Unit 7: Patterns

6-1: Sequences

Sequence: - a list of numbers generated from a pattern.

Arithmetic Sequence: - numbers in a list are related by a common difference.

Example: 5, 8, 11, 14, 17, …… t_n (add 3 to the get the next term)

$t_n = a + (n - 1)d$	a = value of the first term d = common difference	n = number of terms in the sequence t_n = the value of the n^{th} term

Example 1: In the sequence 84, 80, 76, 72, …, find the

a. 20th term.

$a = 84$ $d = -4$ $n = 20$

$t_{20} = 84 + (20 - 1)(-4)$
$t_{20} = 84 + (19)(-4)$

$t_{20} = 8$

b. general equation for t_n.

$a = 84$ $d = -4$

$t_n = 84 + (n - 1)(-4)$
$t_n = 84 + (-4n + 4)$

$t_n = -4n + 88$

Example 2: A salesperson is paid $500 per month plus $35 per encyclopaedia sold.
a. Find the general equation for his monthly earnings.
b. Graph the sequence to the 20 encyclopaedias sold.
c. How much would the salesperson earn if he sold 42 encyclopaedias in one month?
d. Find the number of encyclopaedia sold if the earning is $920 in a month.

a. First, we set up a table.

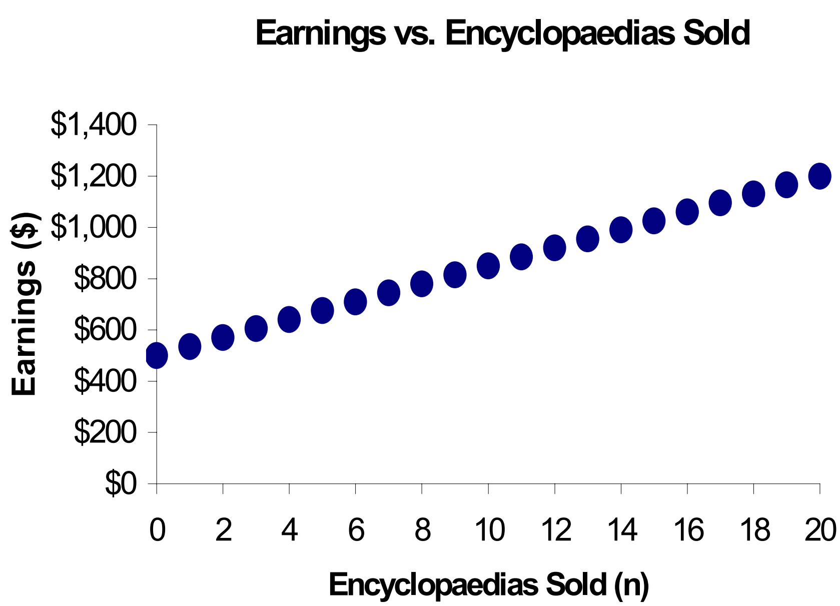
Encyclopaedias Sold	Earning
0	$500
1	$535
2	$570
3	$605

$a = 535$ $d = 35$

$t_n = 535 + (n - 1)(35)$
$t_n = 535 + (35n - 35)$
$t_n = 35n + 500$

$E = 35n + 500$

b.



c. $n = 42$ encyclopedias $E = ?$

$E = 35n + 500$

$E = 35\,(42) + 500$ $\quad$ **$E = \$1970$**

d. $n = ?$ encyclopedias $E = \$920$

$920 = 35n + 500$

$920 - 500 = 35n$

$\dfrac{420}{35} = n$ $\quad$ **$n = 12$**

<u>Geometric Sequence</u>: - numbers in a list that is related by a common ratio.

Example: 3, 6, 12, 24, 48, … t_n (multiply 2 to the get the next term)

$t_n = ar^{(n-1)}$	a = value of the first term r = common ratio	n = number of terms in the sequence t_n = the value of the n^{th} term

Example 3: In the sequence 700, 560, 448, 358.4, …, find the

a. 15th term.

