Unit 2: Statistics

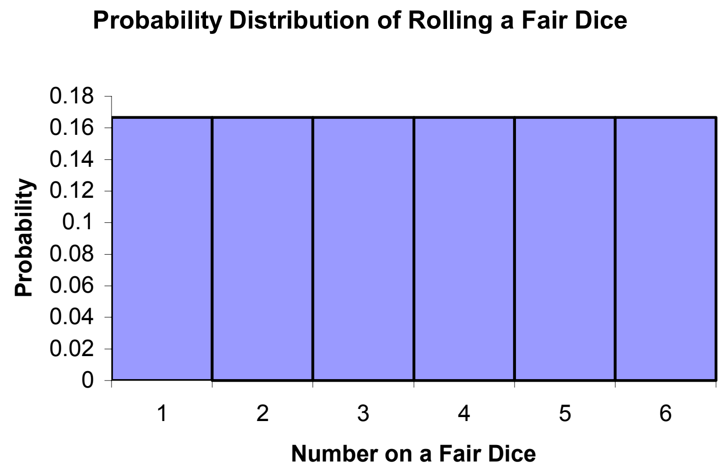
3-1: Distributions

Probability Distribution: - a table or a graph that displays the theoretical probability for each outcome of an experiment.
- *P* (any particular outcome) is between 0 and 1
- the sum of all the probabilities is always 1.

a. **Uniform Probability Distribution**: - a probability distribution where the probability of one outcome is the same as all the others.

Example 1: Rolling a fair dice

$$P\text{ (any particular from 1 to 6)} = \frac{1}{6} = 0.167$$



b. **Binomial Probability Distribution**: - a probability distribution from a binomial experiment (an experiment where there are only two results – favourable and non-favourable).

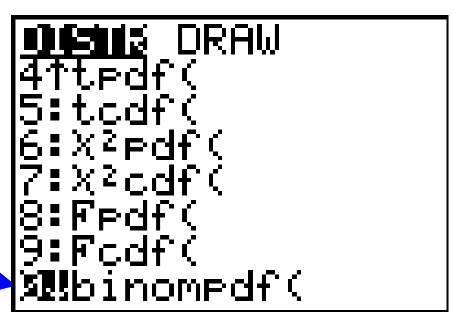
binompdf (Binomial Probability Distribution): displays a binomial probability distribution when the number of trials and the theoretical probability of the favourable outcome are specified.

binompdf (Number of Trials, Theoretical Probability of favorable outcome)

To access binompdf:

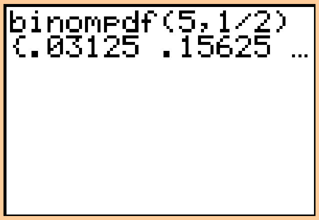
1. Press **2nd** **DISTR VARS**

2. Select Option 0



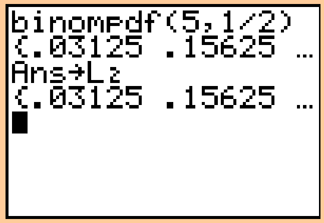
Example 2: Using your graphing calculator, determine the probabilities of having any number of girls in a family of 5 children.

1. **binompdf (5, ½)**



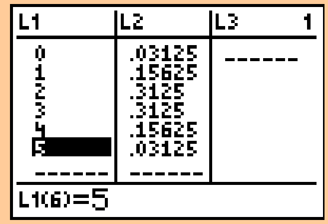
2. Store answer in L_2 of the STAT Editor.

STO➡ **2nd** **L2 2**



3. Enter 0 to 5 in L_1 of the STAT Editor.

STAT **ENTER**



4. WINDOW Settings

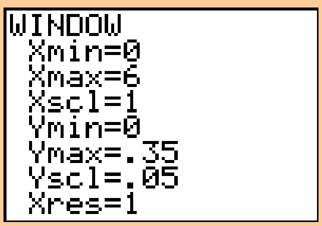
x: $[x_{min}, x_{max}, x_{scl}] = x$: [0, 6, 1]

y: $[y_{min}, y_{max}, y_{scl}] = y$: [0, 0.35, 0.05]

WINDOW

Select a number slightly higher than the maximum in L_2.

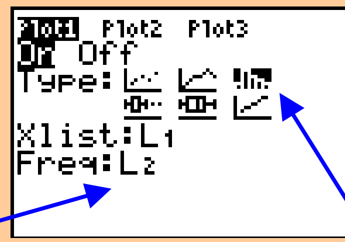
Must be 1 more than the number of trials.

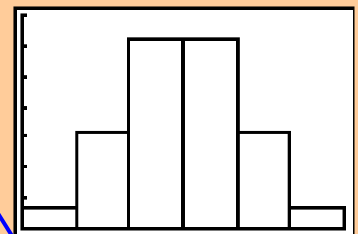


5. Select Histogram in STAT PLOT and graph.

2nd **STAT PLOT Y=** **ENTER** **GRAPH**

Plot1 is ON.

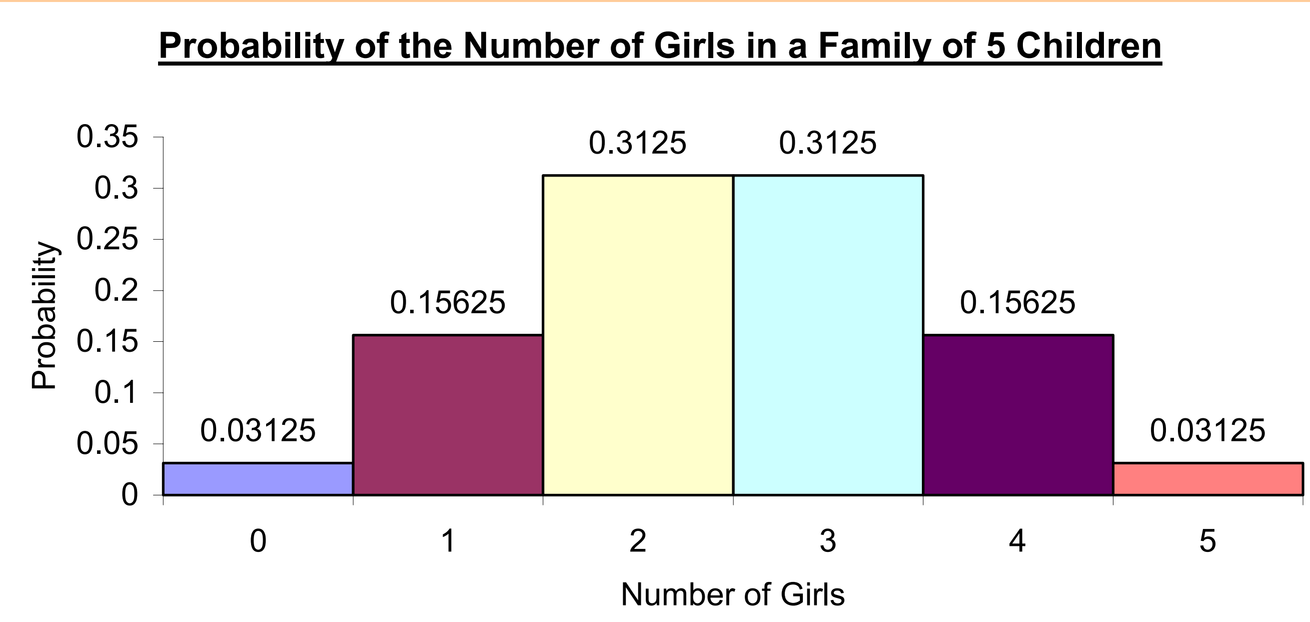




Type in L_2 as Frequency by pressing: **2nd** **L2 2**

Select Histogram

6. Transfer the graph on the calculator to paper.

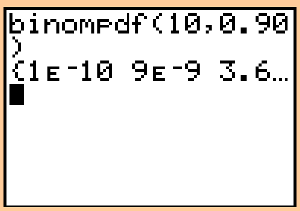


Example 3: The first week of February marks the tradition of Groundhog Day. If the groundhog sees its own shadow, it means 6 more weeks of winter. Otherwise, spring is just around the corner. Recent statistics has shown that the groundhog sees its shadow 90% of the time on Groundhog Day.

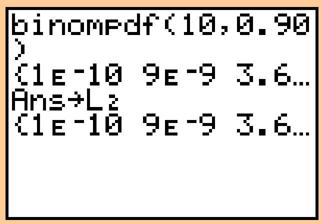
a. Graph a binomial distribution to illustrate the probability that the groundhog will see its shadow for the next ten years.
b. Find the probability that the groundhog will see its shadow 9 time out of the ten years.
c. Calculate the probability that the groundhog will see its shadow at least 6 times out of the next 10 years.
d. Determine the probability that "spring is just around the corner" at least 8 years out of the next ten years.

a. Graph Binomial Distribution

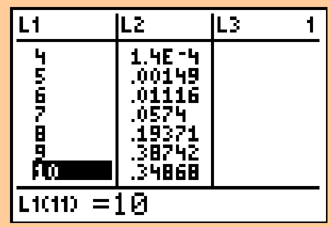
1. **binompdf (10, 0.90)** 2. Store answer in L_2 of the STAT Editor. 3. Enter 0 to 10 in L_1 of the STAT Editor.



STO➔ **2nd** **L2 2**



STAT **ENTER**



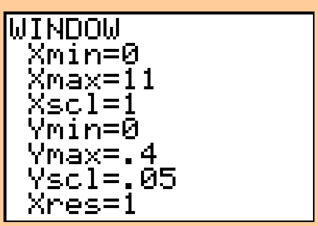
4. WINDOW Settings

x: $[x_{min}, x_{max}, x_{scl}] = x$: [0, 11, 1]

y: $[y_{min}, y_{max}, y_{scl}] = y$: [0, 0.4, 0.05]

WINDOW

Select a number slightly higher than the maximum in L_2.

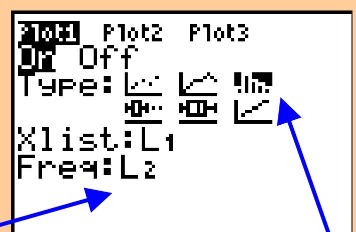


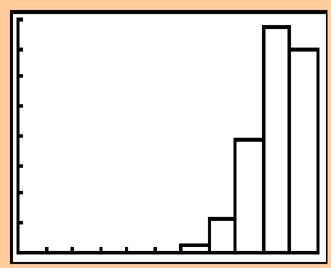
Must be 1 more than the number of trials.

5. Select Histogram in STAT PLOT and graph.

2nd **STAT PLOT Y=** **ENTER** **GRAPH**

Plot1 is ON.

