Unit 5: Vectors

7-1: Vectors and Scalars

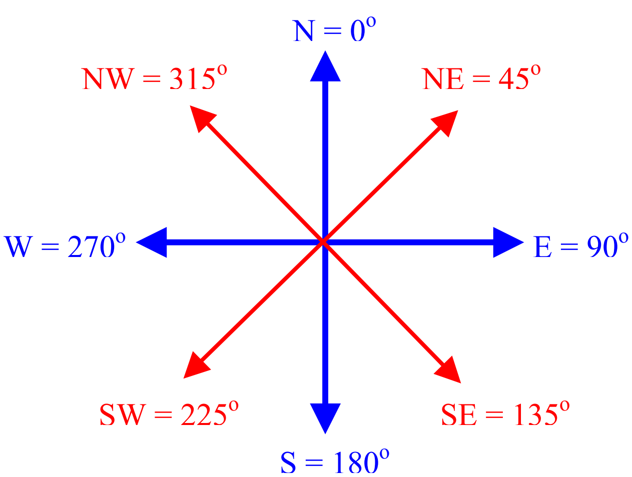
Scalar Quantity: - quantity that involves a **Magnitude** of measurement but **NO Direction**.

Vector Quantity: - quantity that involves a **Magnitude** of measurement **AND** a **Direction**.

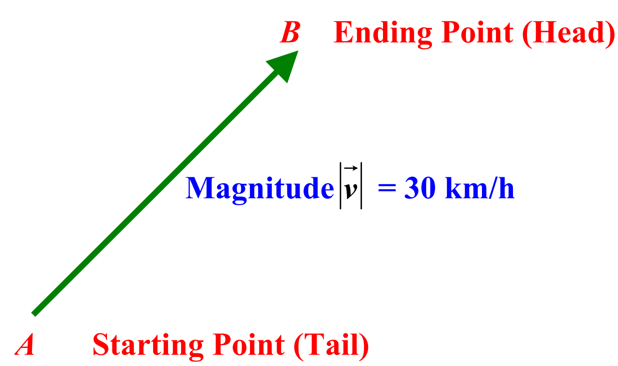
Scalar Quantities	Examples	Vector Quantities	Examples
Distance (Length WITHOUT Direction)	**20 m**	**Displacement** (Length WITH Direction)	**20 m [North]**
Speed (How Fast an object moves WITHOUT Direction)	**50 km/h**	**Velocity** (How Fast an object moves WITH Direction)	**50 km/h [280°]**
Mass (How Much Stuff is IN an object)	**44 kg**	**Force** (Mass × Acceleration)	**40 N downward**
Energy (Emits in ALL Directions)	**500 kJ**	**Weight** (Force due to Gravity)	**500 N (always downward)**
Temperature (Average Kinetic Energy of an object)	**25 °C**	**Friction** (Resistance Force due to Surface Conditions)	**15 N (against the direction of motion)**
Time	**65 minutes**	**Acceleration** (How Fast Velocity Changes over Time)	**5 m/s² [NW]**

Vector Notation: - a method of indicating that the quantity is a vector by placing an arrow → on top of the variable.

Bearing: - compass bearing **STARTS** at the North (0°) and rotates **CLOCKWISE**.

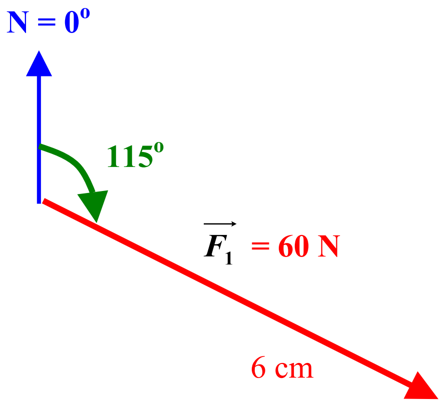


Example: Vector $\overrightarrow{AB}$ or $\vec{v}$ = 30 km/h [NE] or N45°E
(**From** N, move 45° **towards** E)

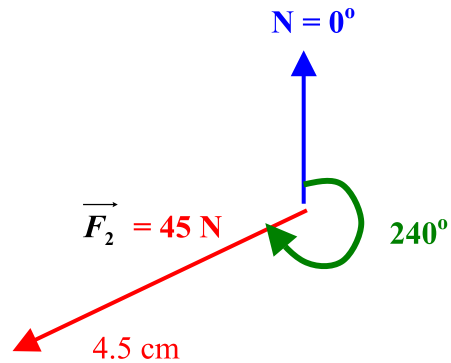


Draw Vectors with Proper Scale

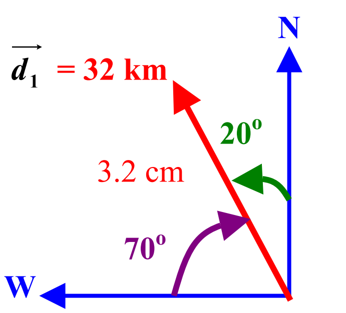
Example 1: Draw $\vec{F_1}$ = 60 N [115°]
(Use 1 cm = 10 N)



Example 2: Draw $\vec{F_2}$ = 45 N [240°]
(Use 1 cm = 10 N)

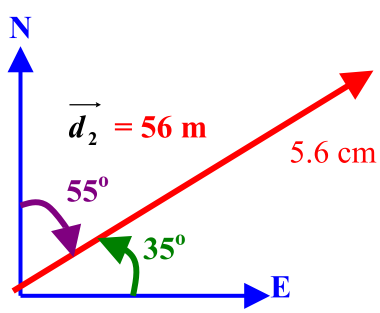


Example 3: Draw $\vec{d_1}$ = 32 km [N20°W]
(Use 1 cm = 10 km)



OR we can say $\vec{d_1}$ = 32 km [W70°N]

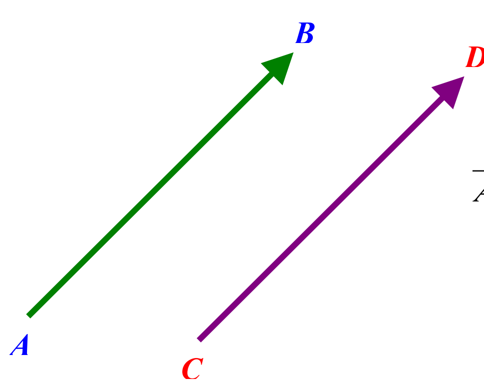
Example 4: Draw $\vec{d_2}$ = 56 m [E35N°]
(Use 1 cm = 10 m)



OR we can say $\vec{d_2}$ = 56 m [N55°E]

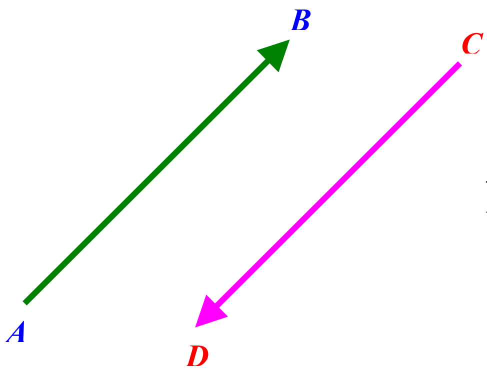
Equal and Opposite Vectors

Equal Vectors: - vectors that have the **SAME Magnitude AND Direction**.



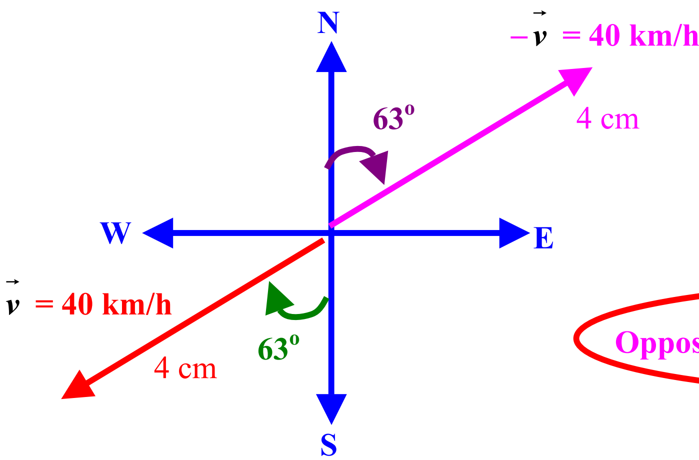
$\vec{AB} = \vec{CD}$ where $|\vec{AB}| = |\vec{CD}|$ and they have the **SAME** Direction.

Opposite Vectors: - vectors that have the **SAME Magnitude but DIFFERENT Direction**.



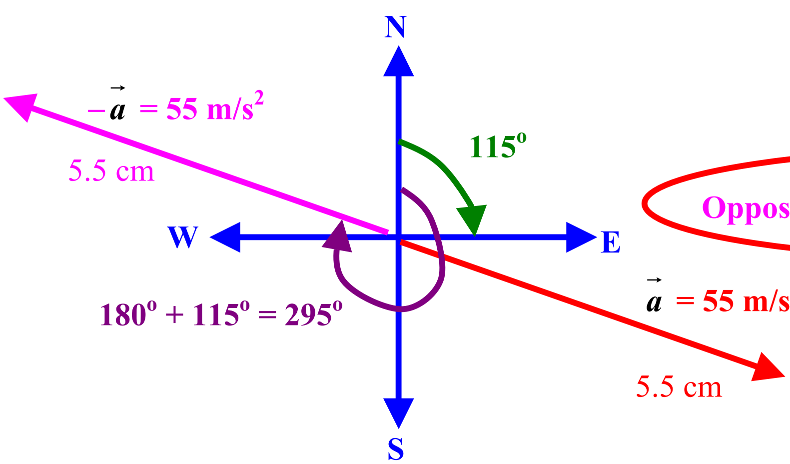
$\overrightarrow{AB} = -\overrightarrow{CD}$ where $|\overrightarrow{AB}| = |\overrightarrow{CD}|$ and they have **OPPOSITE** Direction.

Example 5: Draw the opposite velocity vector to $\vec{v}$ = 40 km/h [S63W°] (Use 1 cm = 10 km/h)



Opposite Velocity Vector = $-\vec{v}$ = 40 km/h [N63°E]

Example 6: Draw the opposite acceleration vector to $\vec{a}$ = 55 m/s^2 [115°] (Use 1 cm = 10 m/s^2)



Opposite Acceleration Vector = $-\vec{a}$ = 55 m/s^2 [295°]

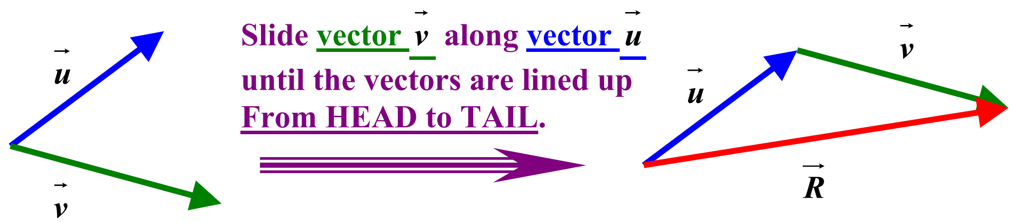
7-1 Assignment: pg. 307 – 309 #1 to 9

7-2: Adding Vectors Using Scale Diagrams

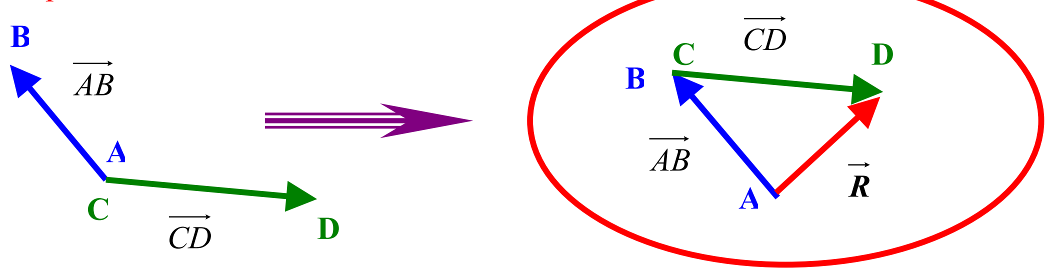
Resultant Vector: - the vector that is the result of vector addition or subtraction.
- **from** the **Starting Point** of the **First** Vector **to** the **Ending Point** of the **Last** Vector.