$a = 700$ $\quad r = \dfrac{560}{700} = 0.8$

$t_{15} = (700)(0.8)^{(15-1)}$

$t_{15} = (700)(0.8)^{14}$

$t_{15} = 30.7863$

b. general equation for t_n.

$a = 700$ $\quad r = \dfrac{448}{560} = 0.8$

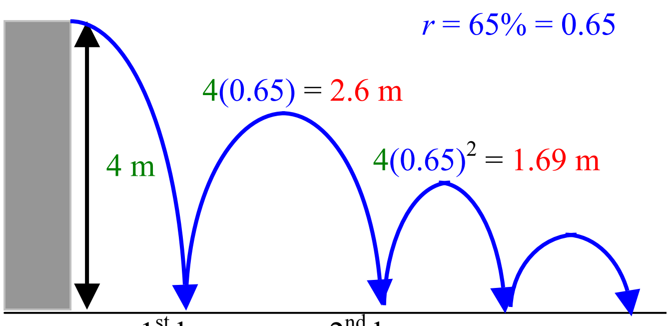
$t_n = (700)(0.8)^{(n-1)}$

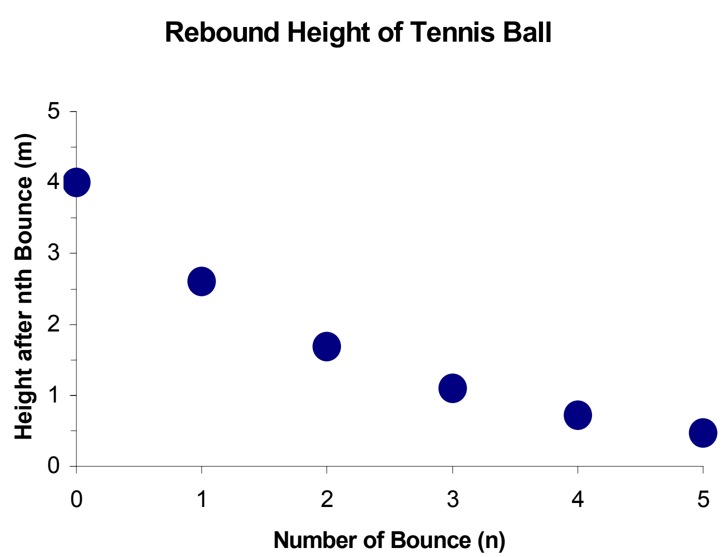
Example 4: A tennis ball is dropped from a height of 4 m and rebound 65% of the previous height.

a. Find the equation that describes the pattern, and graph the rebound height versus the number of bounces.
b. To the nearest, thousandth of a metre, what height did the ball reach after the 6th bounce?
c. How many bounces did it take for the ball to have a rebound height of less than 0.10 m?

a.

$r = 65\% = 0.65$





The sequence is: 4 m, 2.6 m, 1.69 m, …
However, we will define **$a = 2.6$ m** because **t_n = height after nth bounce**.

$t_n = (2.6)(0.65)^{(n-1)}$

b. After the 6th bounce, the ball will have a height of t_6.

$t_n = (2.6)(0.65)^{(n-1)}$
$t_6 = (2.6)(0.65)^{(6-1)}$
$t_6 = (2.6)(0.65)^5$ **$t_6 = 0.302$ m**

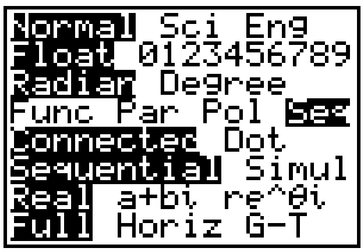
c.

Number of Rebound (n)	Rebound Height (t_n)
6	0.30168 m
7	0.19609 m
8	0.12746 m
9	**0.08285 m**

Recursive Sequence: - a sequence where the value of the next number depends on the value(s) of the previous number(s).
- **all arithmetic and geometric sequences** can be classified as recursive.