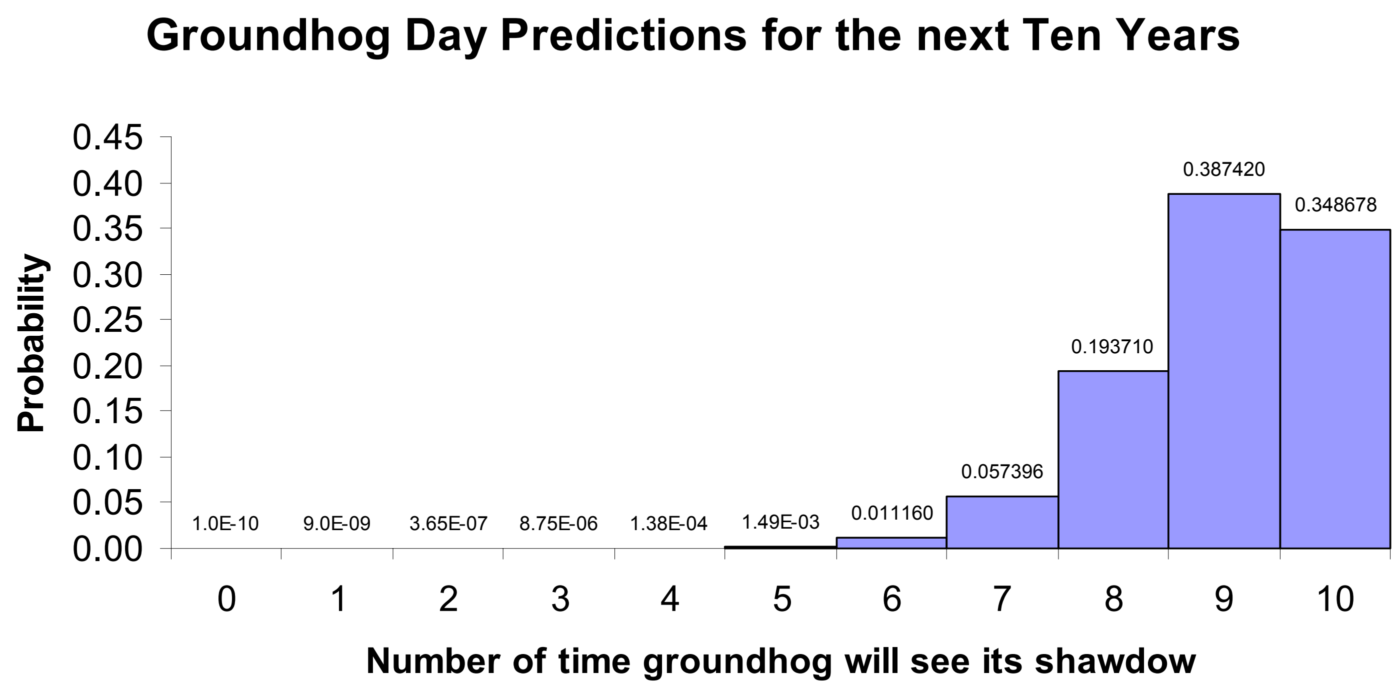




Type in L_2 as Frequency by pressing: **2nd** **L2 2**

Select Histogram

6. Transfer the graph on the calculator to paper.



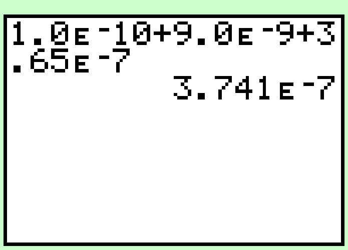
b. ***P* (Sees Shadow 9 times out of 10) = 0.387420 ≈ 38.7%** (read from Table or TRACE on Graph)

c. *P* (Shadow at least 6 times) = *P* (6 times) + *P* (7 times) + *P* (8 times) + *P* (9 times) + *P* (10 times)
= 0.011160 + 0.057396 + 0.193710 + 0.387420 + 0.348678

***P* (Sees Shadow at least 6 times) = 0.998364 ≈ 99.8%**

d. *P* (No Shadow at least 8 times) = *P* (Shadow at most 2 times) = *P* (0 time) + *P* (1 time) + *P* (2 times)
$= (1.0 \times 10^{-10}) + (9.0 \times 10^{-9}) + (3.65 \times 10^{-7})$

(To enter scientific notation, press **2nd** **EE ,**)



***P* (No Shadow at least 8 times) = 3.741×10^{-7} ≈ 0.000 0374%**

<u>3-1 Assignment</u>: pg. 102 – 104 #1 to 7

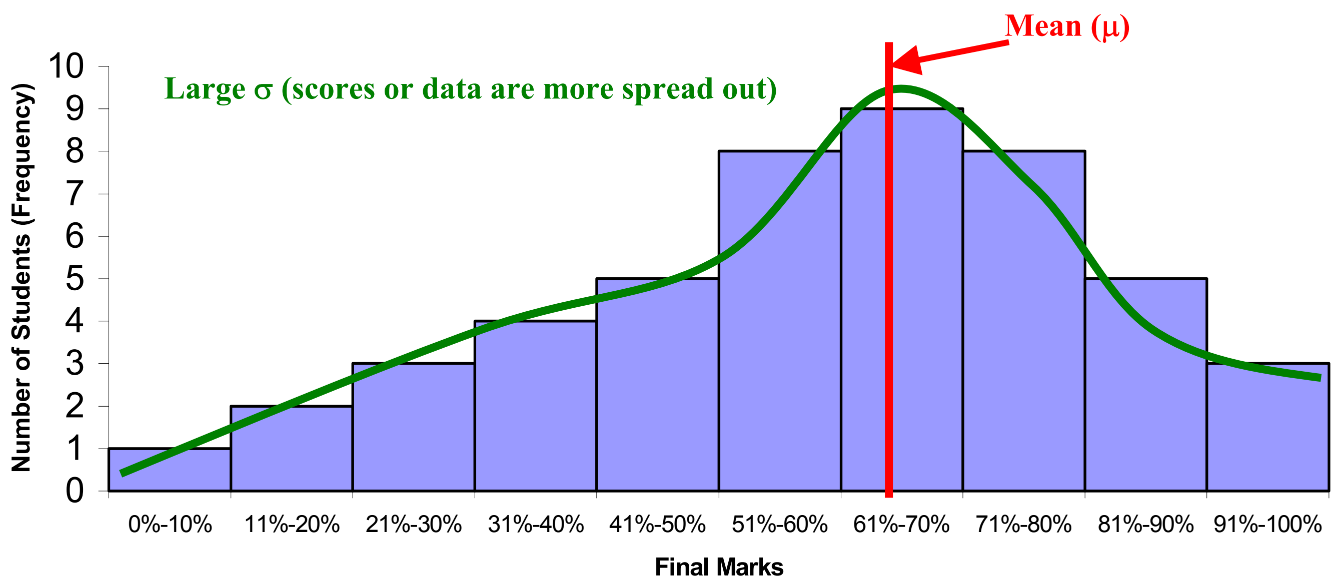
3-2: Mean And Standard Deviation

Mean (μ or $\overline{X}$): - the average of a given set of data.

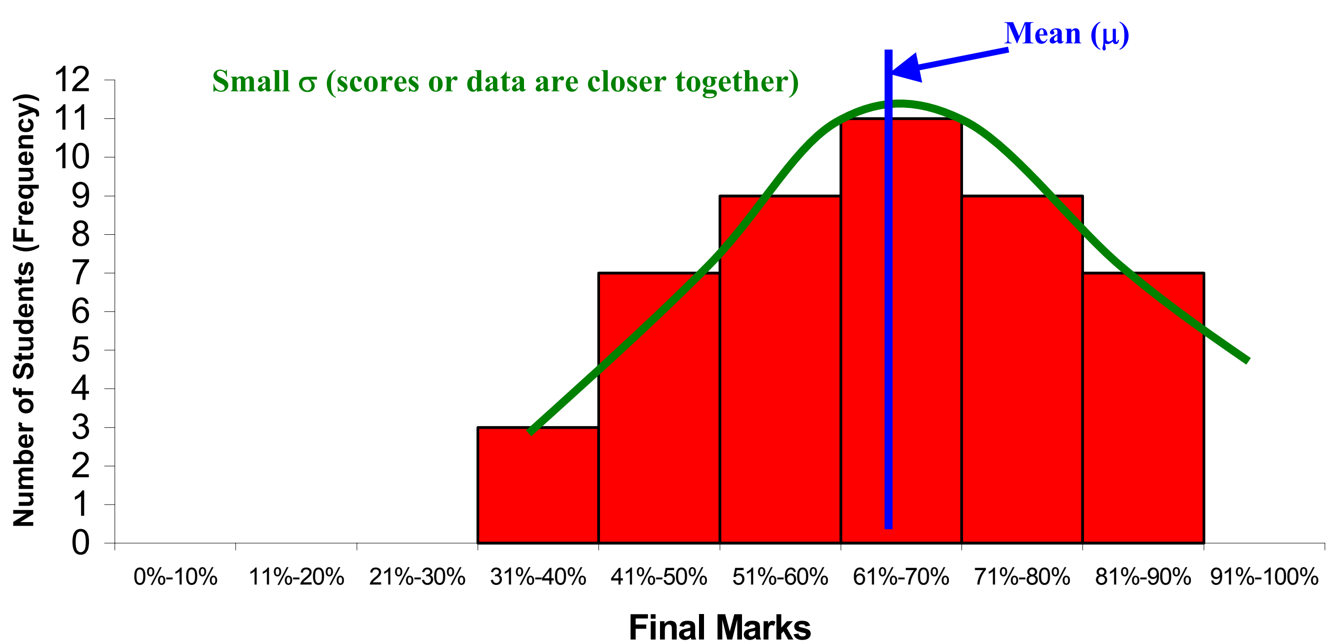
Standard Deviation (σ): - the measure of how far apart are the data spread out from the mean.

Frequency Distribution: - a Histogram (bar graphs with no gaps) OR a Curve showing the frequency of occurrence over the range of values of a data set.

Example of a Large Standard Deviation



Example of a Small Standard Deviation



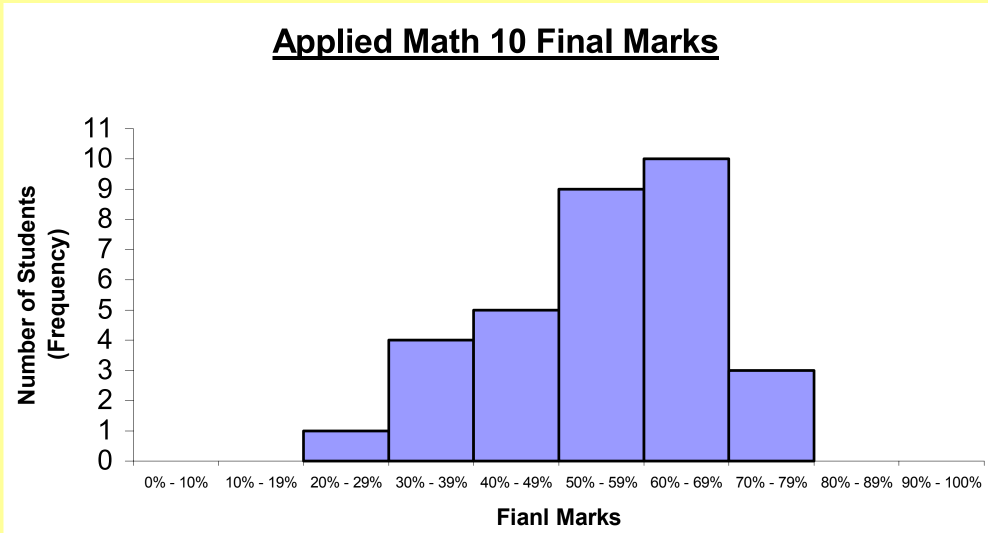
Example 1: The following sets of data are the final marks of an Applied Math 10 class.

56, 32, 50, 29, 60, 45, 43, 50, 34, 63, 72, 67, 70, 50, 68, 42,
65, 50, 50, 65, 34, 60, 45, 61, 65, 45, 60, 55, 50, 32, 77, 59

a. Organize the data into a frequency table below and create a histogram of frequency distribution.
b. Using a graphing calculator, determine the mean and standard deviation of the set of scores above.

a. Frequency Table

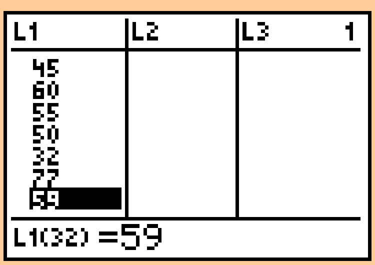
Final Marks	Frequency
90% - 100%	0
80% - 89%	0
70% - 79%	3
60% - 69%	10
50% - 59%	9
40% - 49%	5
30% - 39%	4
20% - 29%	1
10% - 19%	0
0% - 10%	0



b. To find the Mean and Standard Deviation using a Graphing Calculator:

1. Enter the set of data into L_1 of the STAT Editor.

STAT **ENTER**



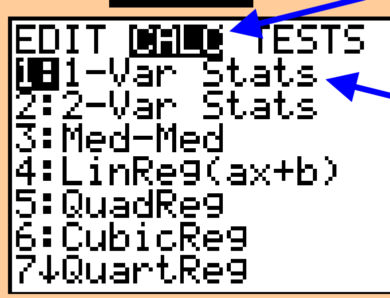
2nd **QUIT** **MODE** after entering the last score.