Adding Vectors (always Connect Vectors FROM HEAD TO TAIL)

$$\vec{u}+\vec{v}=\vec{R}$$

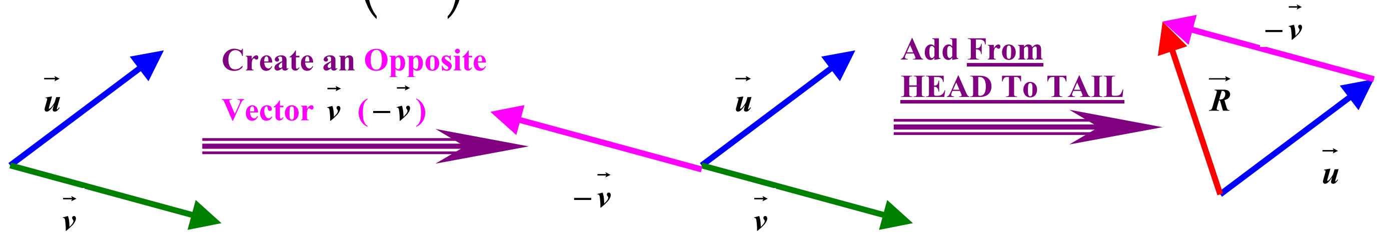


Example 1: Draw $\overrightarrow{AB}+\overrightarrow{CD}$

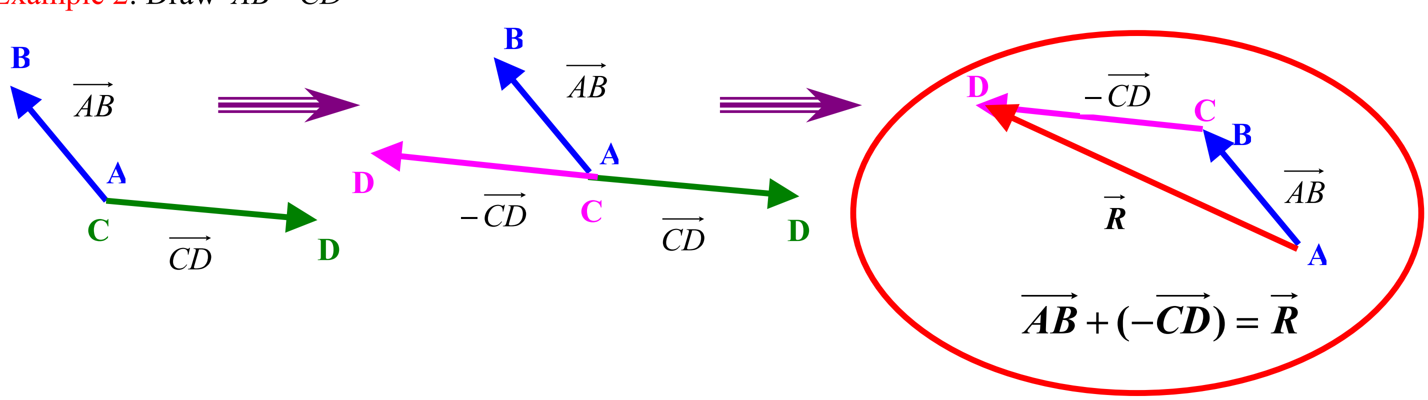


Subtracting Vectors (ADD OPPOSITE VECTORS)

$$\vec{u}-\vec{v}=\vec{u}+\left(-\vec{v}\right)=\vec{R}$$



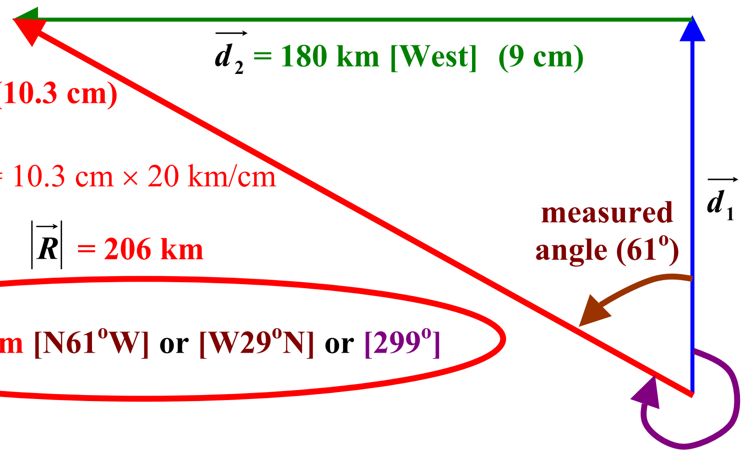
Example 2: Draw $\overrightarrow{AB}-\overrightarrow{CD}$



Example 3: A ship left the dock and traveled north at 50 km/h for 2 hours, then it turned west at 60 km/h for 3 hour.

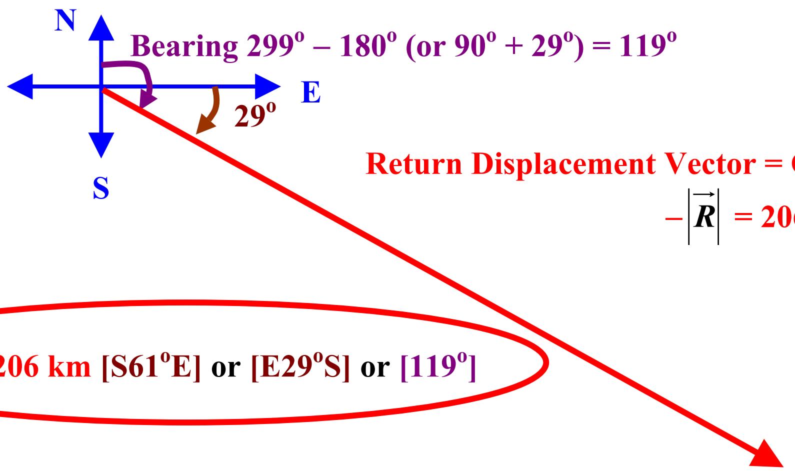
a. What is the net resultant displacement vector, $\vec{R}$? (Use 1 cm = 20 km)

$\vec{d}_1$ = 50 km/h [N] × 2 hr	$\vec{d}_1$ = **100 km [North]**	**(5 cm)**
$\vec{d}_2$ = 60 km/h [N] × 3 hr	$\vec{d}_2$ = **180 km [West]**	**(9 cm)**



$\vec{R}$ = 206 km [N61°W] or [W29°N] or [299°]

b. What is the displacement vector the ship must follow to return to the dock?



Return Displacement Vector = Opposite Resultant Vector

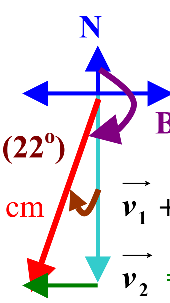
$-|\vec{R}|$ = 206 km

$-\vec{R}$ = 206 km [S61°E] or [E29°S] or [119°]

Example 4: A plane is flying south at 400 km/h and a steady wind is blowing from the east at 100 km/h. If a sudden gust of wind appears from the south at 150 km/h. What is the resultant velocity vector of the plane? (Use 1 cm = 100 km/h)

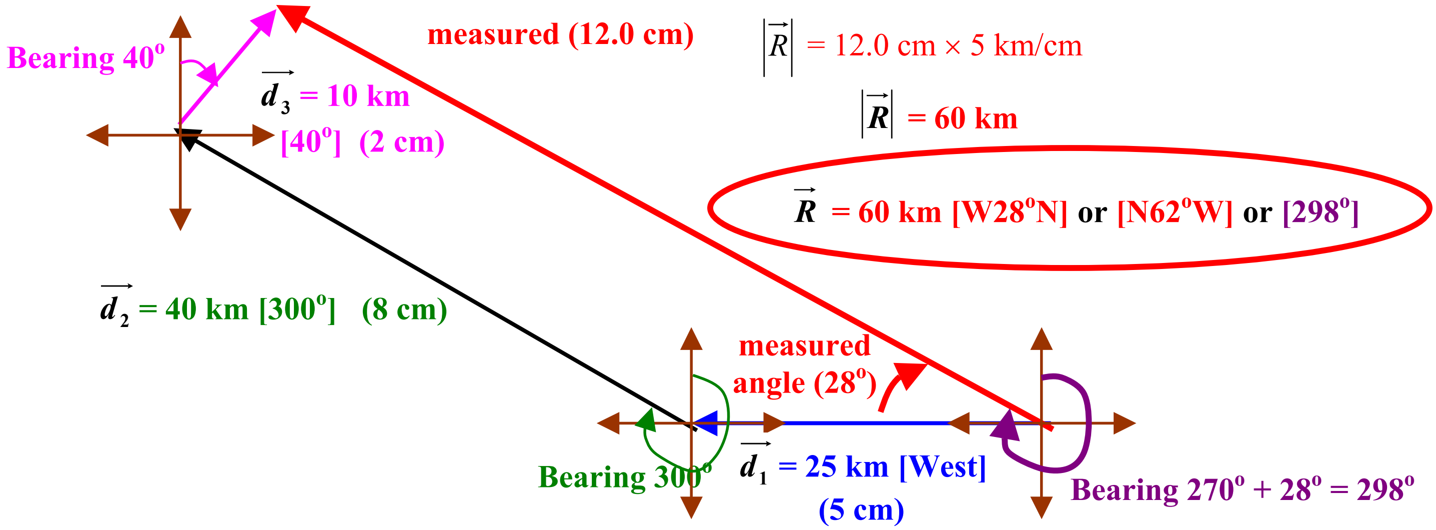
$|\vec{R}|$ = 2.7 cm × 100 km/h / cm

$|\vec{R}|$ = 270 km/h



$\vec{R}$ = 270 km/h [S22°W] or [W68°S] or [202°]

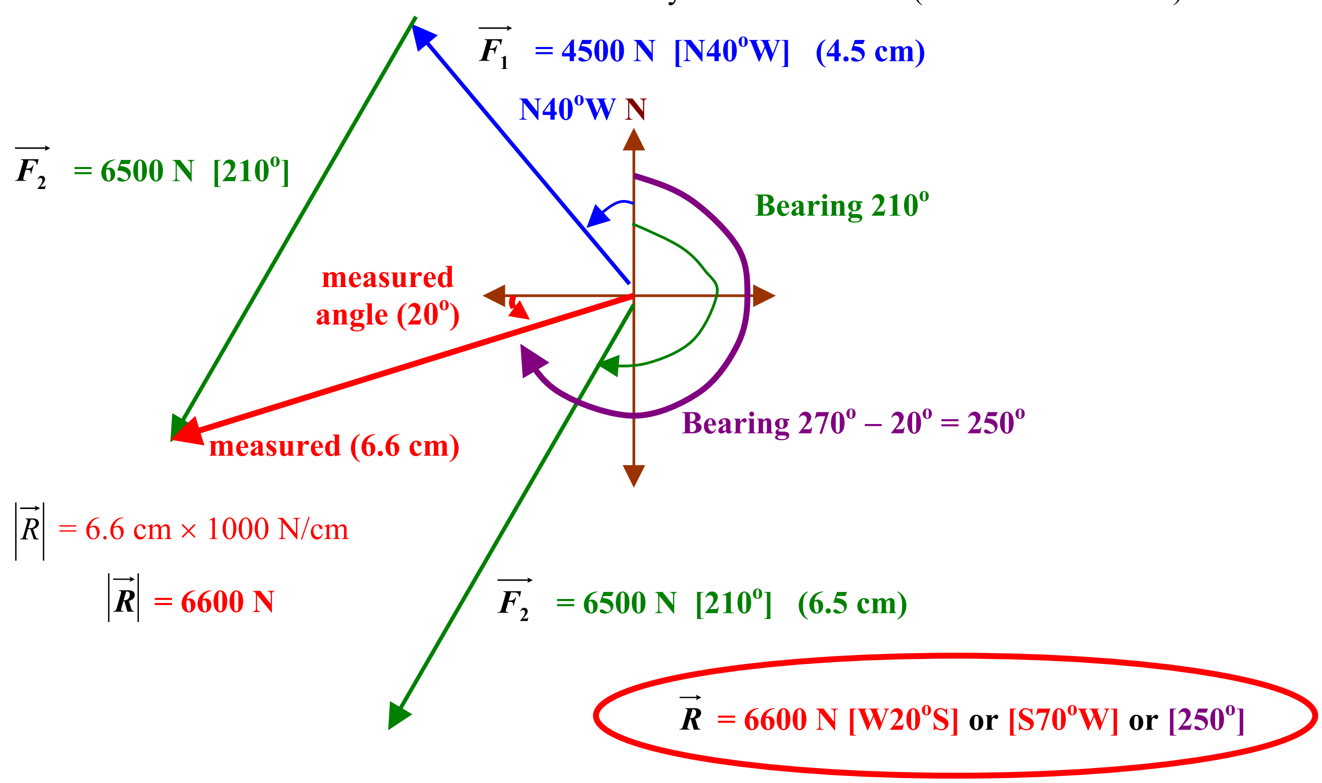
Example 5: A kayak left Port Alberni due west for 25 km. It then turned at a bearing of 300° and traveled on for 40 km. Hearing the sighting of killer whales, it turned at a bearing of 40° for 10 km. What is the kayak's net displacement from Port Alberni? (Use 1 cm = 5 km)



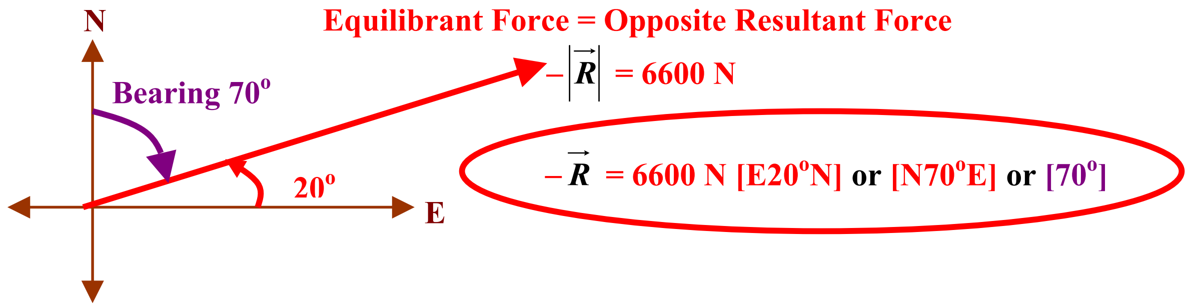
Equilibrant: - the opposite resultant force of an object which does not move, but is acted on by other forces.