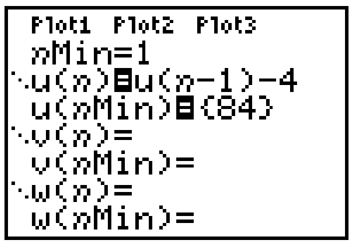
Generating Sequences Using TI-83 Plus Calculator.

1. Set Sequence Mode **MODE**



Select Seq

2. Define Sequence **Y=**



nMin = Starting n of the sequence (usually is set at 1).

u(n) = equation that defines the sequence
We use u(n – 1) to denote the value of the previous term.

u(nMin) = value of the first term (a) in { } brackets

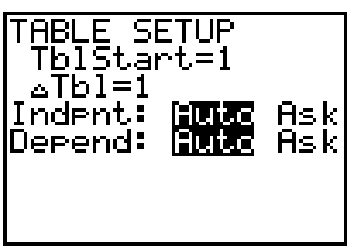
To access u, press **2nd** **u / 7** **To access n, press** **X,T,θ,n**

To access { }, press **2nd** **{ / (** **and** **2nd** **} /)**

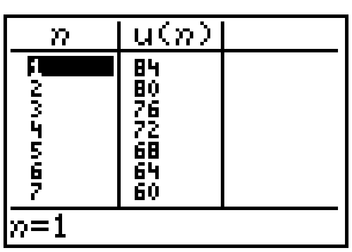
3. Viewing Sequence

a. Using Table:

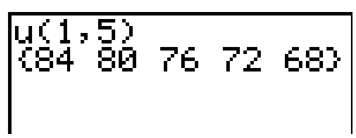
2nd **TBLSET / WINDOW**



2nd **TABLE / GRAPH**



b. As a List: Sequence Name ($n$start, $n$stop)



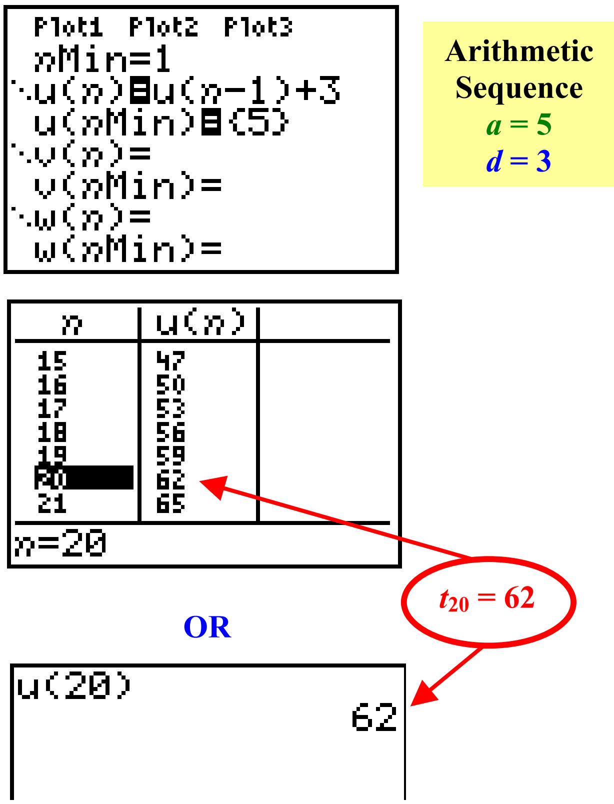
The first five terms of this sequence are 84, 80, 76, 72, and 68.

c. As an individual term: Sequence Name (n)