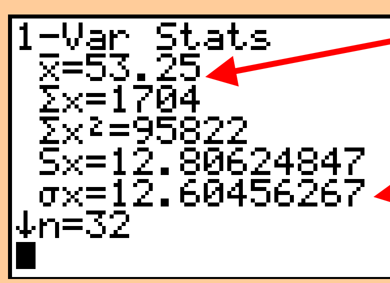
2. To access the "1-Variable Stats" function.

1. Press **STAT**

2. Use  **to access CALC**

3. Press **ENTER** **for Option 1**





$\overline{X}$ = Mean

σx = Standard Deviation

3. Write down the Mean and the Standard Deviation.

$\mu = 53.25$ $\sigma = 12.6$

When the **frequency distribution involves a binomial probability**, we can use the following formulas to estimate the mean and standard deviation.

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}$$

where n = number of trials and p = probability of favourable outcome.

Example 2: Find the mean and standard deviation of the number of male students in a class of 35. Graph the binomial distribution.

$n = 35$ $p = 0.5$ (probability of a male student from any student)

$$\mu = np = 35(0.5)$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{(35)(0.5)(1-0.5)} = \sqrt{35 \times 0.5 \times 0.5}$$

$\mu = 17.5$ $\sigma = 2.96$

To Graph the Binomial Distribution:

1. **binompdf (35, 0.5)**

```
binompdf(35,.5)
{2.910383046E-1…
```

2. Store answer in L_2 of the STAT Editor.

STO➔ 2nd L2 2

```
binompdf(35,.5)
{2.910383046E-1…
Ans→L2
{2.910383046E-1…
```

3. Enter 0 to 35 in L_1 of the STAT Editor.

STAT ENTER

L1	L2	L3
29	4.7E-5	
30	9.4E-6	
31	1.5E-6	
32	1.9E-7	
33	1.7E-8	
34	1E-9	
35	3E-11	

L1(36) =35

4. WINDOW Settings

x: $[x_{min}, x_{max}, x_{scl}] = x$: [0, 36, 1]

y: $[y_{min}, y_{max}, y_{scl}] = y$: [0, 0.14, 0.01]

Select a number slightly higher than the maximum in L_2.

WINDOW

```
WINDOW
 Xmin=0
 Xmax=36
 Xscl=1
 Ymin=0
 Ymax=.14
 Yscl=.01
 Xres=1
```

Must be 1 more than the number of trials.

5. Select Histogram in STAT PLOT and graph.

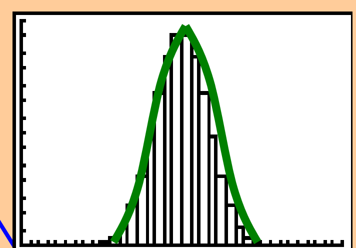
2nd STAT PLOT Y= ENTER GRAPH

Plot1 is ON.

```
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L1
Freq:L2
```

Type in L_2 as Frequency by pressing: 2nd L2 2

Select Histogram



3-2 Assignment: **pg. 108 – 110 #1 to 6, 8 to 10**

3-3: The Normal Distribution

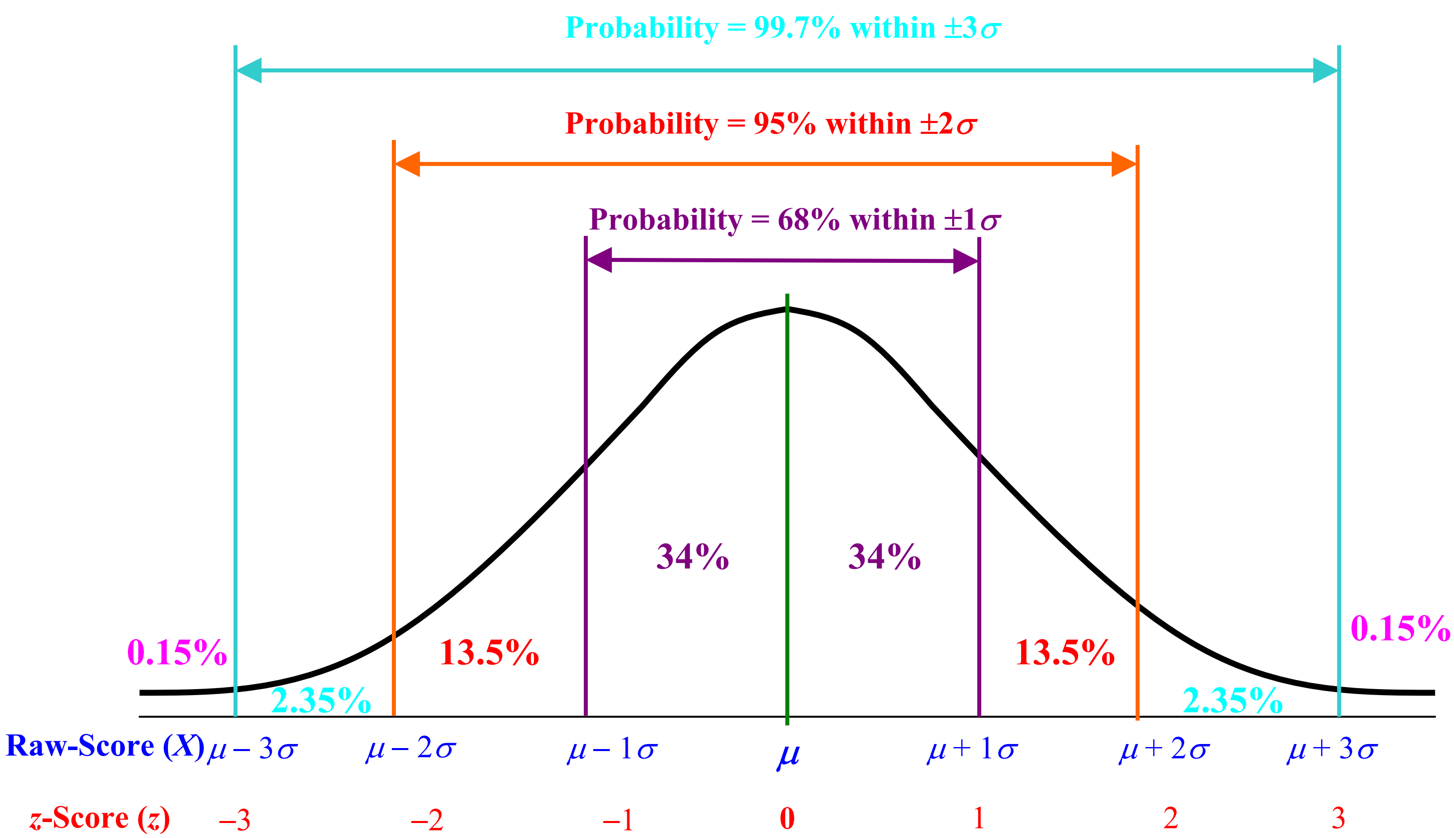
Raw-Scores (X): - the scores as they appear on the original data list.

z-score (z): - the number of standard deviation a particular score is away from the mean in a normal distribution.

Normal Distribution (Bell Curve): a probability distribution that has been normalized for standard use and exhibits the following characteristics.

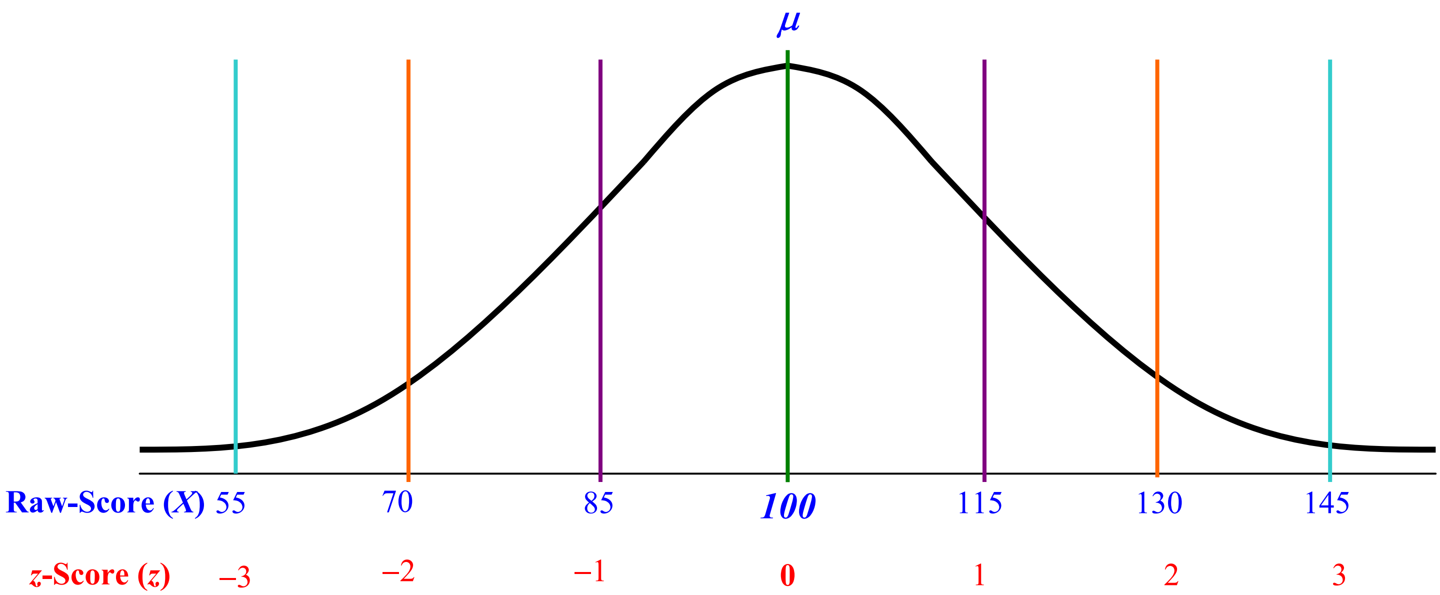
a. The distribution has a mean (μ) and a standard deviation (σ).
b. The curve is symmetrical about the mean.
c. Most of the data is within ±3 standard deviation of the mean.
d. The area under the curve represents probability. The total area under the entire curve is 1 or 100%.
e. The probability under the curve follows the 68-95-99 Rule.
f. The curve gets really close to the x-axis, but never touches it.