Example 6: Two tow trucks are trying to pull a heavy trailer out of a ditch. One tow truck is applying a force of 4500 N at N40°W and the other truck is pulling with a force of 6500 N at 210°. The trailer remained stuck in the ditch.

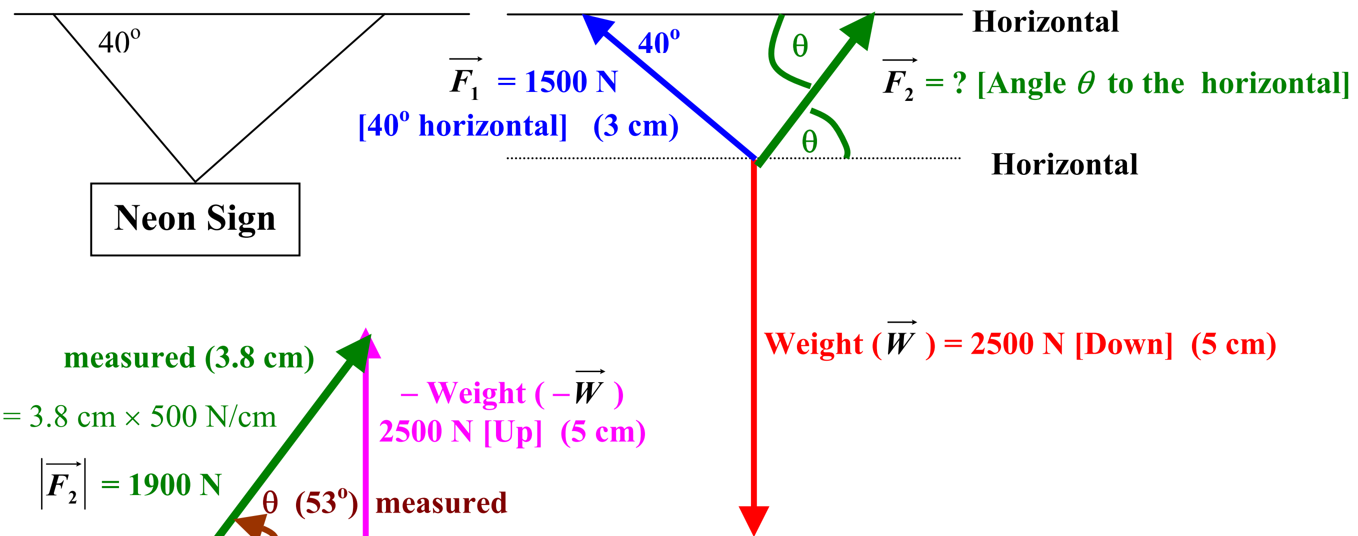
a. Calculate the resultant force on the heavy trailer. (Use 1 cm = 1000 N)

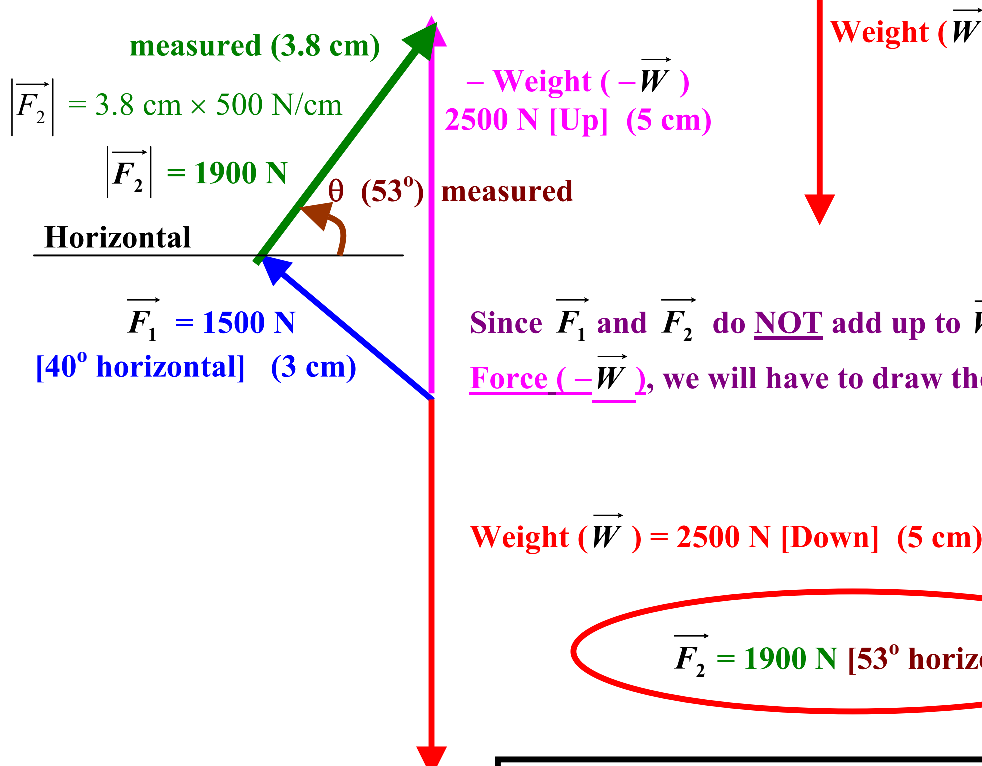


b. What is the equilibrant force on the heavy trailer?



Example 7: A neon sign has a weight of 2500 N is hanged by two ropes as shown below. One rope with a tension force of 1500 N is directed to the left with an angle of 40° to the horizontal. Determine the tension force vector exerted by the other rope to keep the neon sign from moving. (Use 1 cm = 500 N)





Since $\vec{F}_1$ and $\vec{F}_2$ do NOT add up to $\vec{W}$, but ADD up to the Equilibrant Force ($-\vec{W}$), we will have to draw the Opposite Weight Vector.

$\vec{F}_2 = 1900$ N [53° horizontal]

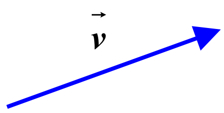
7-2 Assignment: pg. 318 – 319 #1 to 11

7-3: Multiplying a Vector by a Scalar

When multiplying a vector ($\vec{u}$) by a scalar (k), we simply **multiply the magnitude of the vector by the scalar**. There are 3 possibilities with the direction of the resultant vector.

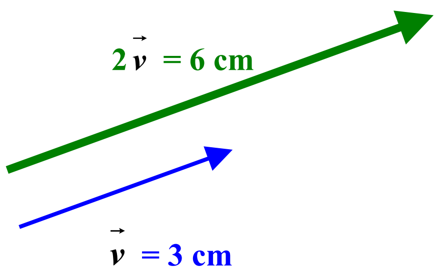
1. If $k > 0$, $k\vec{u} = |k\vec{u}|$ with the **SAME Direction as** $\vec{u}$.
2. If $k < 0$, $k\vec{u} = |k\vec{u}|$ with the **OPPOSITE Direction as** $\vec{u}$.
3. If $k = 0$, $k\vec{u} = 0$ (**NO Resultant Vector**).

Example 1: Given vector $\vec{v}$,

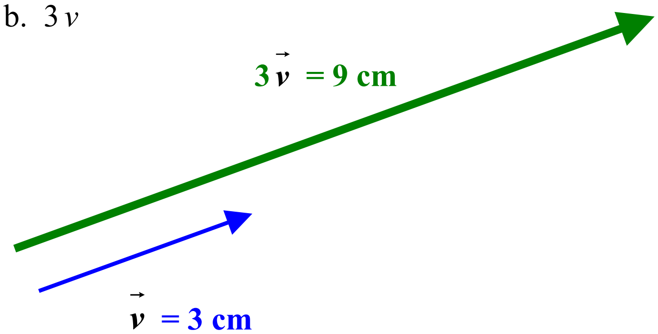


draw:

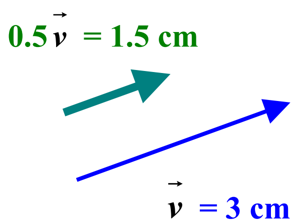
a. $2\vec{v}$



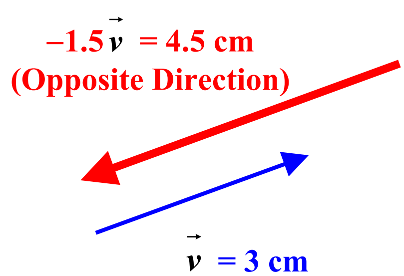
b. $3\vec{v}$



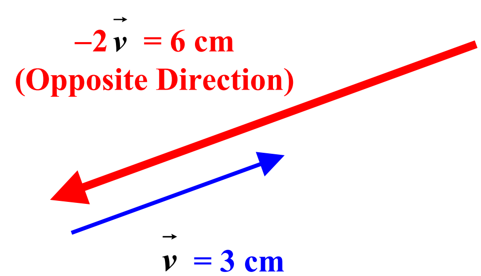
c. $0.5\vec{v}$



d. $-1.5\vec{v}$

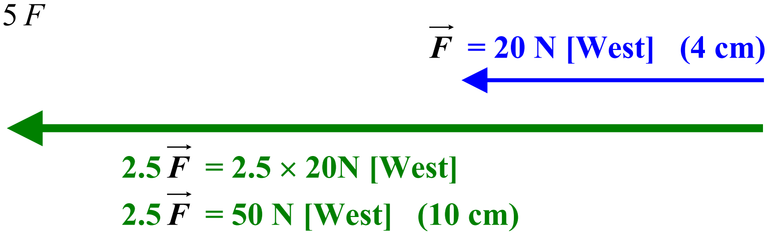


e. $-2\vec{v}$

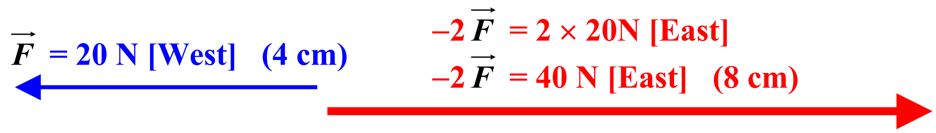


Example 2: Using the scale of 1 cm = 5 N, find the following vectors given that $\vec{F}$ = 20 N [West]

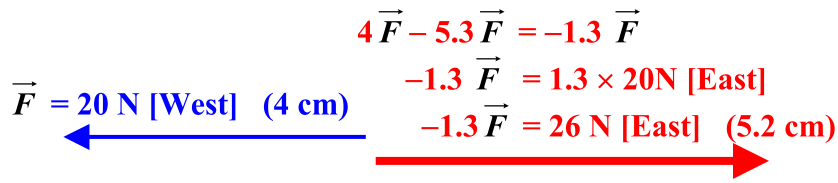
a. $2.5\vec{F}$



b. $-2\vec{F}$



c. $4\vec{F} - 5.3\vec{F}$



Example 3: Find the value of the scalar, *k*, given the following information.

a. $\vec{v}$ = 14 m/s [North] and $k\vec{v}$ = 42 m/s [North]

$k(\vec{v}) = k\vec{v}$

$$k = \frac{k\vec{v}}{\vec{v}} = \frac{42\ \text{m/s [North]}}{14\ \text{m/s [North]}}$$

$k = 3$

Units and **Directions** must be the **SAME** before **Cancellation**.