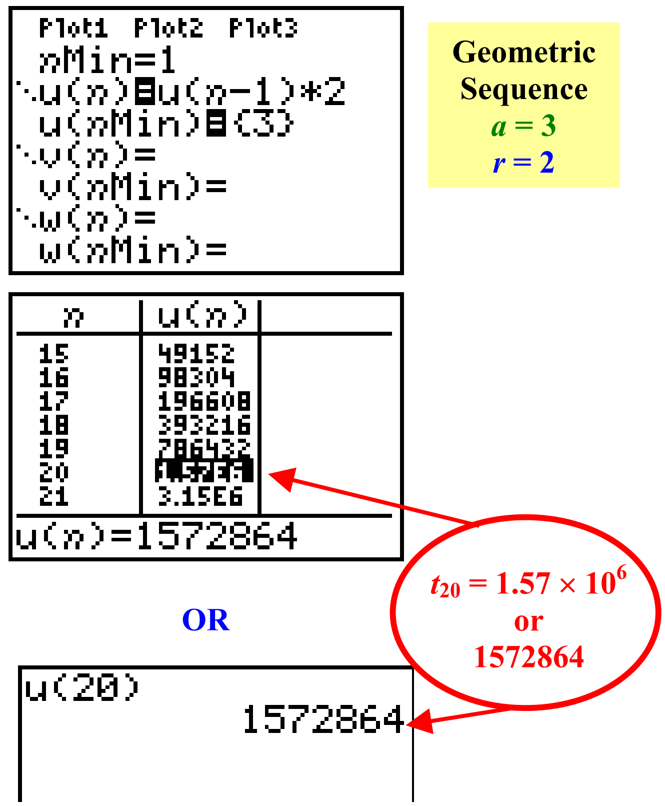
The 7th term of this sequence is 60.

Example 5: Use the graphing calculator in SEQUENCE mode to calculate t_{20}.

a. 5, 8, 11, 14, 17, …



b. 3, 6, 12, 24, 48, …

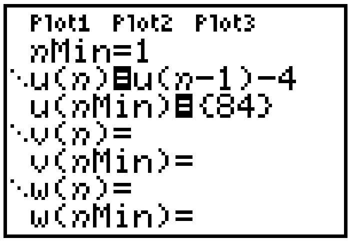


6-1 Assignment: **pg. 264 – 265 #1 to 10**

6-2: Modelling Using Sequences

Graphing Sequences Using TI-83 Plus Calculator.

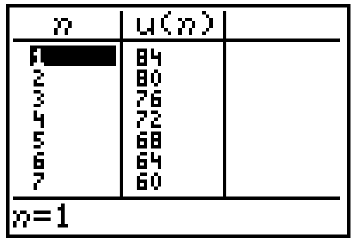
1. Define Sequence **Y=**

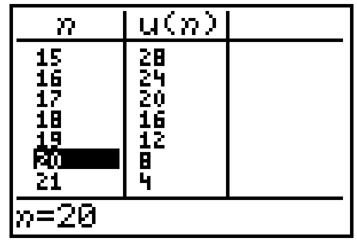


2. Determine Maximum and Minimum *y* values using Table

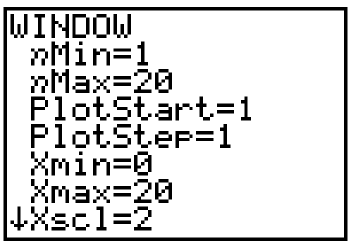
2nd

TABLE
GRAPH

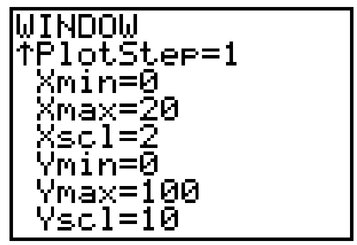




3. Set WINDOW **WINDOW**

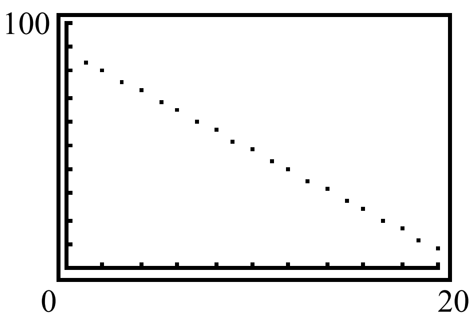


***n*Min = first *n* term to be evaluated**
***n*Max = last *n* term to be evaluated**
Plot Start = first *n* term to be graphed
Plot Step = increments of (*n*) term to be plotted