The 68-95-99 Rule of the Area under the Normal Distribution Curve

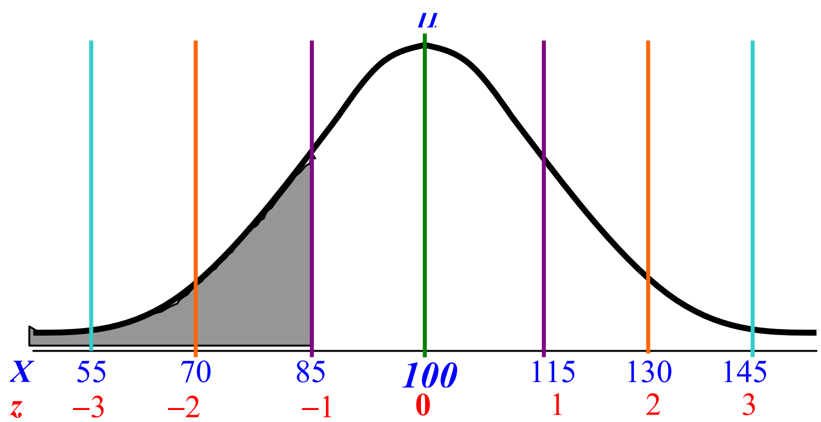


Example 1: The standard IQ test has a mean of 100 and a standard deviation of 15.

a. Draw the normal distribution curve for the standard IQ test.



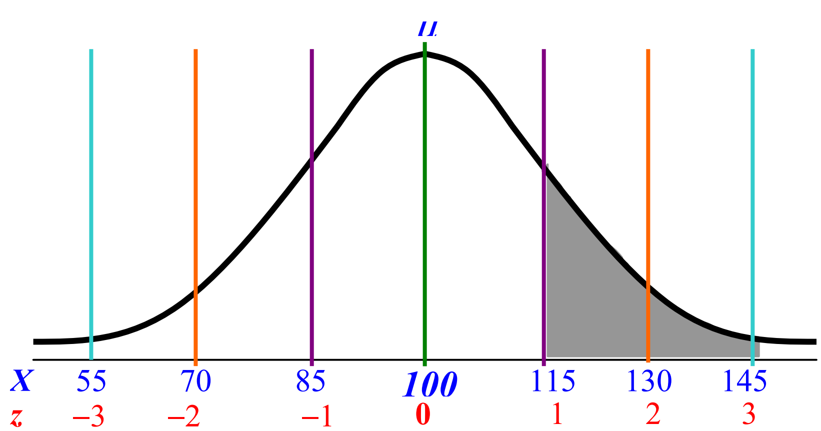
b. What is the probability that a randomly selected person will have a IQ score of 85 and below?



$P(X \le 85) = P(z \le -1)$
$= 13.5\% + 2.35\% + 0.15\%$
$= 16\%$

$\mathbf{P(X \le 85) = 0.16}$

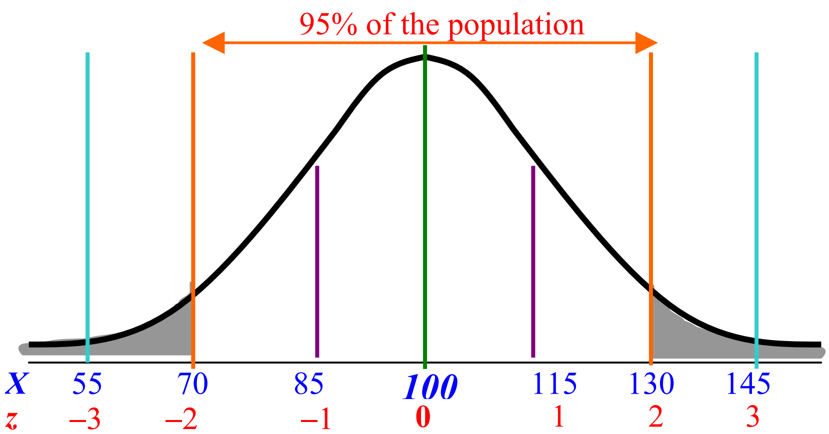
c. What is the probability that a randomly selected person will have a IQ score between 115 to 145?



$P(115 \le X \le 145) = P(1 \le z \le 3)$
$= 13.5\% + 2.35\%$
$= 15.85\%$

$\mathbf{P(115 \le X \le 145) = 0.1585}$

d. Find the percentage of the population who has an IQ test score outside of the 2 standard deviations of the mean. Determine the range of the IQ test scores.



$P(z \le -2 \text{ and } z \ge 2) = 100\% - 95\%$

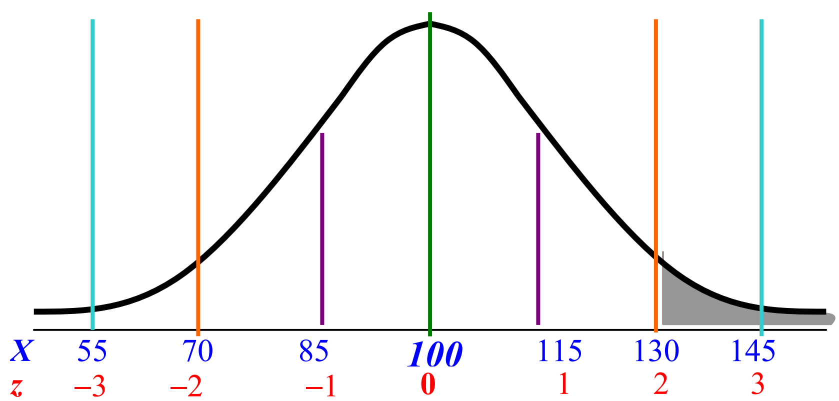
$\mathbf{P(z \le -2 \text{ and } z \ge 2) = 5\%}$

From the bell curve, we can see that the ranges are

$P(z \le -2 \text{ and } z \ge 2) = P(X \le 70 \text{ and } X \ge 130)$

The ranges of IQ test scores are $X \le 70$ and $X \ge 130$

e. In a school of 1500 students, how many students should have an IQ test score above 130?



$P(X > 130) = P(z > 2)$
$= 2.35\% + 0.15\%$
$P(X > 130) = 2.5\%$

Number of students with IQ score > 130 = Total × Probability
= 1500 × 0.025
= 37.5

Number of students with IQ score > 130 = 38 students

<u>3-3 Assignment</u>: pg. 116 – 118 #1 to 10

3-4: Standard Normal Distribution

The 68-95-99 Rule in the previous section provides an approximate value to the probability of the normal distribution (area under the bell-curve) for 1, 2, and 3 standard deviations from the mean. For *z*-scores other than ±1, 2, and 3, we can use a variety of ways to determine the probability under the normal distribution curve from the raw-score (*X*) and vice versa.

$$z = \frac{X - \mu}{\sigma}$$

where μ = mean, σ = standard deviation, X = Raw-Score, z = *z*-Score

Example 1: To the nearest hundredth, find the *z*-score of the followings.

a. $X = 52$, $\mu = 41$, and $\sigma = 6.4$

$$z = \frac{X - \mu}{\sigma} \qquad z = \frac{52 - 41}{6.4} = \frac{11}{6.4}$$

$z = 1.72$

b. $X = 75$, $\mu = 82$, and $\sigma = 9.1$

$$z = \frac{X - \mu}{\sigma} \qquad z = \frac{75 - 82}{9.1} = \frac{-7}{9.1}$$

$z = -0.77$

Example 2: To the nearest tenth, find the raw-score of the followings.

a. $z = 1.34$, $\mu = 16.2$, and $\sigma = 3.8$

$$z = \frac{X - \mu}{\sigma}$$
$$1.34 = \frac{X - 16.2}{3.8}$$
$$(1.34)(3.8) = X - 16.2$$
$$5.092 = X - 16.2$$
$$5.092 + 16.2 = X$$

$X = 21.3$

b. $z = -1.85$, $\mu = 65$, and $\sigma = 12.7$

$$z = \frac{X - \mu}{\sigma}$$
$$-1.85 = \frac{X - 65}{12.7}$$
$$(-1.85)(12.7) = X - 65$$
$$-23.495 = X - 65$$
$$-23.495 + 65 = X$$

$X = 41.5$

Example 3: Find the unknown mean or standard deviation to the nearest tenth.

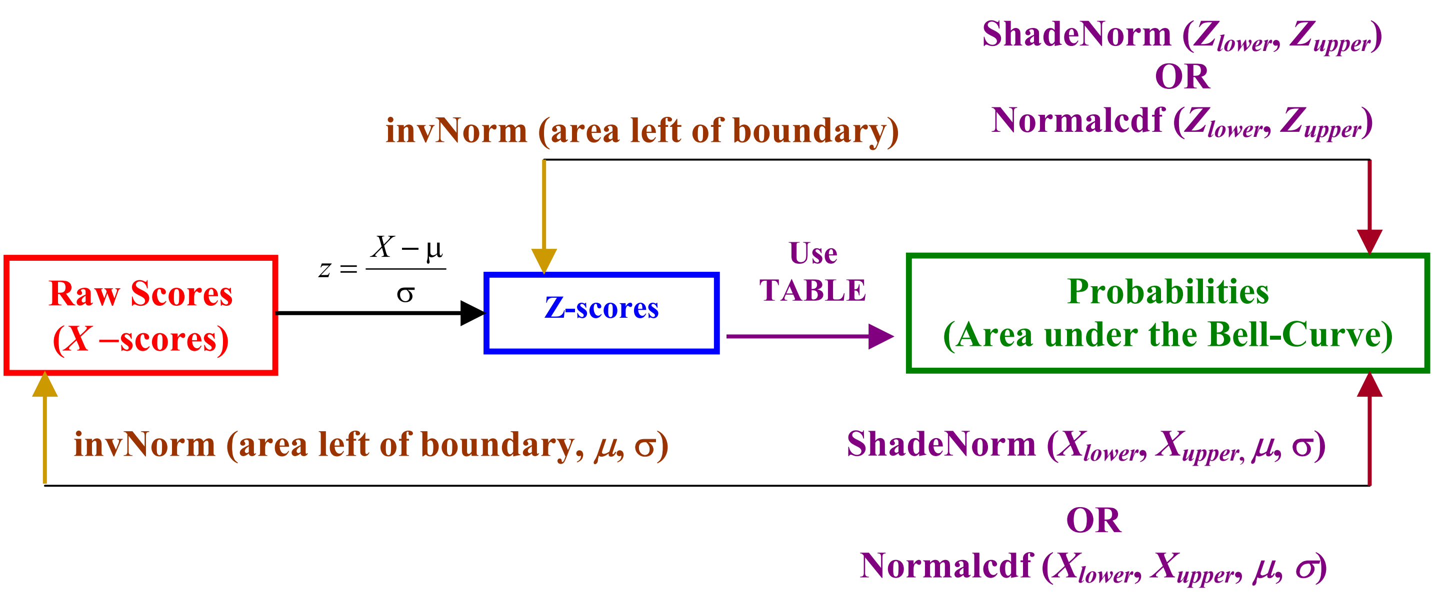
a. $z = -2.33$, $X = 47$, and $\mu = 84$ $\sigma = ?$

$$z = \frac{X - \mu}{\sigma}$$
$$-2.33 = \frac{47 - 84}{\sigma}$$
$$\sigma = \frac{47 - 84}{-2.33}$$
$$\sigma = \frac{-37}{-2.33}$$
$$\sigma = 15.9$$

b $z = 1.78$, $X = 38$, and $\sigma = 8.2$ $\mu = ?$

$$z = \frac{X - \mu}{\sigma}$$
$$1.78 = \frac{38 - \mu}{8.2}$$
$$(1.78)(8.2) = 38 - \mu$$
$$14.596 = 38 - \mu$$
$$\mu = 38 - 14.596$$
$$\mu = 23.4$$

Normal Distribution Curve Summary



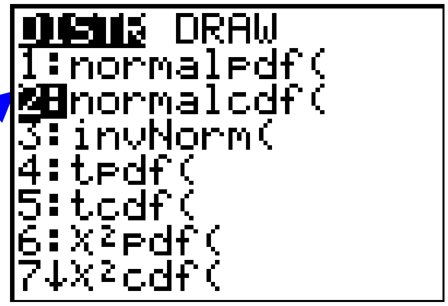
Normalcdf (X_{lower}, X_{upper}, μ, σ) : - use to convert Raw-Score directly to probability with NO graphics.