The scalar, *k*, is always UNITLESS

b. $\vec{a}$ = 9.81 m/s^2 [Down] and $k\vec{a}$ = 4.905 m/s^2 [Up]

$k(\vec{a}) = k\vec{a}$

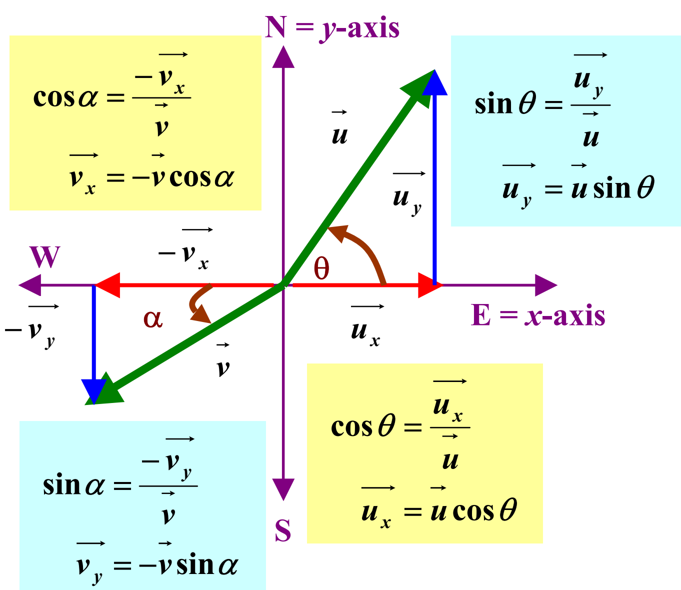
$$k = \frac{k\vec{a}}{\vec{a}} = \frac{4.905\ \text{m/s}^2\ \text{[Down]}}{9.81\ \text{m/s}^2\ \text{[Up]}} = \frac{-4.905\ \text{m/s}^2\ \text{[Up]}}{9.81\ \text{m/s}^2\ \text{[Up]}}$$

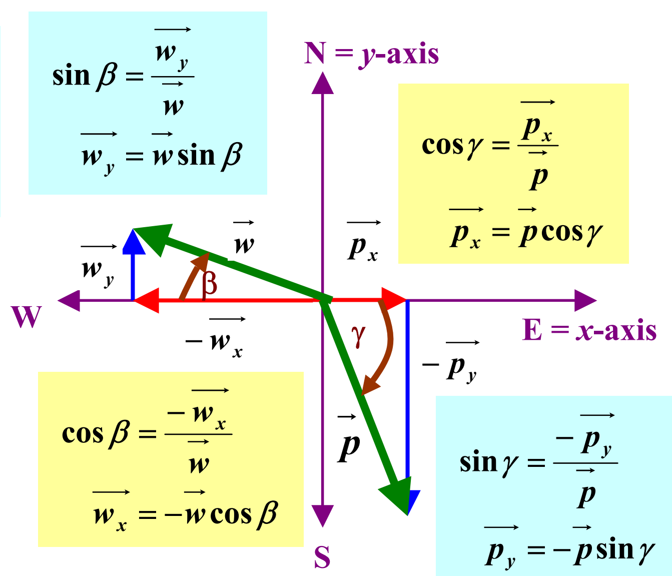
Opposite Vectors is used to bring about the **SAME Direction**.

$k = -0.5$

Analyzing Vector by its Horizontal and Vertical Components

For any given vector, we can form a **Right Angle Triangle** by taking the **Vector as a HYPOTENUSE**.

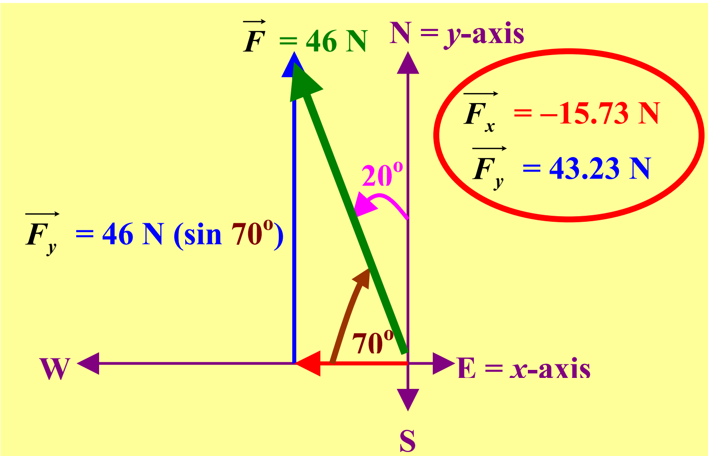




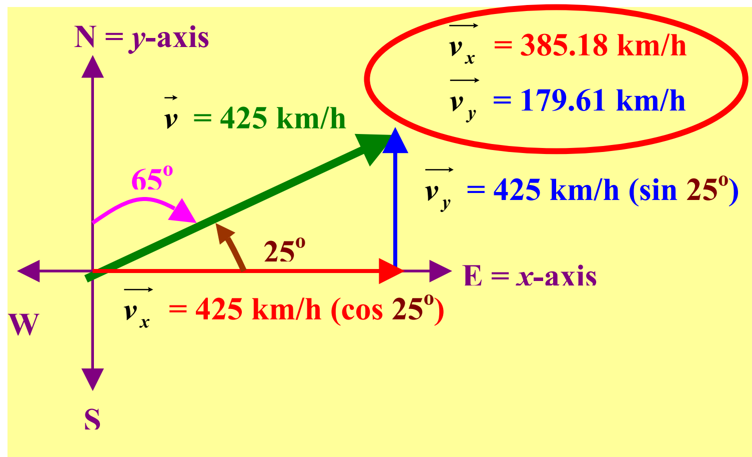
Calculator must be in **DEGREE Mode**!

Example 4: Find the horizontal and vertical components of the following vectors.

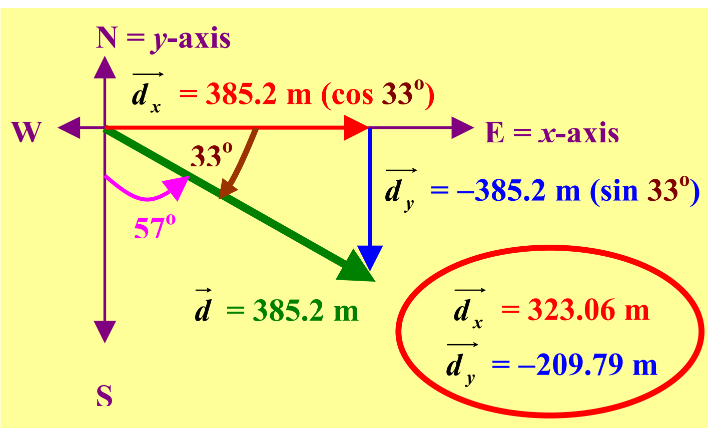
a. $\vec{F}$ = 46 N [N20°W]



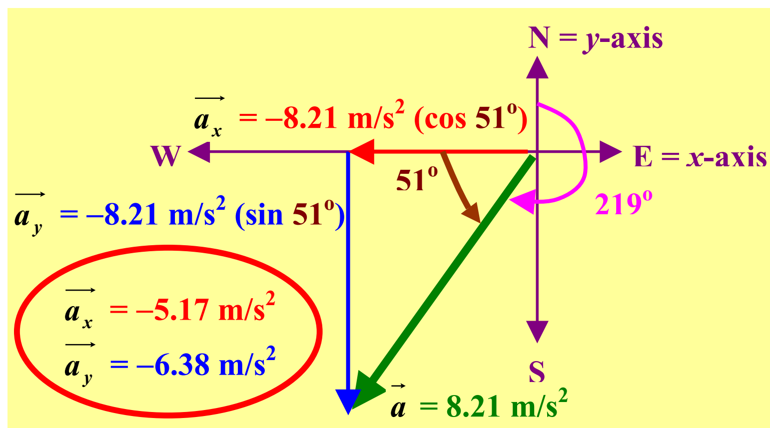
b. $\vec{v}$ = 425 km/h [65°]



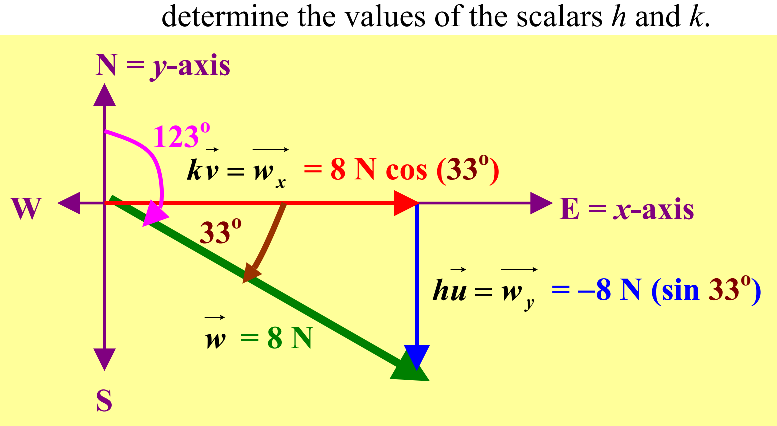
c. $\vec{d}$ = 385.2 m [S57°E]



d. $\vec{a}$ = 8.21 m/s² [219°]



Example 5: Given $\vec{u}$ = 1 N [North], $\vec{v}$ = 1 N [East], scalars h and k, and $\vec{w} = h\vec{u} + k\vec{v}$. For $\vec{w}$ = 8 N [123°], determine the values of the scalars h and k.



$k\vec{v}$ = 6.71 N

$$k = \frac{k\vec{v}}{\vec{v}} = \frac{6.71\ \text{N [East]}}{1\ \text{N [East]}}$$

$k = 6.71$

$h\vec{u}$ = –4.36 N

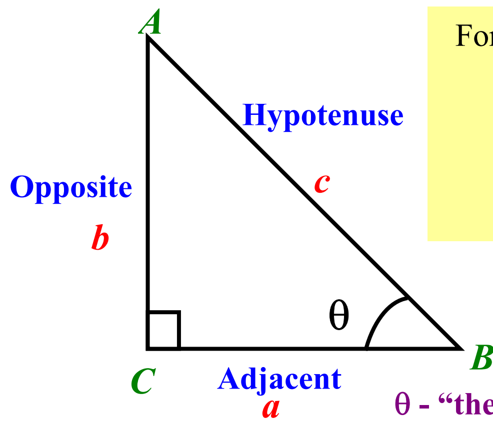
$$h = \frac{h\vec{u}}{\vec{u}} = \frac{-4.36\ \text{N [North]}}{1\ \text{N [North]}}$$

$h = -4.36$

7-3 Assignment: **pg. 322 – 323 #1 to 5**

7-4: Solving Vector Problems by Computation

Basic Trigonometry Review



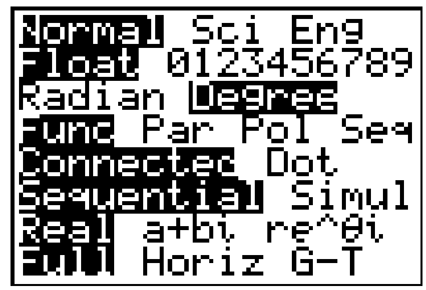
θ - "theta" – variable for angle

For any **right angle triangles**, we can use the simple trigonometric ratios.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

SOH **CAH** **TOA**

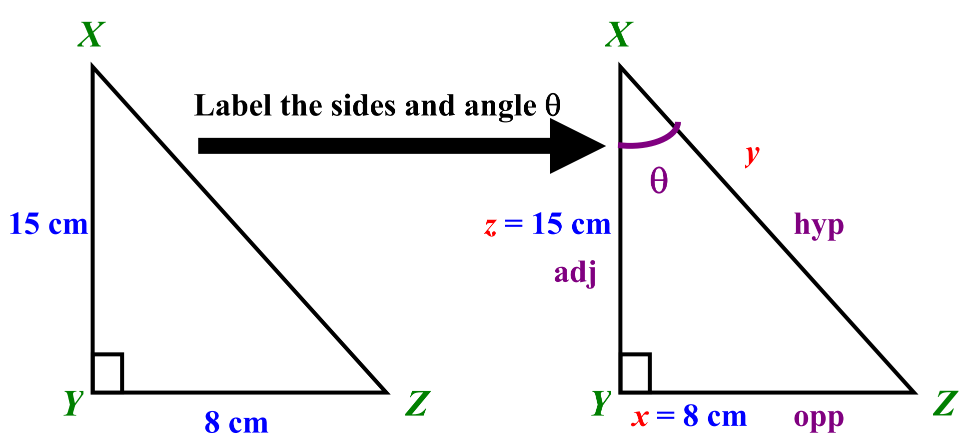
Be sure that your calculator is set in DEGREE under the settings in your **MODE** menu!



Capital letter is always used to label the angle.

The name for the side that is opposite to the angle has the corresponding letter in small case.

Example 1: Find tan X, $\angle X$ and $\overline{XZ}$.



