 23 to 37, 40.

AP: pg. 298 to 299 #2 to 22 (even), 23 to 43.