Normalcdf (Z_{lower}, Z_{upper}) : - use to convert z-Score to probability with NO graphics

- if X_{lower} or Z_{lower} is at the very left edge of the curve and is not obvious, use -1×10^{99} (–1E99 on calculator).
- if X_{upper} or Z_{upper} is at the very right edge of the curve and is not obvious, use 1×10^{99} (1E99 on calculator).

To access normalcdf:

1. Press **2nd** **DISTR VARS**



2. Select Option 2

ShadeNorm (X_{lower}, X_{upper}, μ, σ) : - use to convert Raw-Score directly to probability with graphics.

ShadeNorm (Z_{lower}, Z_{upper}) : - use to convert z-Score to probability with graphics

- if X_{lower} or Z_{lower} is at the very left edge of the curve and is not obvious, use -1×10^{99} (–1E99 on calculator).
- if X_{upper} or Z_{upper} is at the very right edge of the curve and is not obvious, use 1×10^{99} (1E99 on calculator).

Before accessing ShadeNorm, we need to select the *WINDOW* setting.

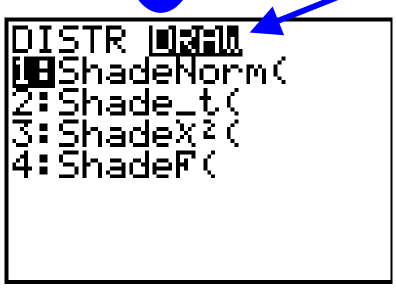
For **ShadeNorm (X_{lower}, X_{upper}, μ, σ)**, select a reasonable setting based on the information provided.

For **ShadeNorm (Z_{lower}, Z_{upper})**, use x: [–5, 5, 1] and y: [–0.15, 0.5, 0].

To access ShadeNorm:

1. Press **2nd** **DISTR VARS**

2. Use ▶ to access DRAW

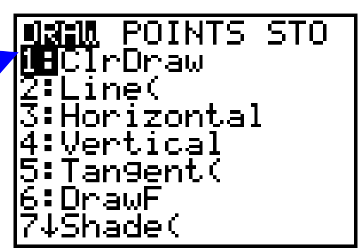


3. Select Option 1

Must Clear the Drawing (ClrDraw) before drawing or graphing again!

To access ClrDraw:

1. Press **2nd** **DRAW PRGM**



2. Select Option 1

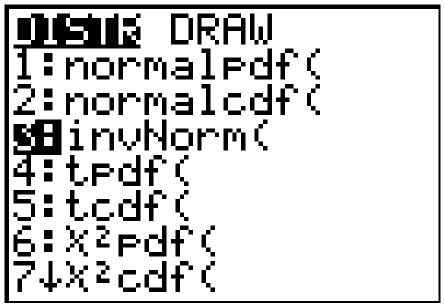
invNorm (area left of boundary, μ, σ) : - use to convert Area under the curve (Probability directly back to Raw-Score with NO graphics.

invNorm (area left of boundary) : - use to convert Area under the curve (Probability) back to *z*-Score with NO graphics.

To access invNorm:

1. Press **2nd** **DISTR VARS**

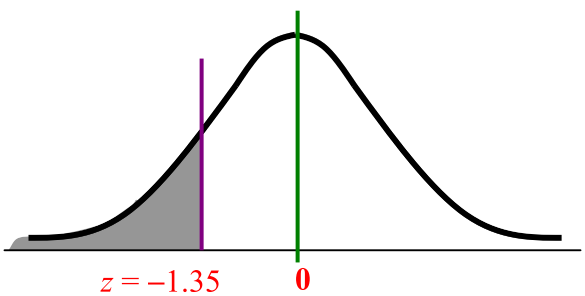
2. Select Option 3

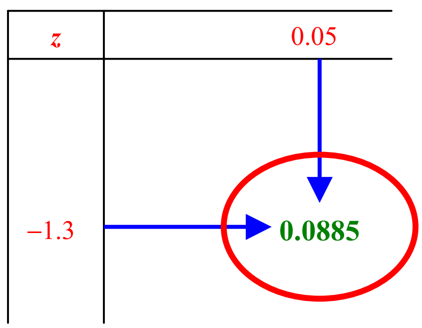


Using the Table of Areas under the Standard Normal Curve (Complete table on the next 2 pages)

1. **Converting *z*-score to Area under the curve (LEFT of the *z*-score boundary)**
   a. Look up the *z*-score from the column and row headings.
   b. Follow that row and column to find the area.

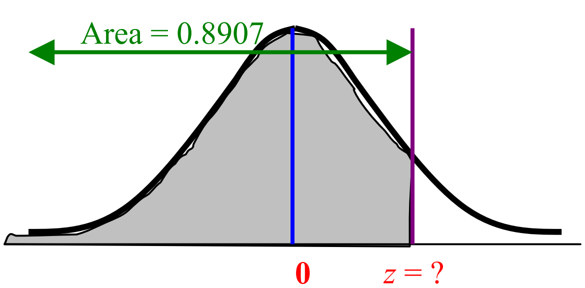
Example: Find $P(z \leq -1.35)$.

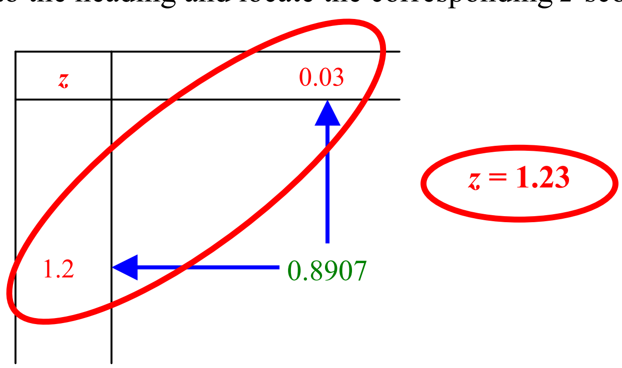




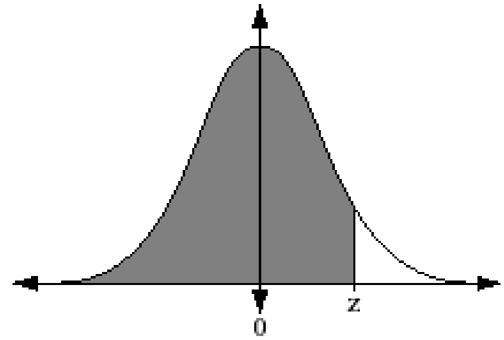
2. **Converting Area under the curve back to *z*-score (LEFT of the *z*-score boundary)**
   a. Look up the Area LEFT of the boundary from INSIDE the table.
   b. Follow that row and column back to the heading and locate the corresponding *z*-score.

Example: $P(z \leq ?) = 0.8907$





$$z = \frac{x - \mu}{\sigma}$$



Areas under the Standard Normal Curve

z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
−3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
−3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
−3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
−3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
−3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
−2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
−2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
−2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
−2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
−2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
−2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
−2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
−2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
−2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
−2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
−1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
−1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
−1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
−1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
−1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
−1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
−1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
−1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
−1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
−1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
−0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
−0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
−0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
−0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
−0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
−0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
−0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
−0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
−0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
−0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

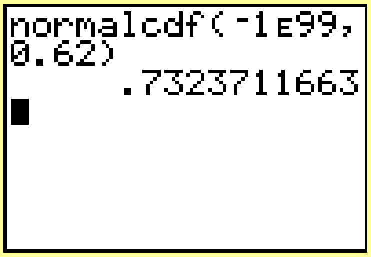
Areas under the Standard Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.5	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Example 4: To the nearest hundredth of a percent, find the probability of the following.

a. $P(z < 0.62)$

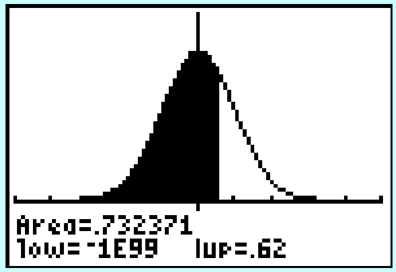
normalcdf (-1×10^{99}, 0.62)



***P* (*z* < 0.62) = 73.24%**

OR

ShadeNorm (-1×10^{99}, 0.62)

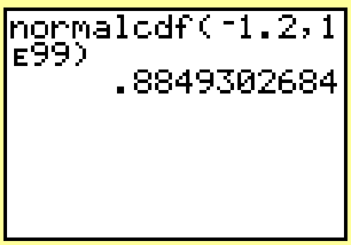


x: [–5, 5, 1]
y: [–0.15, 0.5, 0]

***P* (*z* < 0.62) = 73.24%**

b. $P(z > -1.2)$

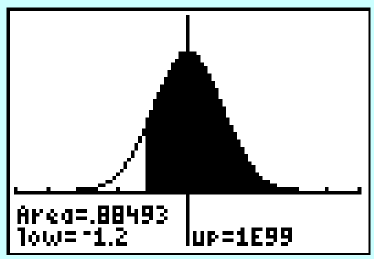
normalcdf (–1.2, 1×10^{99})



***P* (*z* > –1.2) = 88.49%**

OR

ShadeNorm (–1.2, 1×10^{99})

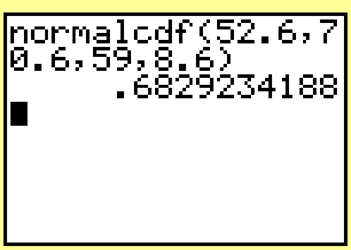


x: [–5, 5, 1]
y: [–0.15, 0.5, 0]

***P* (*z* > –1.2) = 88.49%**

c. $P(52.6 < X < 70.6)$ given $\mu = 59$ and $\sigma = 8.6$

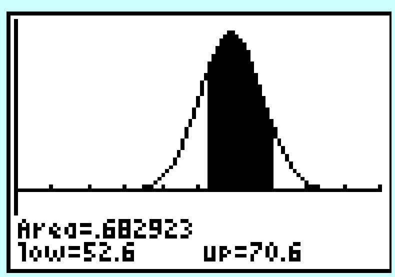
normalcdf (52.6, 70.6, 59, 8.6)



***P* (52.6 < *X* < 70.6) = 68.29%**

OR

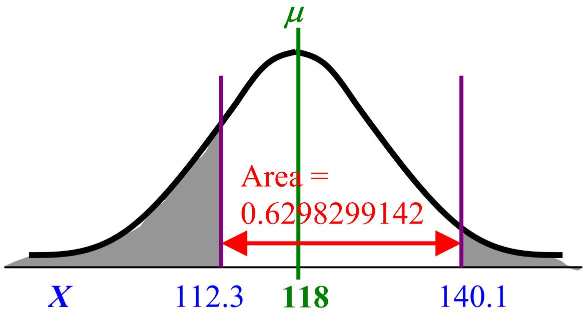
ShadeNorm (52.6, 70.6, 59, 8.6)



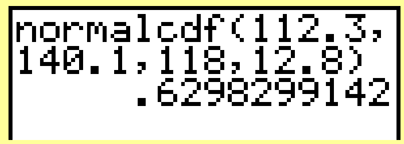
x: [0, 100, 10]
y: [–0.02, 0.05, 0]

***P* (52.6 < *X* < 70.6) = 68.29%**

d. $P\ (X < 112.3 \text{ and } X > 140.1)$ given $\mu = 118$ and $\sigma = 12.8$



normalcdf (112.3, 140.1, 118, 12.8)