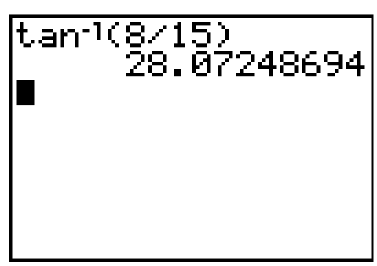
$$y^2 = 8^2 + 15^2$$
$$y^2 = 64 + 225$$
$$y^2 = 289$$
$$y = \sqrt{289}$$
$$y = 17cm$$

$$\tan X = \frac{\text{opp}}{\text{adj}}$$
$$\tan X = \frac{8}{15}$$

$$\tan X = \frac{8}{15}$$
$$X = 28^\circ$$

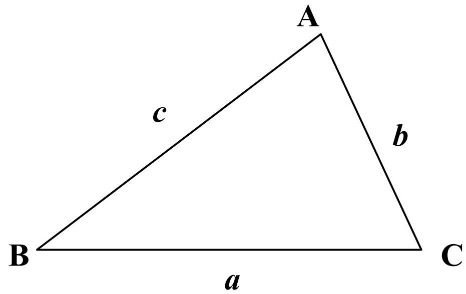
2nd

TAN



The Sine Law

For any triangle, the **Law of Sines** allows us to solve the rest of the triangle if we know the measure of an angle and the length of its opposite side, plus one other angle or side.

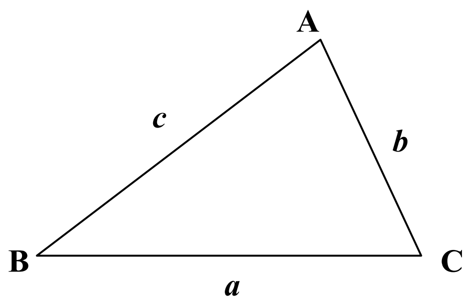


$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

When using the Sine Law, we only use a ratio of two fractions at one time.

The Cosine Law

For any triangle, the **Law of Cosines** allows us to solve the triangle if we know the measure of an angle and the length of its two adjacent sides (Case SAS), or if we know the lengths of all three sides (Case SSS).



$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

Solving for cos A:

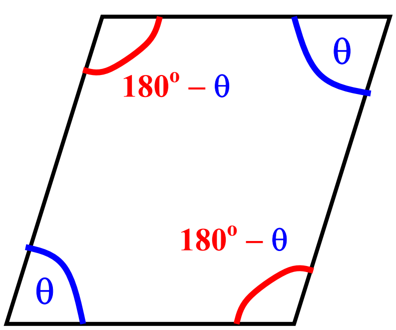
$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$2bc(\cos A) = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

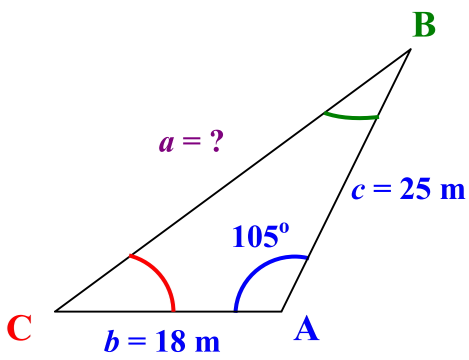
When given the length for all three sides, solve for the largest angle first!
This will eliminate any chance for an ambiguous situation.

Angles in Parallelogram



In any parallelogram, **the Opposite Angles always have the SAME measure**. The **Adjacent Angles will always ADD up to 180°**.

Example 2: In ΔABC, ∠A = 105°, b = 18 m and c = 25 m. Solve the triangle to the nearest degree and to the nearest tenth of a metre.



$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$a^2 = 18^2 + 25^2 - 2(18)(25)(\cos 105^\circ)$$
$$a^2 = 1181.937141$$
$$a = \sqrt{1181.937141}$$

a = 34.4 m

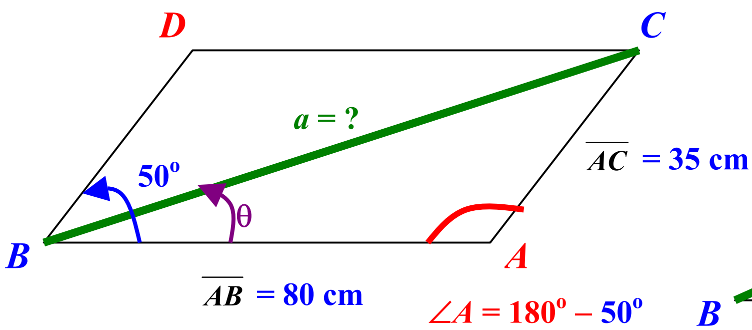
∠C = 180° − 105° − 30°

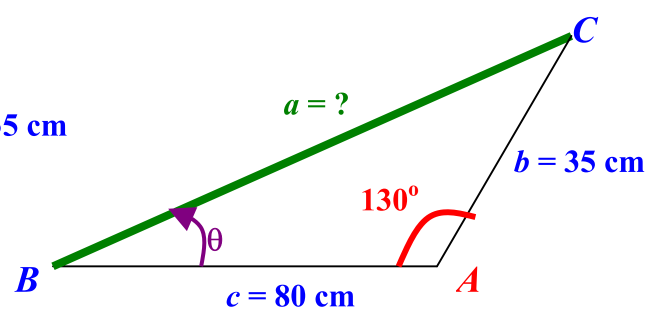
∠C = 45°

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 105^\circ}{34.4} = \frac{\sin B}{18}$$
$$\sin B = \frac{18(\sin 105^\circ)}{34.4}$$
$$\sin B = 0.5054263045$$

∠B = 30°

Example 3: In parallelogram $BACD$, $\angle B$ = 50°, $\overline{AB}$ = 80 cm and $\overline{AC}$ = 35 cm. Find the length of the diagonal $\overline{BC}$ to the nearest tenth of a centimetre. Determine $\angle CBA$ to the nearest degree.





$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$a^2 = 35^2 + 80^2 - 2(35)(80)(\cos 130^\circ)$$
$$a^2 = 11224.61061$$
$$a = \sqrt{11224.61061}$$

a = 105.9 cm

```
35²+80²-2*35*80c
os(130)
       11224.61061
√(Ans)
       105.9462629
```

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 130^\circ}{105.9} = \frac{\sin B}{35}$$
$$\sin B = \frac{35(\sin 130^\circ)}{105.9}$$
$$\sin B = 0.2531780501$$

∠B = 15°

```
35sin(130)/105.9
       .2531780501
sin⁻¹(Ans)
       14.66565285
```

Solving Vectors Algebraically

1. Draw a **Vector Diagram** with everything properly labeled.
2. For vectors that form a **Right Angle Triangle**, line up vectors from **Head to Tail**. Use the **Pythagorean Theorem** and the **Tangent Ratio** to solve for the **Magnitude** of the **Resultant Vector** and its **Compass Bearing or Heading**.
3. For vectors that do **NOT** form a **Right Angle Triangle**, use any one of the two methods below.

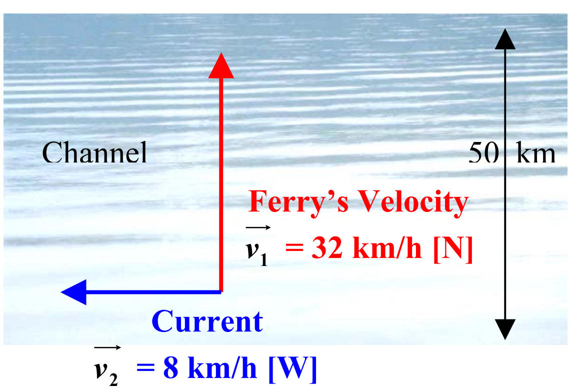
Parallelogram Method: (can only be use to Add TWO Vectors)

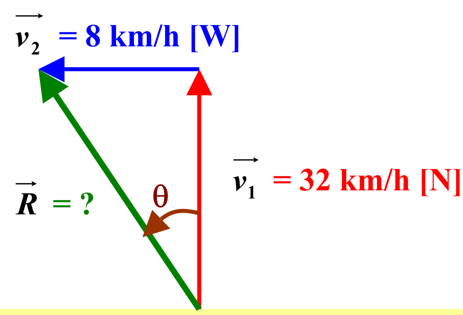
a. Line up vectors from **Head to Tail** by sliding the second vector along the first.
b. **Complete** the remaining side to form a **Parallelogram**.
c. Using the **Law of Cosine**, find the **Magnitude of the Resultant Vector**.
d. Using the **Law of Sine**, find the proper **Compass Heading or Bearing of the Resultant Vector**.

Vector Components Method: (can be use to Add TWO or MORE Vectors)

a. Find the **horizontal** and **vertical** components of **ALL Vectors**.
b. **Add up all the horizontal vectors** (be careful with the signs). This is the **Resultant Vector's Horizontal Component**.
c. **Add up all the vertical vectors** (be careful with the signs). This is the **Resultant Vector's Vertical Component**.
d. Using the **horizontal** and **vertical** components of the **Resultant Vector** with the **Pythagorean Theorem**, determine the **Magnitude of the Resultant Vector**.
e. Using the **Tangent Ratio**, find the proper **Compass Heading or Bearing of the Resultant Vector**.

Example 4: A ferry is crossing a channel 50 km wide with a velocity of 32 km/h [North]. The channel has a current that is 8 km/h [West]. What would be the magnitude and direction of the ferry's resultant velocity?





$$R^2 = 8^2 + 32^2$$
$$R^2 = 64 + 1024$$
$$R^2 = 1088$$
$$R = \sqrt{1088}$$

R = 33 km/h

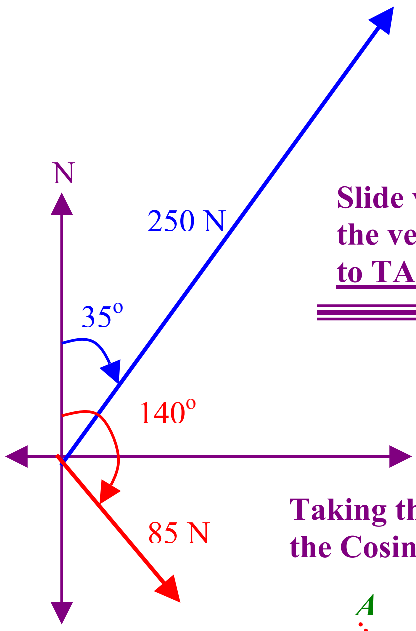
$$\tan\theta = \frac{8}{32}$$

$\theta = 14^{\circ}$

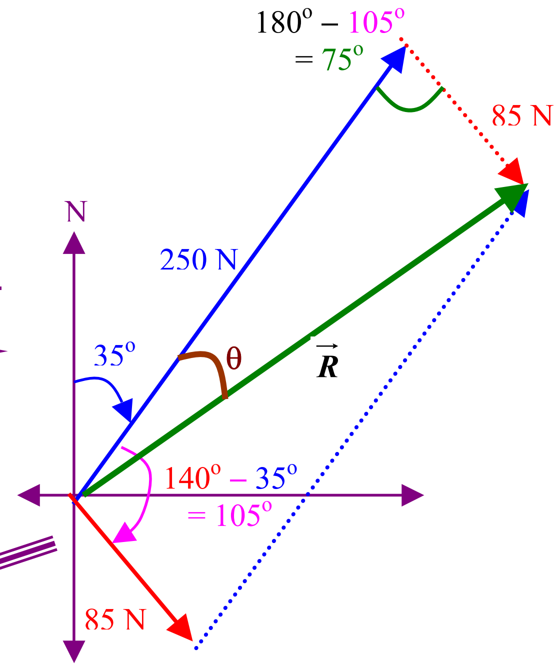
$\vec{R}$ = 33 km/h [N14°W] or 346°

Example 5: 250 N [35°] + 85 N [140°]

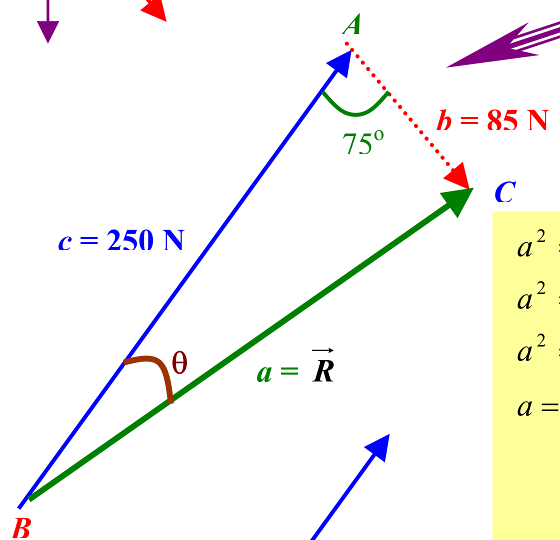
Parallelogram Method



Slide vectors along each other until the vectors are lined up <u>From HEAD to TAIL</u> to form a parallelogram.