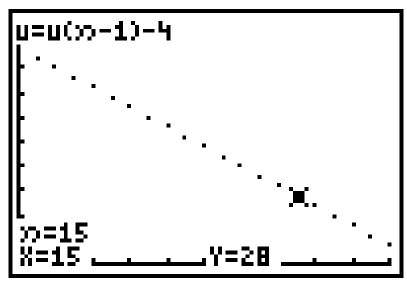
4. Graph

GRAPH



5. Tracing will allow us to find value at t_n

TRACE



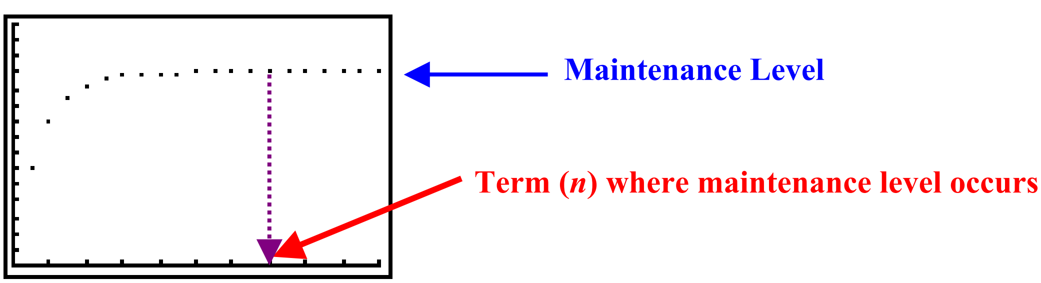
Sometimes sequences may involve More Than One Operation to go from one term to the next.

Example: 3, 7, 15, 31, … t_n (Multiply 2 and Add 1)

We can write $t_1 = 3$

$t_n = 2(t_{n-1}) + 1$

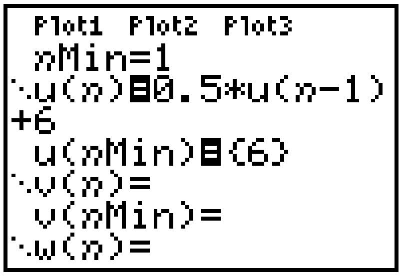
Maintenance Level: - where the sequence levels off on a graph.



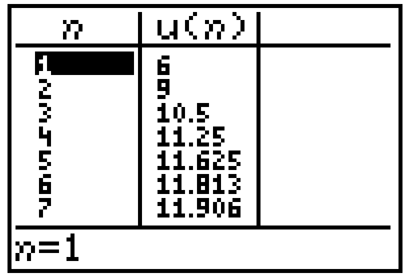
Example 1: Given $t_1 = 6$ and $t_n = \frac{1}{2}(t_{n-1}) + 6$, and using the graphing calculator

a. Find the first five terms of the sequence.
b. Graph the first thirty-five of the sequence.
c. Find the maintenance level of the sequence