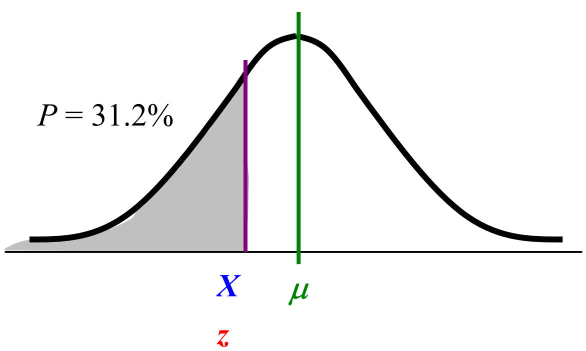
$P\ (112.3 < X < 140.1) = 0.6298299142$

$P\ (X < 112.3 \text{ and } X > 140.1) = 1 - 0.6298299142$

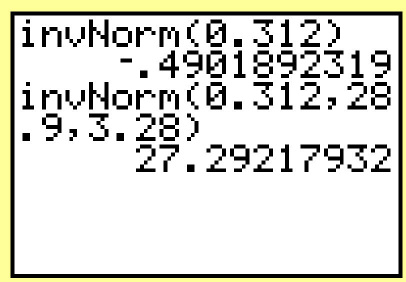
$\boldsymbol{P\ (X < 112.3 \text{ and } X > 140.1) = 37.0\%}$

Example 5: To the nearest hundredth, find the z-score and the raw-score from the following probability.

a. $\mu = 28.9$ and $\sigma = 3.28$

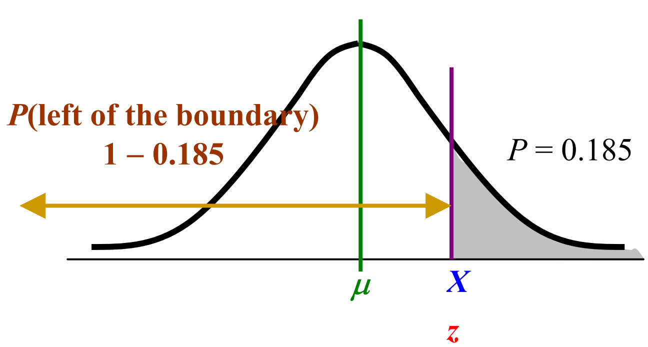


z = invNorm (0.312)
X = invNorm (0.312, 28.9, 3.28)

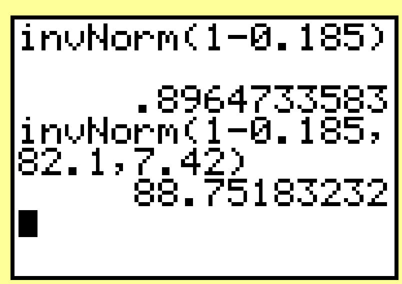


$z = -0.49$ and $X = 27.29$

b. $\mu = 82.1$ and $\sigma = 7.42$



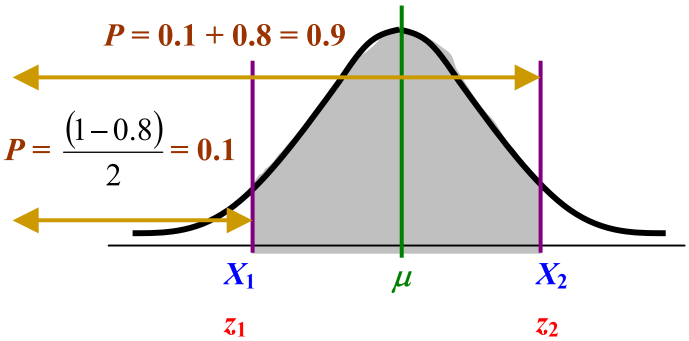
z = invNorm (1 – 0.185)
X = invNorm (1 – 0.185, 82.1, 7.42)



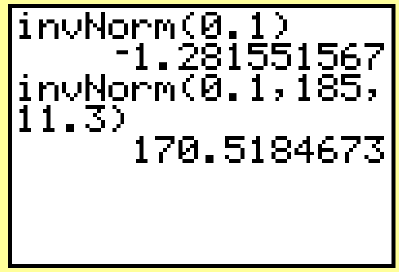
$z = 0.90$ and $X = 88.75$

c. $\mu = 185$ and $\sigma = 11.3$

P = 80% symmetrical about the mean



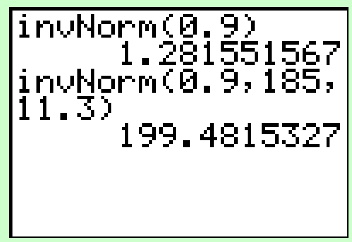
z_1 = **invNorm (0.1)**
X_1 = **invNorm (0.1, 185, 11.3)**



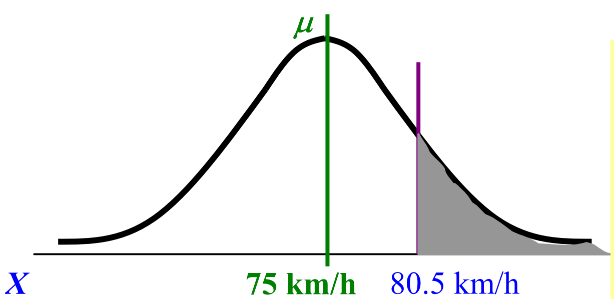
$z_1 = -1.28$ and $X_1 = 170.52$

z_2 = **invNorm (0.9)** X_2 = **invNorm (0.1, 185, 11.3)**

$z_2 = 1.28$ and $X_2 = 199.48$



Example 6: There are approximately 5000 vehicles travelling on 14th Street SW during non-rush hours everyday. The average speed of these vehicles is 75 km/h with a standard deviation of 8 km/h. If the posted speed limit on 14th Street is 70 km/h and the police will pull people over when they are 15% above the speed limit, how many people will the police pull over on any given day?



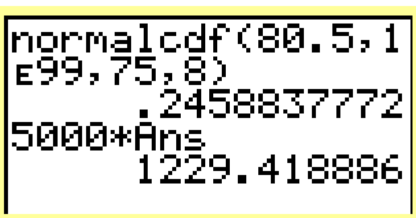
15% above 70 km/h = 70 × 115% = 80.5 km/h

X = 80.5 km/h, μ = 75 km/h, σ = 8 km/h

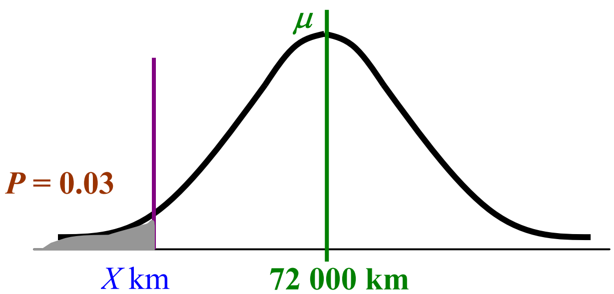
$P(X \geq 80.5$ km/h$)$ = **normalcdf (80.5, 1×10^{99}, 75, 8)**
= 0.2458837772

Number of drivers pulled over = Total × Probability
= 5000 × 0.2458837772

1229 drivers can be pulled over by the police

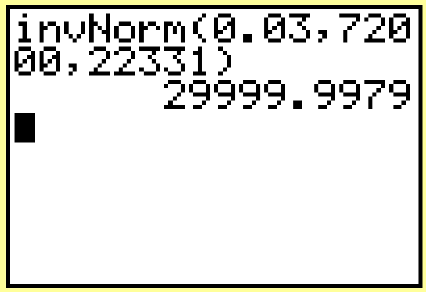


Example 7: A tire manufacturer finds that the mean life of the tires produced is 72000 km with a standard deviation of 22331 km. To the nearest kilometre, what should the manufacturer's warranty be set at if it can only accept a return rate of 3% of all tires sold?



P = 0.03, μ = 72000 km, σ = 22331 km

X = invNorm(0.03, 720000, 22331)



The warranty should be set at 30000 km

3-4 Assignment: **pg. 123 – 125 #1 to 10**

3-5: The Normal Approximation to a Binomial Distribution

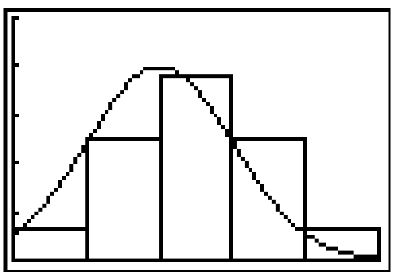
Binomial Distribution: - a histogram that shows the probabilities of an experiment repeated many times (only success or failure – desirable or undesirable outcomes).

When the conditions $np > 5$ and $n(1 - p) > 5$ are met, we can use the normal approximation for the binomial distribution. **ONLY IN THAT CASE**, the mean and the standard deviation can be calculated by:

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}$$

where n = number of trials and p = probability of favourable outcome

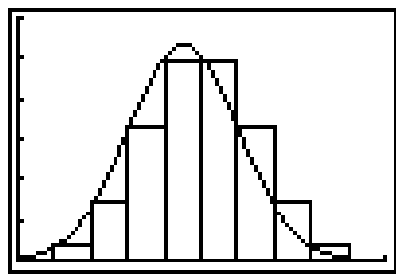
Using the bell curve to approximate a binomial distribution really depends on the number of trials, n. When n is small, there are very few bars on the binomial distribution and the bell curve does not fit the graph well. However, when n is large, the bell curve fits the binomial distribution much better. Therefore, we can use the area under normal bell curve to approximate the cumulative sum of the binomial probabilities.



$n = 4$ $p = 0.5$

$np = 4 \times 0.5 = 2$ (less than 5)
$n(1 - p) = 4 \times (1 - 0.5) = 2$ (less than 5)

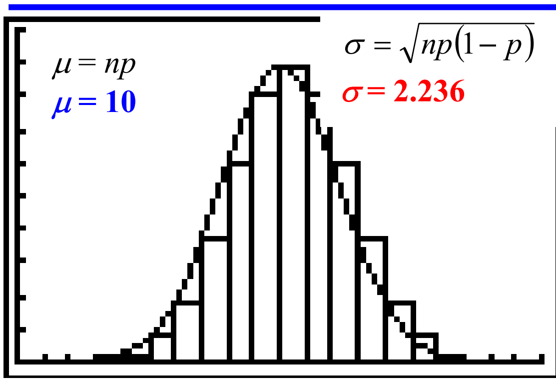
CANNOT use Normal Approximation
Bell-Curve does NOT fit the binomial distribution well.



$n = 9$ $p = 0.5$

$np = 9 \times 0.5 = 4.5$ (less than 5)
$n(1 - p) = 9 \times (1 - 0.5) = 4.5$ (less than 5)

CANNOT use Normal Approximation
Bell-Curve does NOT fit the binomial distribution well. It's still not good enough.



$n = 20$ $p = 0.5$

$np = 20 \times 0.5 = 10$ (greater than 5)
$n(1 - p) = 20 \times (1 - 0.5) = 10$ (greater than 5)

CAN use Normal Approximation
Bell-Curve fits the binomial distribution well.
(MORE bars means better fit!)