Taking the Top Triangle and apply the Cosine Law and Sine Law.

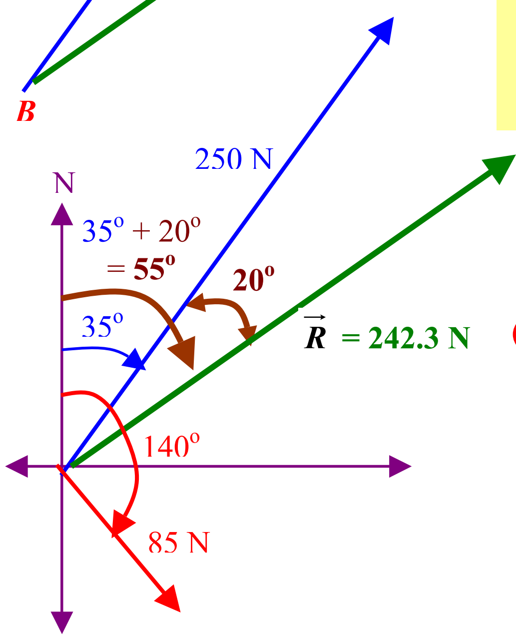


$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$a^2 = 250^2 + 85^2 - 2(250)(85)(\cos 75^\circ)$$
$$a^2 = 58725.19058$$
$$a = \sqrt{58725.19058}$$

$\vec{R}$ = 242.3 N

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 75^\circ}{242.3} = \frac{\sin B}{85}$$
$$\sin B = \frac{85(\sin 75^\circ)}{242.3}$$
$$\sin B = 0.3388514042$$

∠B = θ = 20°



$\vec{R}$ = 242.3 N [N55°E] or [55°]

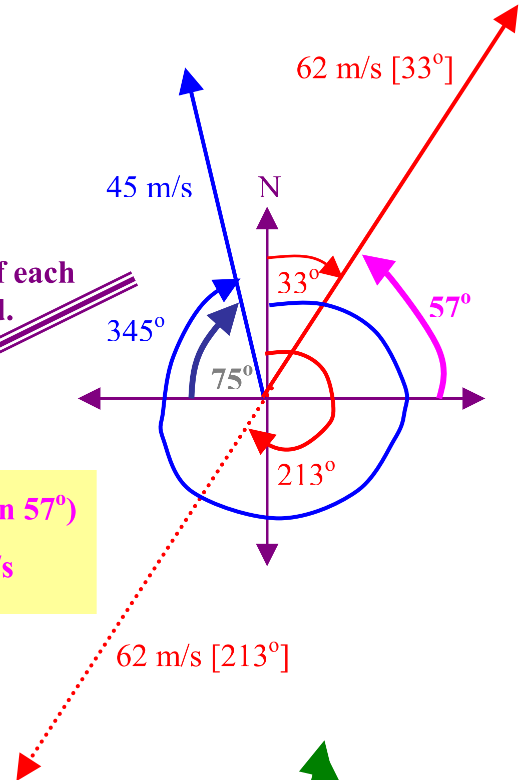
Example 6: 45 m/s [345°] – 62 m/s [213°]

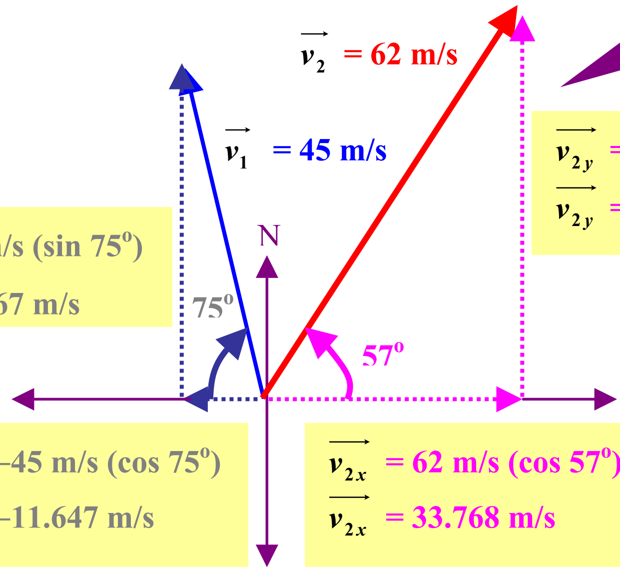
Vector Components Method

45 m/s [345°] – 62 m/s [213°]
= 45 m/s [345°] + (– 62 m/s [213°])
= 45 m/s [345°] + 62 m/s [33°]

213° – 180°

Obtain the Vertical and Horizontal Components of each vector we are about to add.





$\vec{v}_{1y}$ = 45 m/s (sin 75°)
$\vec{v}_{1y}$ = 43.467 m/s

$\vec{v}_{2y}$ = 62 m/s (sin 57°)
$\vec{v}_{2y}$ = 51.998 m/s

$\vec{v}_{1x}$ = –45 m/s (cos 75°)
$\vec{v}_{1x}$ = –11.647 m/s

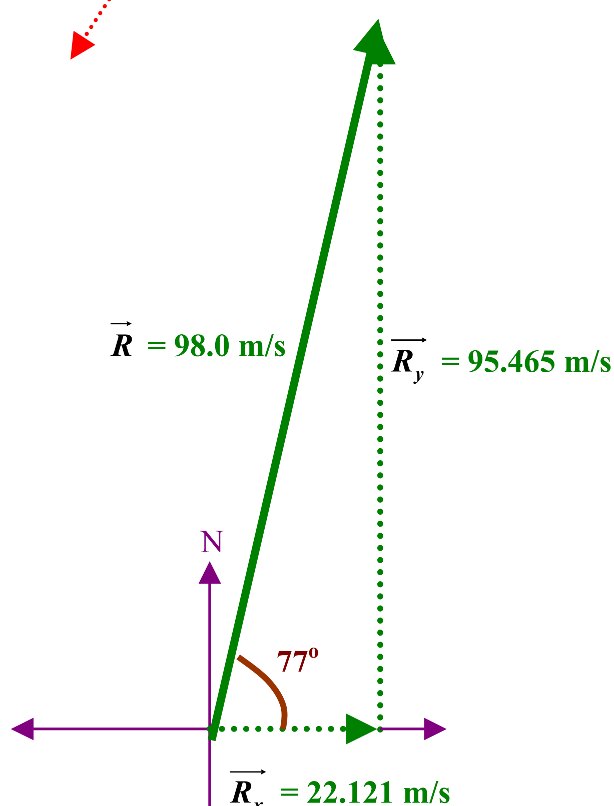
$\vec{v}_{2x}$ = 62 m/s (cos 57°)
$\vec{v}_{2x}$ = 33.768 m/s

$$\vec{R}_x = \vec{v}_{1x} + \vec{v}_{2x} = -11.647 \text{ m/s} + 33.768 \text{ m/s}$$
$$\vec{R}_x = 22.121 \text{ m/s}$$

$$\vec{R}_y = \vec{v}_{1y} + \vec{v}_{2y} = 43.467 \text{ m/s} + 51.998 \text{ m/s}$$
$$\vec{R}_y = 95.465 \text{ m/s}$$

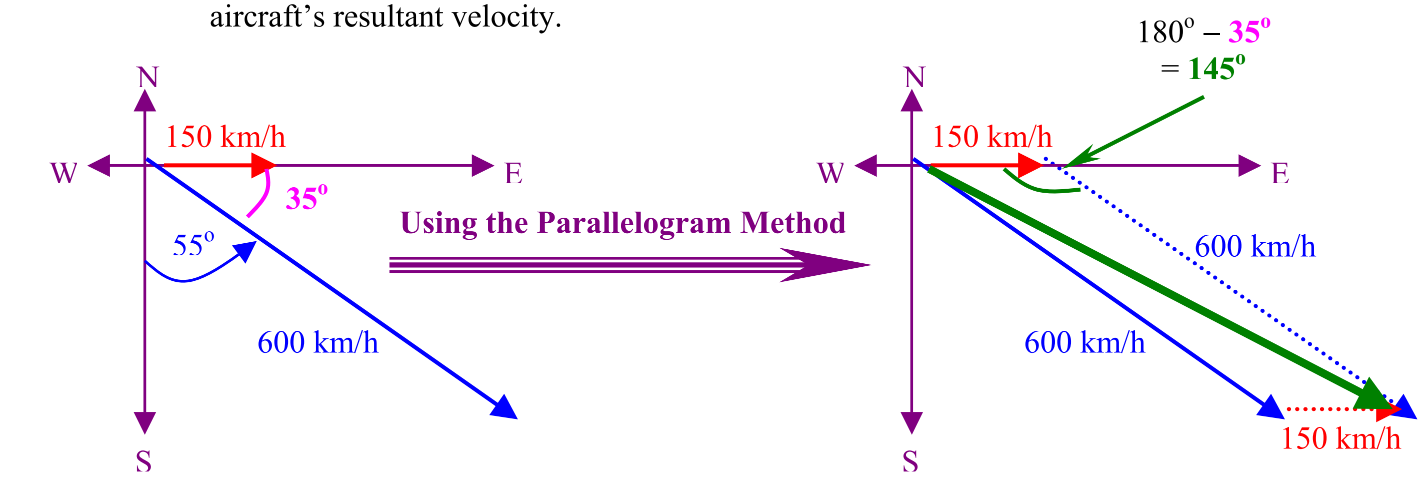
$$|\vec{R}|^2 = |\vec{R}_x|^2 + |\vec{R}_y|^2$$
$$|\vec{R}| = \sqrt{|\vec{R}_x|^2 + |\vec{R}_y|^2}$$
$$|\vec{R}| = \sqrt{22.121^2 + 95.465^2}$$
$$|\vec{R}| = \sqrt{9602.904866}$$
$$|\vec{R}| = 98.0 \text{ m/s}$$

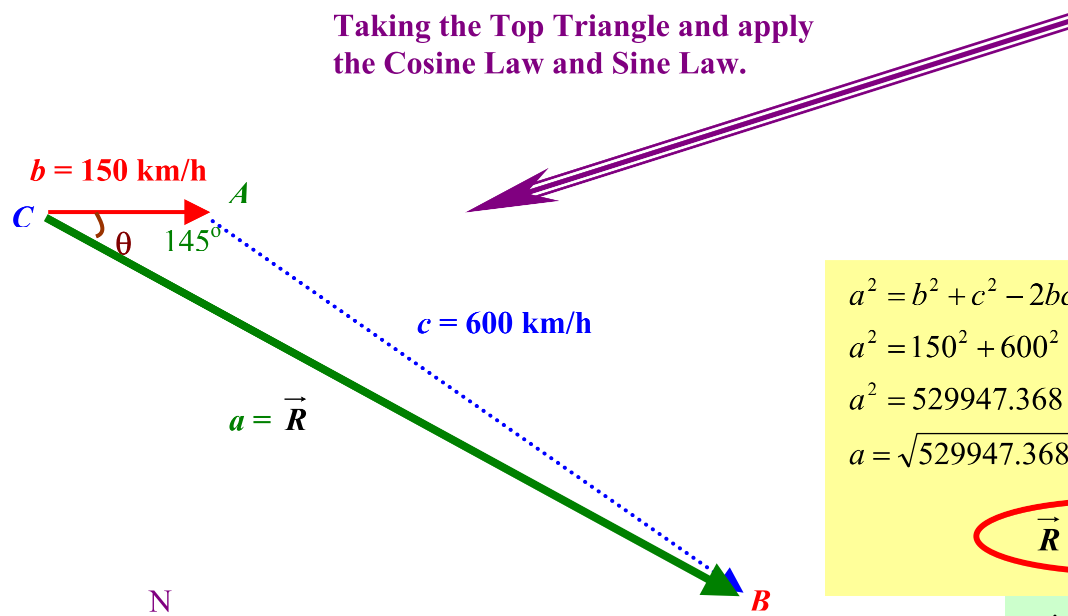
$$\tan\theta = \frac{|\vec{R}_y|}{|\vec{R}_x|}$$
$$\tan\theta = \frac{95.465}{22.121}$$
$$\theta = 77^\circ$$



$\vec{R}$ = 98.0 m/s [E77°N] or [N13°E] or [13°]

Example 7: A plane is flying at 1200 feet with a cruising speed of 600 km/h heading S55°E. If the velocity of the wind is 150 km/h from the west, determine the magnitude and the direction of the aircraft's resultant velocity.