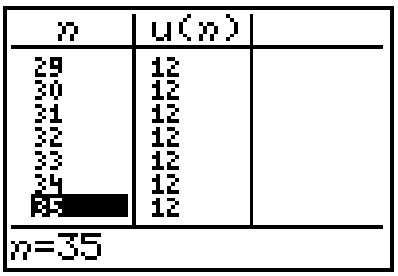
a. **Y=**



2nd **TABLE** **GRAPH**

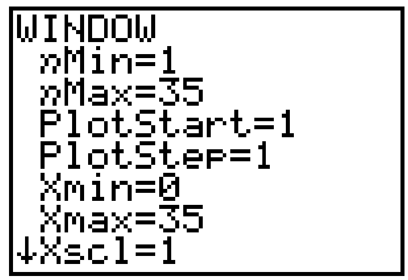


b. **2nd** **TABLE** **GRAPH**

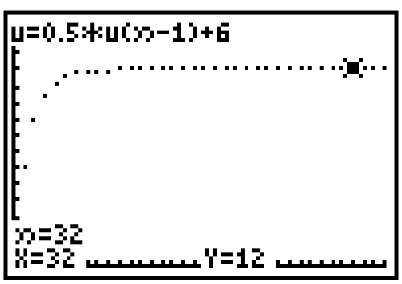


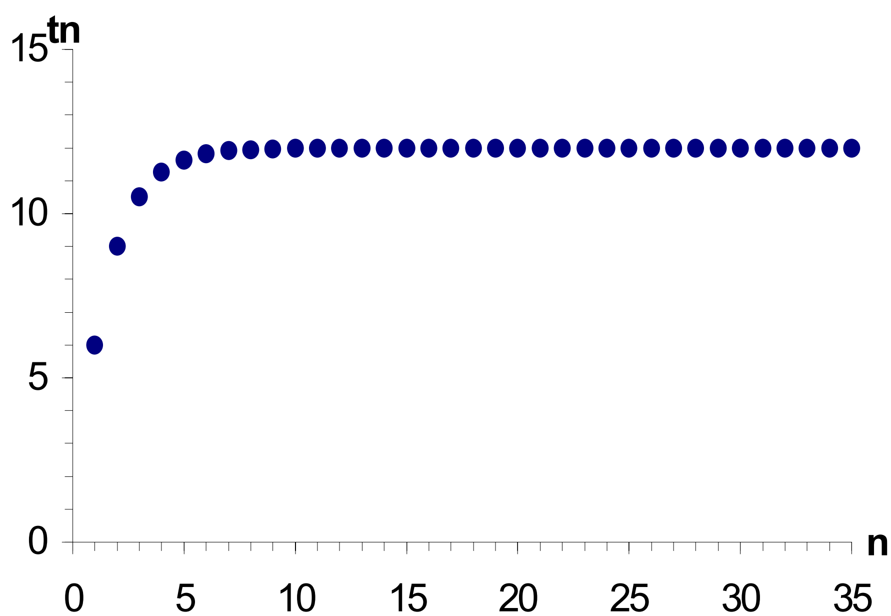
WINDOW

x: [0, 35, 1]
y: [0, 15, 1]

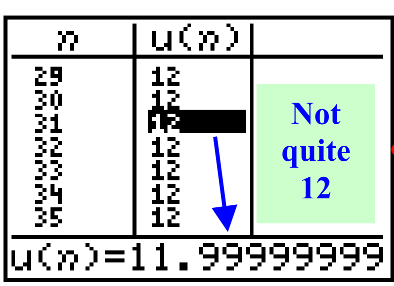


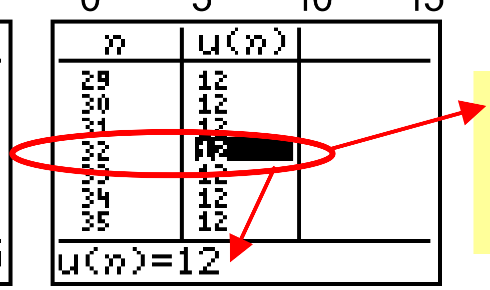
GRAPH **TRACE**





c.





Maintenance Level at 12 when n = 32. (This is where the whole number 12 FIRST APPEARS on the table.)

Example 2: Due to higher than expected fish stocks at Sylvan Lake, the government decided to limit the recreational fishing quota to 20% of the fish available. In the meantime, scientists predict that the fish stock at Sylvan Lake will increase by 675 every year. The current fish population at Sylvan Lake is approximately at 4500.

a. Complete the table below.
b. Define a recursive sequence to be entered into the calculator.
c. Using the calculator, determine the fish stock at Sylvan Lake at the beginning of the 10th year.
d. Graph the fish stock population versus time. What is the maintenance level of Sylvan Lake's fish population?

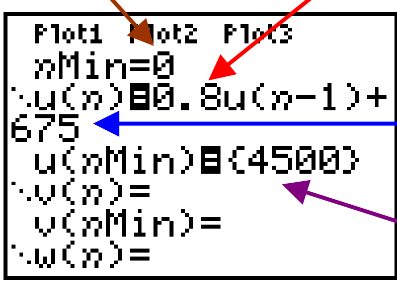
a.