Example 1: A multiple-choice test has 10 questions. Each question has 4 possible choices.

a. Determine whether the conditions for normal approximation are met.
b. Graph the resulting binomial distribution.
c. Find the probability that a student will score **exactly** 6 out of 10 on the test.
d. Calculate the probability that a student will **at least** pass the test.

a. Determining Condition for Normal Approximation

n = 10 questions $\quad\quad p = \frac{1}{4} = 0.25$ (probability of guessing a question correct)

$np = 10 \times 0.25$ $\quad\quad n(1 - p) = 10 \times (1 - 0.25)$

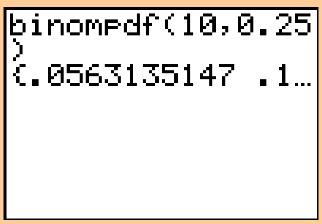
$np = 2.5$ **(less than 5)** $\quad\quad = 10 \times 0.75$

$n(1 - p) = 7.5$ (greater than 5)

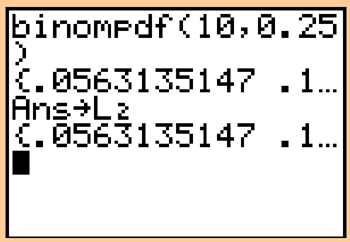
Since the np condition is **NOT** met, we **CANNOT** use the normal approximation for this question.

b. To Graph the Binomial Distribution:

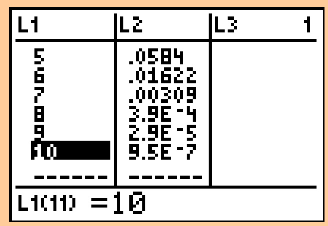
1. **binompdf (10, 0.25)** 2. Store answer in L_2 of the STAT Editor. 3. Enter 0 to 10 in L_1 of the STAT Editor.



STO➡ **2nd** **L2 2**



STAT **ENTER**



4. WINDOW Settings

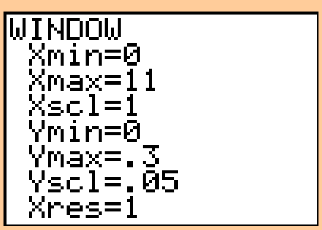
x: [x_{min}, x_{max}, x_{scl}] = x: [0, 11, 1]

y: [y_{min}, y_{max}, y_{scl}] = y: [0, 0.30, 0.05]

Select a number slightly higher than the maximum in L_2.

Must be 1 more than the number of trials.

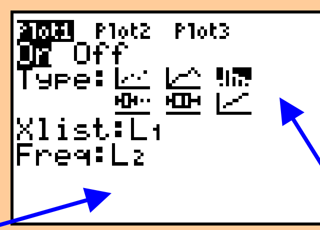
WINDOW

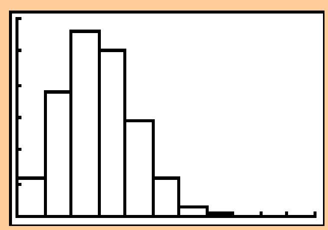


5. Select Histogram in STAT PLOT and graph.

2nd **STAT PLOT Y=** **ENTER** **GRAPH**

Plot1 is ON.





Type in L_2 as Frequency by pressing: **2nd** **L2 2**

Select Histogram

c. $P(6)$ = **binompdf (10, 0.25, 6)**

```
binompdf(10,0.25
,6)
        .0162220001
```

exactly 6 successes

$P(6) = 0.01622$

To access sum function

1. Press **2nd** **LIST STAT**
2. Use ▶ to access MATH
3. Select Option 5

```
NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:prod(
7↓stdDev(
```

d. P (passing) = P (at least 5 out of 10)

$= P(X \geq 5)$

$P(X \geq 5)$ = **sum (binompdf (10, 0.25 {5,6,7,8,9,10}))**

sum up binomial probabilities of 5 through 10 successes

list up binomial probabilities of 5 through 10 successes

```
sum(binompdf(10,
0.25,{5,6,7,8,9,
10}))
         .0781269073
```

P (passing) = 0.07813 = 7.813%

Example 2: A multiple-choice test has 30 questions. Each question has 4 possible choices.

a. Determine whether the conditions for normal approximation are met.
b. Find the mean and standard deviation
c. Graph the resulting binomial distribution..
d. Find the probability that a student will score **exactly** 17 out of 30 on the test.
e. Calculate the probability that a student will **at least** pass the test.

a. Determining Condition for Normal Approximation

$n = 30$ questions $\qquad p = \frac{1}{4} = 0.25$ (probability of guessing a question correct)

$np = 30 \times 0.25$

$np = 7.5$ (greater than 5)

$n(1-p) = 30 \times (1 - 0.25)$

$= 30 \times 0.75$

$n(1-p) = 22.5$ (greater than 5)

Since both np and $n(1-p)$ condition are met, we **CAN** use the normal approximation for this question.

b. Mean and Standard Deviation

$$\mu = np = 30(0.25) = 7.5$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{(30)(0.25)(1-0.25)} = \sqrt{35 \times 0.25 \times 0.75} = 2.372$$

$\mu = 7.5$
$\sigma = 2.372$

c. To Graph the Binomial Distribution:

1. **binompdf (30, 0.25)**
2. Store answer in L_2 of the STAT Editor.
3. Enter 0 to 30 in L_1 of the STAT Editor.

```
binompdf(30,0.25
)
{1.785820902E-4…
```

STO➡ **2nd** **L2 / 2**

```
binompdf(30,0.25
)
{1.785820902E-4…
Ans→L2
{1.785820902E-4…
```

STAT **ENTER**

L1	L2	L3
25	3E-11	
26	2E-12	
27	1E-13	
28	3E-15	
29	8E-17	
30	9E-19	

L1(32) =

4. WINDOW Settings

x: $[x_{min}, x_{max}, x_{scl}] = x$: [0, 31, 1]

y: $[y_{min}, y_{max}, y_{scl}] = y$: [0, 0.18, 0.02]

Select a number slightly higher than the maximum in L_2.

WINDOW

```
WINDOW
 Xmin=0
 Xmax=31
 Xscl=1
 Ymin=0
 Ymax=.18
 Yscl=.02
 Xres=1
```

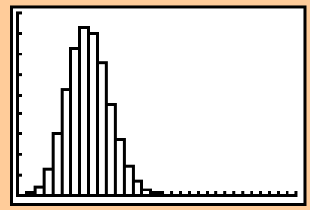
Must be 1 more than the number of trials.

5. Select Histogram in STAT PLOT and graph.

2nd **STAT PLOT / Y=** **ENTER** **GRAPH**

Plot1 is ON.

```
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L1
Freq:L2
```



Type in L_2 as Frequency by pressing: **2nd** **L2 / 2**

Select Histogram

6. To Overlay the Binomial Distribution with a Normal Bell Curve
   [Enter equation: normalpdf (X, μ, σ)]

Y=

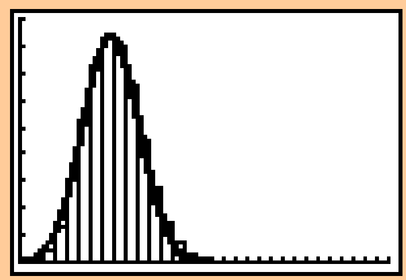
```
Plot1 Plot2 Plot3
\Y1=normalpdf(X,
7.5,√(30*.25*.75
))
\Y2=
\Y3=
\Y4=
\Y5=
```

$\mu = 7.5$

$\sigma = \sqrt{30 \times 0.25 \times 0.75}$

use exact value for σ

GRAPH



d. P (17) = **binompdf (30, 0.25, 17)**

```
binompdf(30,0.25
,17)
  1.656104796E-4
```

$P(17) = 1.656 \times 10^{-4}$

***P* (17) = 0.0001656**

e. P (passing) = P (at least 15 out of 30)

= $P(15 \leq X \leq 30)$ ← 30 is the maximum score

Since Normal Approximation is allowed, we can use **normalcdf** to calculate area under the bell curve.

$P(15 \leq X \leq 30)$ = **normalcdf (15, 30, 7.5, 2.372)** $= 7.84 \times 10^{-4}$

$\mathbf{P(15 \leq X \leq 30) = 0.000\ 784}$

3-5 Assignment: **pg. 130 – 131 #1 to 9**

3-6: Confidence Intervals

Confidence Intervals: - the level of assurance from a statistical report.
- symmetrical area around the mean.

EXTRA!!! The Times EXTRA!!!

POLL SHOW TORIES ARE STILL POPULAR

A recent poll conducted with 2000 Albertans shows that if there is a provincial election today, the Conservative Party will win it by 65% of the popular vote. The poll is said to be accurate within 1.5%, 19 times out of 20.

n = 2000 Albertans

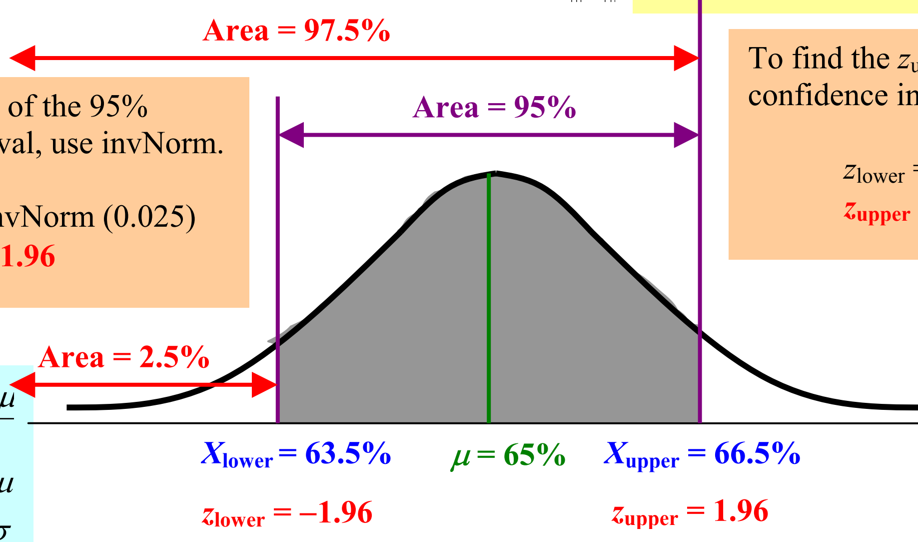
μ = 65%

***Margin of Error* = ±1.5%**

X_{lower} = 65% − 1.5% = 63.5%

X_{upper} = 65% + 1.5% = 66.5%

Confidence Interval (Area) = $\frac{19}{20}$ = 95%



To find the z_{lower} of the 95% confidence interval, use invNorm.

z_{lower} = invNorm (0.025)
z_{lower} = −1.96

To find the z_{upper} of the 95% confidence interval, use invNorm.

z_{lower} = invNorm (0.975)
z_{upper} = 1.96

$$-1.96 = \frac{X_{lower} - \mu}{\sigma}$$
$$-1.96\sigma = X_{lower} - \mu$$
$$X_{lower} = \mu - 1.96\sigma$$

$$1.96 = \frac{X_{upper} - \mu}{\sigma}$$
$$1.96\sigma = X_{upper} - \mu$$
$$X_{upper} = \mu + 1.96\sigma$$

95% Confidence Interval

95% *conf-int.* = $\mu \pm (1.96\sigma)$

95% *conf-int.* Margin of Error

95% *conf-int.* Margin of Error (Percent)

$$\pm\frac{1.96\sigma}{n}\times 100\%$$

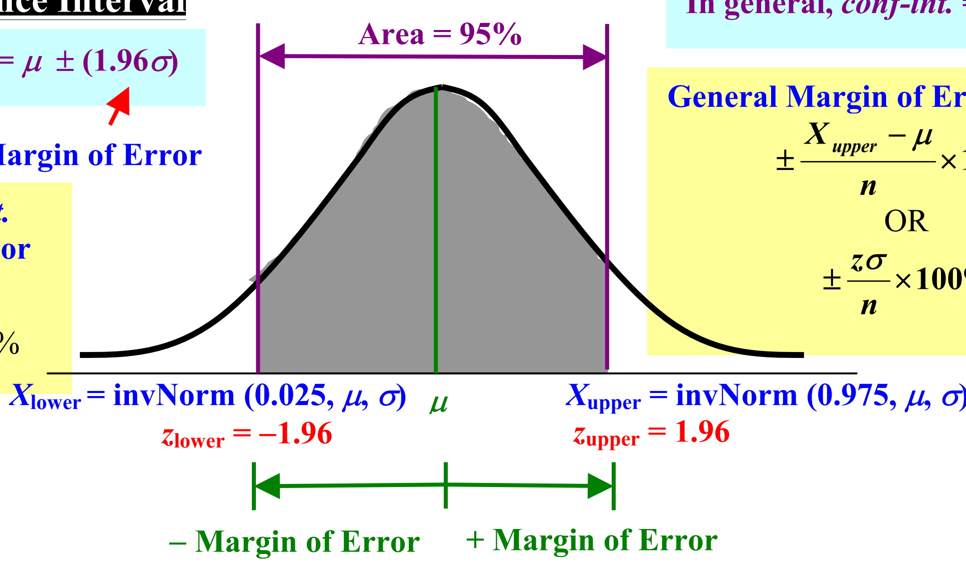
In general, *conf-int.* = $\mu \pm z\sigma$