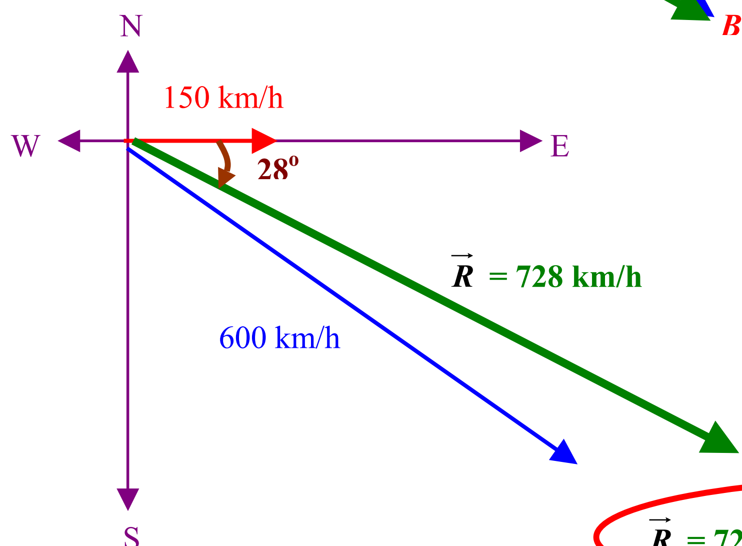


$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$a^2 = 150^2 + 600^2 - 2(150)(600)(\cos 145^\circ)$$
$$a^2 = 529947.368$$
$$a = \sqrt{529947.368}$$

$\vec{R}$ = **728 km/h**

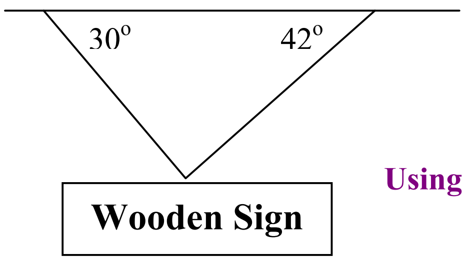
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 145^\circ}{728} = \frac{\sin C}{600}$$
$$\sin C = \frac{600(\sin 145^\circ)}{728}$$
$$\sin C = 0.4727278322$$

∠C = θ = 28°

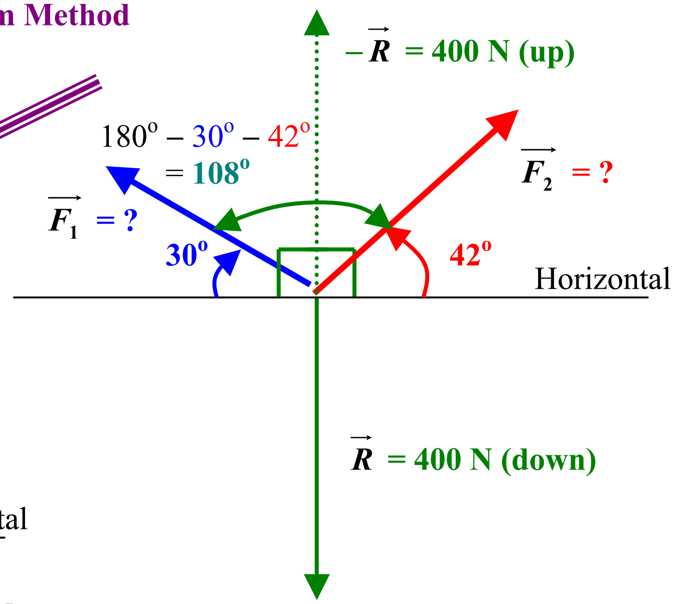


$\vec{R}$ = 728 km/h [118°] or [E28°S] or [S62°E]

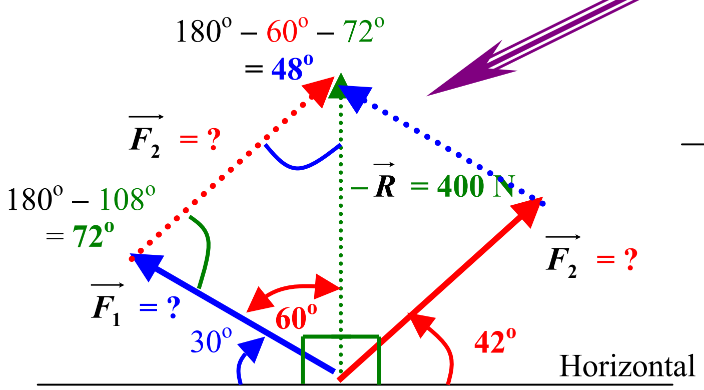
Example 8: A wooden sign with a weight of 400 N is hung from the ledge of a building by two ropes. One rope pulls at 30° to the horizontal from the left, and the other pulls at 42° to the horizontal from the right as illustrated on the diagram below. What are the magnitudes of the tension forces exerted by the two ropes?



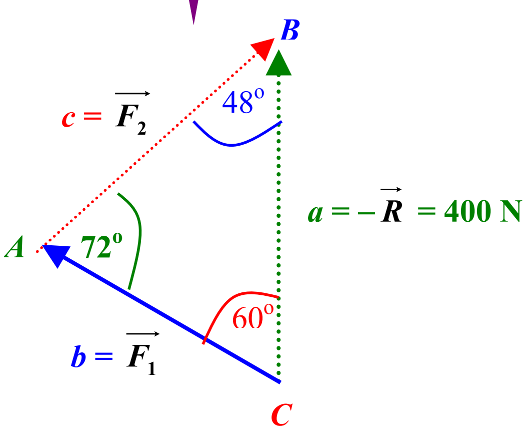
The two ropes exert a combined tension force equal to the opposite vector of the sign's weight.



Using the Parallelogram Method



Taking the Left Triangle and apply the Sine Law.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 72^\circ}{400} = \frac{\sin 48^\circ}{b}$$
$$b = \frac{400(\sin 48^\circ)}{(\sin 72^\circ)}$$
$$b = \vec{F_1} = 312.6 \text{ N}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 72^\circ}{400} = \frac{\sin 60^\circ}{c}$$
$$c = \frac{400(\sin 60^\circ)}{(\sin 72^\circ)}$$
$$c = \vec{F_2} = 364.2 \text{ N}$$

7-4 Assignment: pg. 328 – 329 #1 to 11
REDO: pg. 318 – 319 #1 to 10 (without using scale diagrams)

Vector Project: Vehicle Collision Analysis

Purpose: To reconstruct accident details using the law of conservation of momentum.

Background Information:

Momentum: - a vector quantity which is the product of an object's mass (kg) and velocity (m/s or km/h)

Momentum = Mass × Velocity

$$\vec{p} = m\vec{v}$$

Example: Calculate the momentum of a 500 kg car is moving N20°W at 60 km/h.

$$\vec{p} = m\vec{v} = (500\text{ kg})(60\text{ km/h})$$

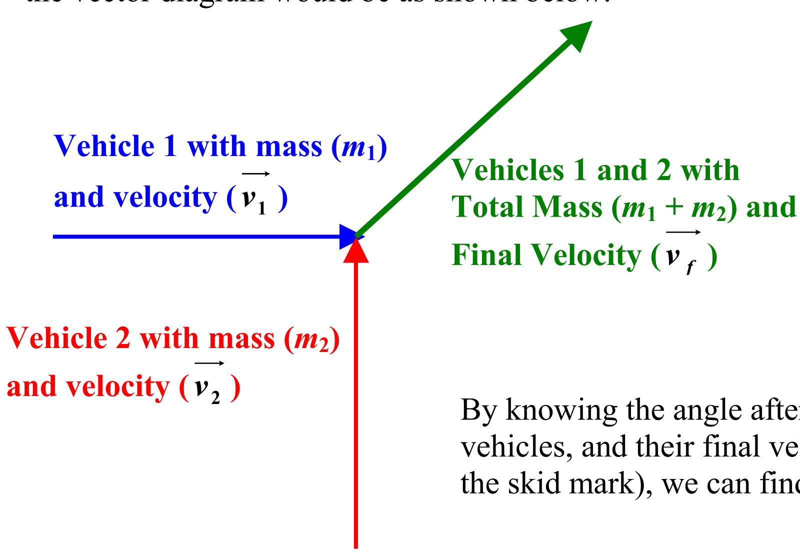
$$\vec{p} = 30000\text{ kg}\bullet\text{km/h [N20°W]}$$

Conservation of Momentum: - when objects collide, the total momentum before the collision is the same as the total momentum after the collision.

Total Initial Momentum = Total Final Momentum

$$\vec{p_i} = \vec{p_f}$$

If two vehicles collided at a right angle, and the vehicles got **STUCK TOGETHER After the Collision**, the vector diagram would be as shown below.

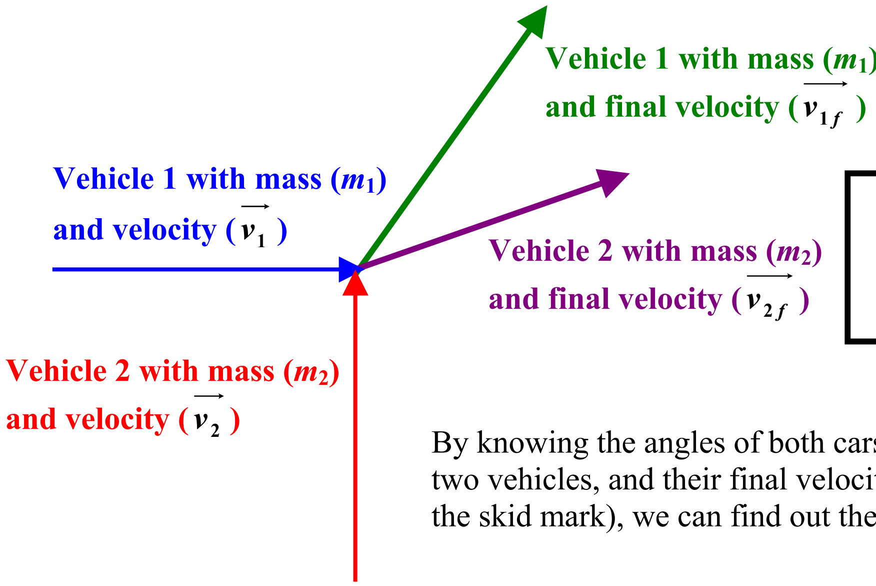


$$\vec{p_i} = \vec{p_f}$$

$$m_1\vec{v_1} + m_2\vec{v_2} = (m_1 + m_2)\,\vec{v_f}$$

By knowing the angle after the collision, the masses of the two vehicles, and their final velocity (usually by measuring the length of the skid mark), we can find out their velocities before the collision.

If two vehicles collided at a right angle, and the vehicles **REMAIN SEPARATED After the Collision**, the vector diagram would be as shown below.



$$\vec{p_i} = \vec{p_f}$$
$$m_1\vec{v_1} + m_2\vec{v_2} = m_1\vec{v_{1f}} + m_2\vec{v_{2f}}$$

By knowing the angles of both cars after the collision, the masses of the two vehicles, and their final velocities (usually by measuring the length of the skid mark), we can find out their velocities before the collision.

Procedure:

Part A: Collision of Two Vehicles that got STUCK TOGETHER Afterwards

1. Two vehicles collided at an uncontrolled intersection. Vehicle A was traveling eastbound and Vehicle B was traveling northbound. No one knew how fast they were going but both vehicles got stuck together traveling at N (some angle) E direction after the collision. Eventually they stopped and left a skid mark.

2. Copy and fill out the following table. Pick numbers for the values from the brackets below.

Mass of Vehicle A (500 kg to 1000 kg)	
Mass of Vehicle B (500 kg to 1000 kg)	
Angle both vehicles made after the collision (N15°E to N75°E)	
Velocity of both vehicles after the collision (40 km/h to 130 km/h)	

3. Draw a vector diagram with the appropriate vectors and proper labeling. The diagram should be drawn to scale and the scale factor labeled.

4. Find the velocities of both vehicles just before the collision using:
a. your scale drawing.
b. the algebraic method.

Part B: Collision of Two Vehicles that REMAIN SEPARATED Afterwards

1. Two vehicles collided at an uncontrolled intersection. Vehicle C was traveling westbound and Vehicle D was traveling southbound. No one knew how fast they were going. After the collision, both vehicles were traveling at different S (some angle) W directions after the collision. Eventually they stopped and left two different skid marks.

2. Copy and fill out the following table. Pick numbers for the values from the brackets below.

Mass of Vehicle C (500 kg to 1000 kg)	
Mass of Vehicle D (500 kg to 1000 kg)	
Angle Vehicle C made after the collision (S10°W to S44°W)	
Angle Vehicle D made after the collision (S46°W to S80°W)	
Velocity of Vehicle C after the collision (40 km/h to 130 km/h)	
Velocity of Vehicle D after the collision (40 km/h to 130 km/h)	

3. Draw a vector diagram with the appropriate vectors and proper labeling. The diagram should be drawn to scale and the scale factor labeled. (You might have to draw more than one diagram for this part.)