General Margin of Error (Percent)

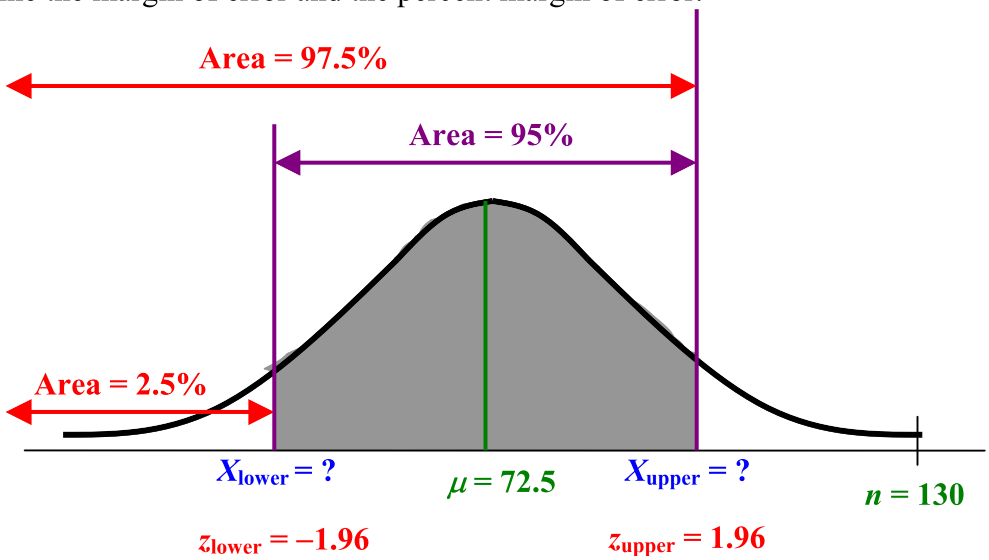
$$\pm\frac{X_{upper} - \mu}{n}\times 100\%$$

OR

$$\pm\frac{z\sigma}{n}\times 100\%$$



Example 1: Given that $\mu = 72.5$, $\sigma = 5.24$ and $n = 130$, draw a 95% confidence interval curve and determine the margin of error and the percent margin of error.



X_{lower} = invNorm (0.025, 72.5, 5.24)
X_{upper} = invNorm (0.975, 72.5, 5.24)

$X_{\text{lower}} = 62.2$ **$X_{\text{upper}} = 82.8$**

```
invNorm(0.025,72
.5,5.24)
      62.22978871
invNorm(0.975,72
.5,5.24)
      82.77021129
```

Margin of Error = $\mu \pm (1.96\sigma)$
$= 72.5 \pm 1.96 \times 5.24$

OR

Margin of Error = $\mu \pm (\mu - X_{\text{lower}})$
$= 72.5 \pm (72.5 - 62.2)$

Margin of Error = 72.5 ± 10.3

Percent Margin of Error $= \pm \dfrac{1.96\sigma}{n} \times 100\%$
$= \pm \dfrac{1.96(5.24)}{130} \times 100\%$

OR

Percent Margin of Error $= \pm \dfrac{X_{upper} - \mu}{n} \times 100\%$
$= \pm \dfrac{82.8 - 72.5}{130} \times 100\%$

Percent Margin of Error = 55.8% ± 7.9%

Mean expressed in percent

$\mu = \dfrac{72.5}{130} \times 100\%$

$\mu = 55.8\%$

We can say that we are 95% confident that the scores are in the range of 72.5 ± 10.3 out of a total of 130.

OR

We can say that we are 95% confident that the scores are in the range of 55.8% ± 7.9% out of a total of 130.

Example 2: Given that $n = 250$ and $p = 0.4$, draw a 95% confidence interval curve and determine the margin of error and the percent margin of error.

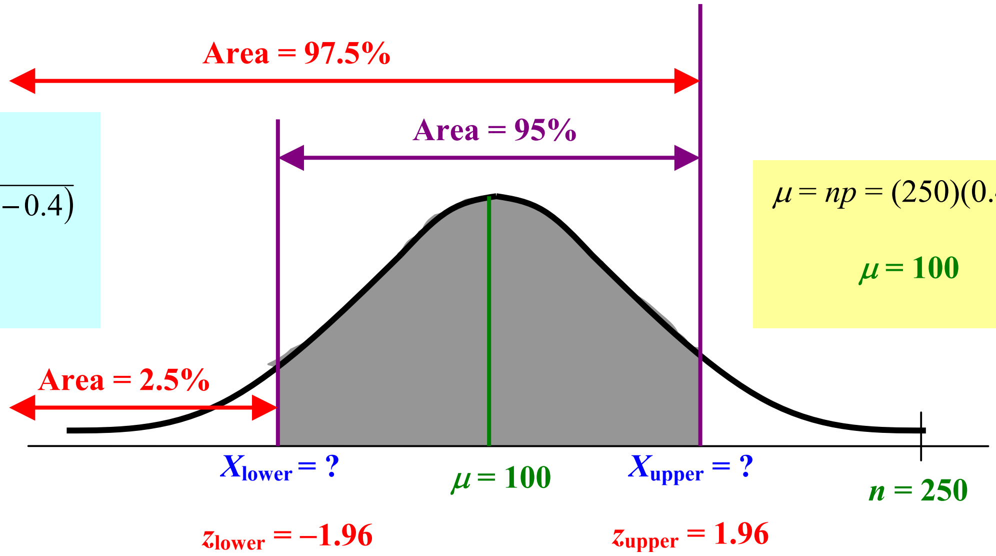
This is a binomial distribution. We need to use the formulas $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.

$\sigma = \sqrt{np(1-p)}$
$= \sqrt{250 \times 0.4 \times (1 - 0.4)}$

$\sigma = 7.746$

$\mu = np = (250)(0.4)$

$\mu = 100$



X_{lower} = invNorm (0.025, 100, 7.746)
X_{upper} = invNorm (0.975, 100, 7.746)

$X_{\text{lower}} = 85$ $X_{\text{upper}} = 115$

```
invNorm(0.025,10
0,7.746)
         84.81811896
invNorm(0.975,10
0,7.746)
           115.181881
```

(Answers are round to whole number because it is a binomial distribution.)

Margin of Error = $\mu \pm (1.96\sigma)$
$= 100 \pm 1.96 \times 7.746$ OR

Margin of Error = $\mu \pm (\mu - X_{\text{lower}})$
$= 100 \pm (100 - 85)$

Margin of Error = 100 ± 15

Percent Margin of Error $= \pm \dfrac{1.96\sigma}{n} \times 100\%$
$= \pm \dfrac{1.96(7.746)}{250} \times 100\%$

OR

Percent Margin of Error $= \pm \dfrac{X_{upper} - \mu}{n} \times 100\%$
$= \pm \dfrac{115 - 100}{250} \times 100\%$

Percent Margin of Error = 40% ± 6%

We can say that we are 95% confident that the scores are in the range of 100 ± 15 out of a total of 250.
OR
We can say that we are 95% confident that the scores are in the range of 40% ± 6% out of a total of 250.

Example 3: From a random survey of 1000 people, 852 of them believe that the government should regulate the electricity industry. Calculate the 95% confidence intervals and the margin of error in percent. Report your final answer in complete sentences.

This is a binomial distribution. We need to use the formulas $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.

$p = \dfrac{852}{1000}$

$p = 0.852$

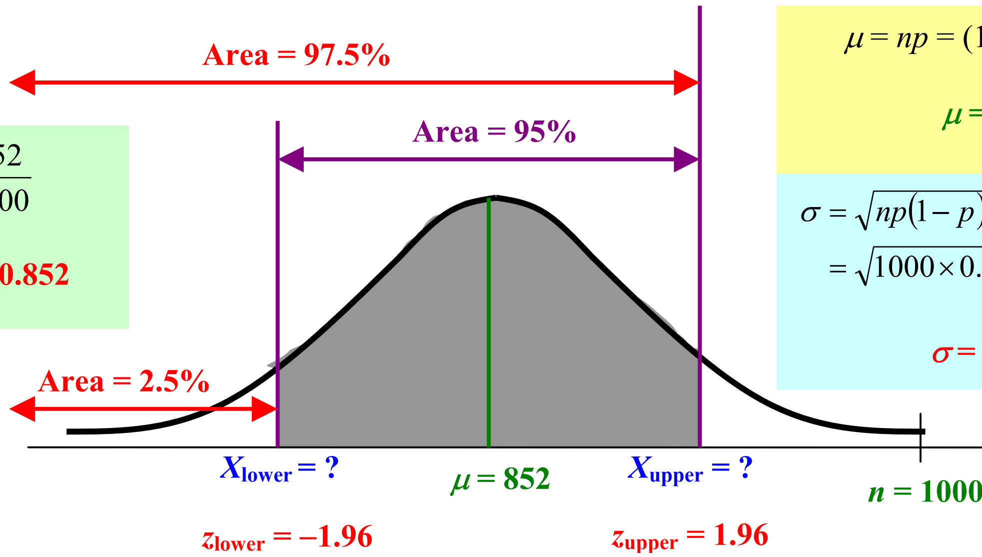
$\mu = np = (1000)(0.852)$

$\mu = 852$

$\sigma = \sqrt{np(1-p)}$

$= \sqrt{1000 \times 0.852 \times (1 - 0.852)}$

$\sigma = 11.23$



X_{lower} = invNorm (0.025, 852, 11.23)
X_{upper} = invNorm (0.975, 852, 11.23)

$X_{lower} = 830$ $X_{upper} = 874$

```
invNorm(0.025,85
2,11.23)
         829.9896044
invNorm(0.975,85
2,11.23)
         874.0103956
```

(Answers are round to whole number because it is a binomial distribution.)

Margin of Error = $\mu \pm (1.96\sigma)$
= $852 \pm 1.96 \times 11.23$

OR

Margin of Error = $\mu \pm (\mu - X_{lower})$
= $852 \pm (852 - 830)$

Margin of Error = 852 ± 22

Percent Margin of Error = $\pm \dfrac{1.96\sigma}{n} \times 100\%$
$= \pm \dfrac{1.96(11.23)}{1000} \times 100\%$

OR

Percent Margin of Error = $\pm \dfrac{X_{upper} - \mu}{n} \times 100\%$
$= \pm \dfrac{874 - 852}{1000} \times 100\%$

Percent Margin of Error = 85.2% ± 2.2%

We can say that we are 95% confident that the scores are in the range of 852 ± 22 out of a total of 1000.
OR
We can say that we are 95% confident that the scores are in the range of 85.2% ± 2.2% out of a total of 1000.

3-6 Assignment:
pg. 139 – 141 #1 to 11