4. Find the velocities of both vehicles just before the collision using:
   a. your scale drawing.
   b. the algebraic method (either by parallelogram method or vector components method).

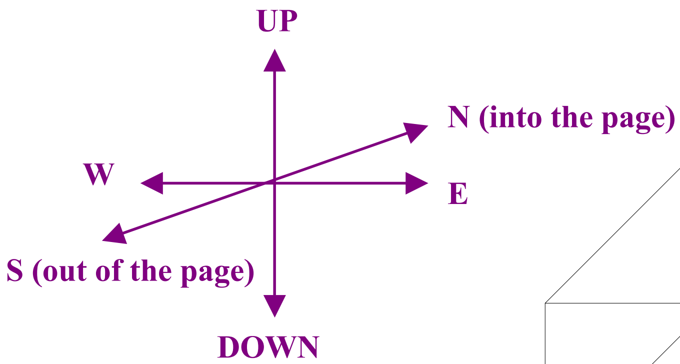
Note:

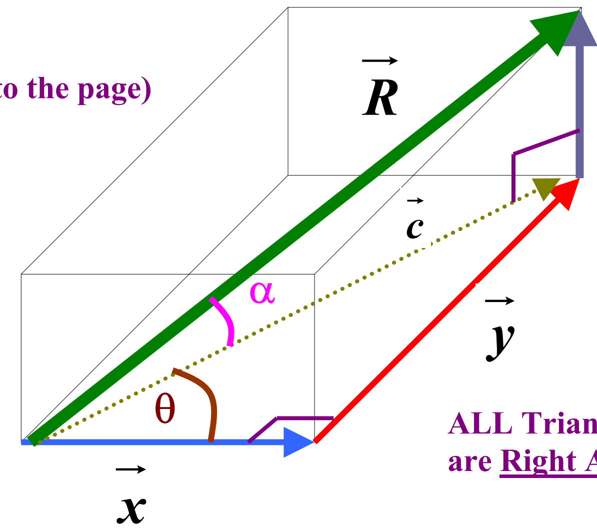
1. All diagrams drawn must be to scale and properly label.
2. All work and calculations must be shown clearly.
3. Students can work together to discuss the project, but they should each have their own numbers to work with and do their own calculations. Students who copied from each other will end up sharing the mark. Let's say the mark was 70% and two students were involved in copying each other's work. They each get 35%.
4. Late Project handed in one day after the due date is counted as 30% off the total mark. Project handed in two days and later will not be marked.

Due Date: ____________________

7-5: Vector Problems in 3-Dimensions

When answering vector problems in 3-dimensions, it is helpful to make a box to show all the angles involved.





$\vec{c} = \vec{x} + \vec{y}$

$\vec{R} = \vec{x} + \vec{y} + \vec{z}$

$\vec{R}$ has the direction of East (θ) North and α as the **Angle of Elevation**.

ALL Triangles in the box are **Right Angle Triangles**!

The diagonal of the box ($\vec{R}$) can be found by the 3-Dimensional Pythagorean Theorem, $|\vec{R}|^2 = |\vec{x}|^2 + |\vec{y}|^2 + |\vec{z}|^2$

$$|\vec{R}| = \sqrt{|\vec{x}|^2 + |\vec{y}|^2 + |\vec{z}|^2}$$

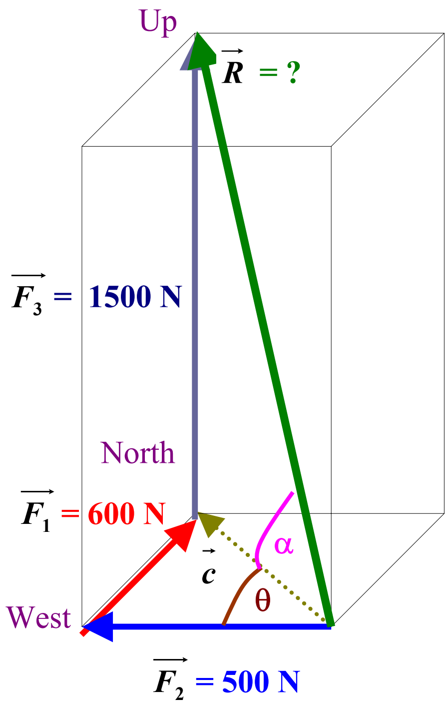
OR

$$|\vec{c}| = \sqrt{|\vec{x}|^2 + |\vec{y}|^2} \quad \text{and} \quad |\vec{R}| = \sqrt{|\vec{c}|^2 + |\vec{z}|^2}$$

The angles θ and α can be determined by using the **Tangent Ratio**.

$$\tan\theta = \frac{|\vec{y}|}{|\vec{x}|} \qquad \sin\alpha = \frac{|\vec{z}|}{|\vec{R}|}$$

Example 1: A crane is lifting up a wall with a force of 1500 N. One worker pulls it with 600N of force towards the north and another worker pulls it 500 N towards the west. What is the resulatnt force on the wall?



$$|\vec{R}| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2 + |\vec{F}_3|^2}$$

$$|\vec{R}| = \sqrt{600^2 + 500^2 + 1500^2}$$

$$|\vec{R}| = \sqrt{2860000}$$

$$|\vec{R}| = 1691.2 \text{ N}$$

$$\tan\theta = \frac{|\vec{F}_1|}{|\vec{F}_2|} = \frac{600 \text{ N}}{500 \text{ N}}$$

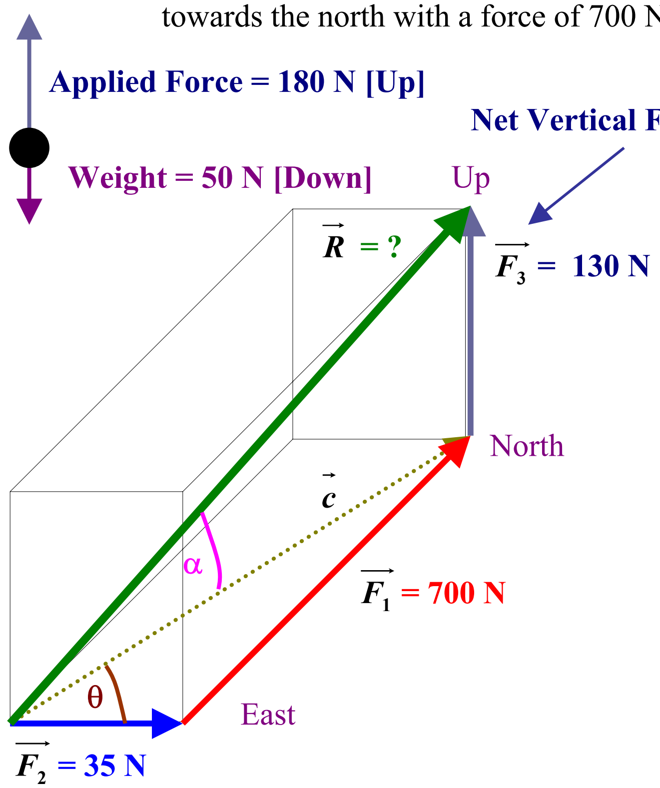
$$\theta = 50^\circ$$

$$\sin\alpha = \frac{|\vec{F}_3|}{|\vec{R}|} = \frac{1500 \text{ N}}{1691.2 \text{ N}}$$

$$\alpha = 62.5^\circ$$

$\vec{R}$ = 1691.2 N [W50°N] with an Angle of Elevation of 62.5°

Example 2: A tennis ball has a weight of 50 N and is thrown up during a serve with an applied force of 180 N. If the wind exerts a force of 35 N towards east, and the player's racket hit the ball towards the north with a force of 700 N, what is the resulatant force on the ball?



$$|\vec{R}| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2 + |\vec{F}_3|^2} = \sqrt{700^2 + 35^2 + 130^2}$$

$$|\vec{R}| = \sqrt{508125}$$

$$|\vec{R}| = 712.8 \text{ N}$$

$$\tan\theta = \frac{|\vec{F}_1|}{|\vec{F}_2|} = \frac{700 \text{ N}}{35 \text{ N}}$$

$$\theta = 87^\circ$$

$$\sin\alpha = \frac{|\vec{F}_3|}{|\vec{R}|} = \frac{130 \text{ N}}{712.8 \text{ N}}$$

$$\alpha = 10.5^\circ$$

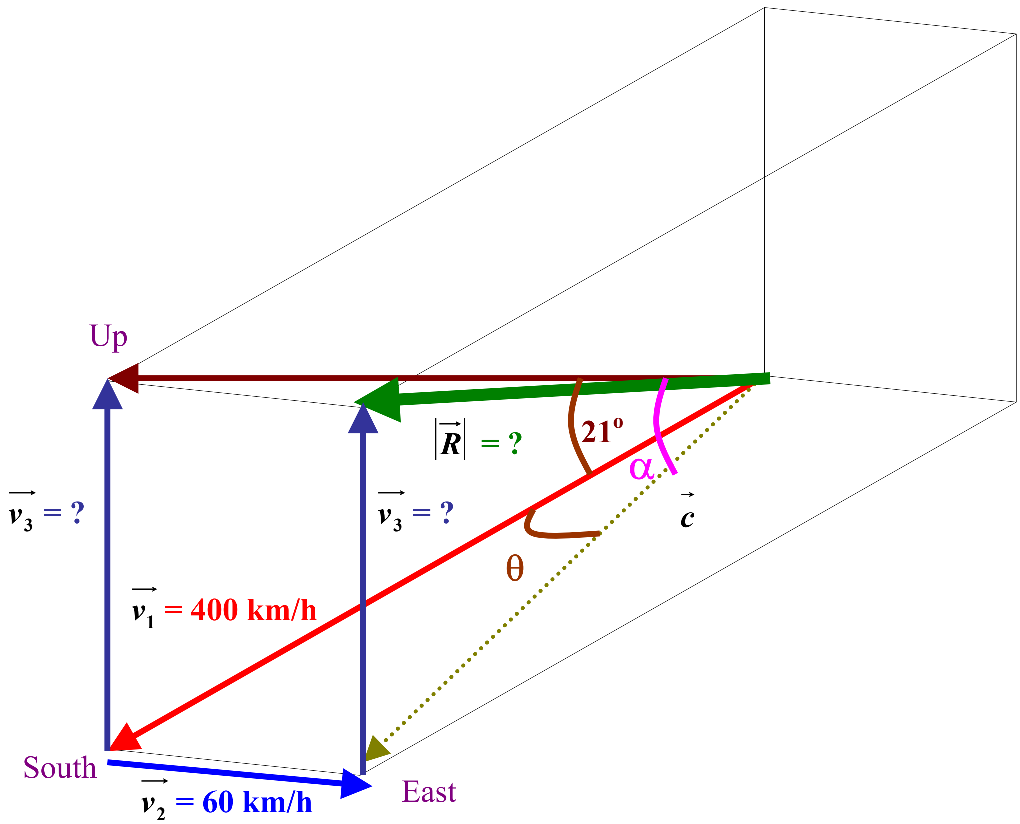
$\vec{R}$ = 712.8 N [E87°N] with an Angle of Elevation of 10.5°

Example 3: A plane is taking off with a speed of 400 km/h towards the south and climbs at an angle of 21° with the horizontal. A steady wind is blowing from the west at 60 km/h. Determine the speed of the plane relative to the ground, and the angles the plane's track make with the horizontal and the ground.

$|\vec{R}|$ = **Resultant Speed (relative to the ground)**

α = Angle the Track of the Plane makes with the Ground

θ = Angle the Track of the Plane makes with the Horizontal



$$\tan 21° = \frac{|\vec{v}_3|}{|\vec{v}_1|}$$

$$\tan 21° = \frac{|\vec{v}_3|}{400 \text{ km/h}}$$

$$|\vec{v}_3| = (400)\tan 21°$$

$$|\vec{v}_3| = 153.546 \text{ km/h}$$

$$|\vec{R}| = \sqrt{|\vec{v}_1|^2 + |\vec{v}_2|^2 + |\vec{v}_3|^2} = \sqrt{400^2 + 60^2 + 153.546^2}$$

$$|\vec{R}| = \sqrt{187176.2556}$$

$$|\vec{R}| = 432.6 \text{ km/h}$$

$$\tan\theta = \frac{|\vec{v}_2|}{|\vec{v}_1|} = \frac{60 \text{ km/h}}{400 \text{ km/h}}$$

$$\theta = 8.5°$$

$$\sin\alpha = \frac{|\vec{v}_3|}{|\vec{R}|}$$

$$\tan\alpha = \frac{153.546 \text{ km/h}}{432.6 \text{ km/h}}$$

$$\alpha = 20.8°$$

7-5 Assignment: **pg. 336 – 337 #1 to 8**