Time (Beginning of the Year)	Current Population	Fishing Quota	Natural Population Increase	Population Remaining
0	4500	900	675	4275
1	4275	**855**	**675**	**4095**
2	**4095**	**819**	**675**	**3951**
3	**3951**	**790**	**675**	**3836**
4	**3836**	**767**	**675**	**3744**
5	**3744**	**749**	**675**	**3670**

b.

Starting Time

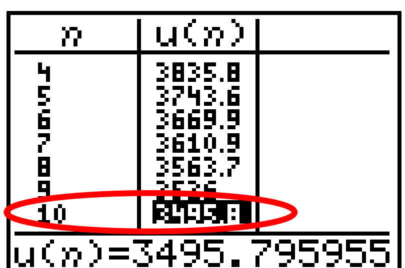
1- 20% = 80% Remaining



Annual Increase

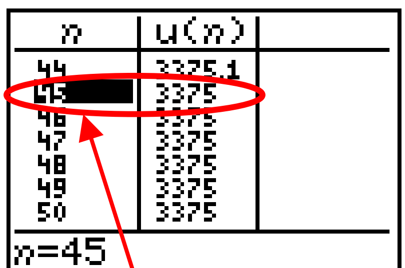
Starting Amount

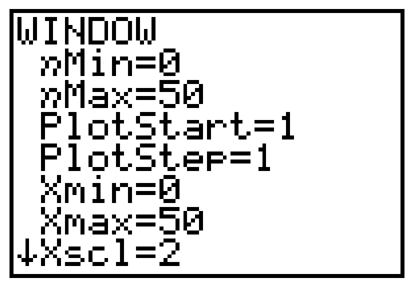
c.

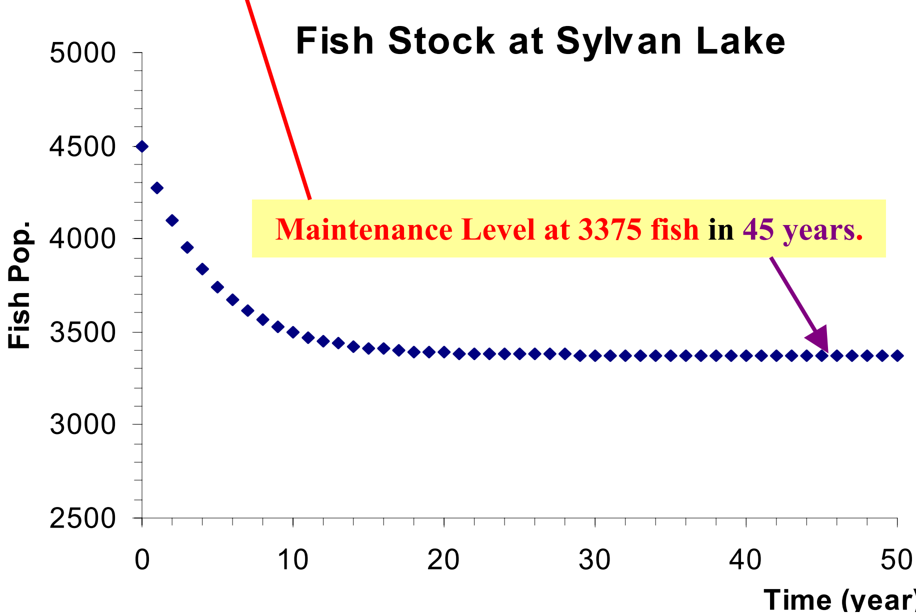


In 10 years, there will be 3496 fish in Sylvan Lake.

d. **2nd** **TABLE** **GRAPH** **WINDOW** *x*: [0, 50, 2] *y*: [0, 4500, 500]







Maintenance Level at 3375 fish in 45 years.

Half-life: - the amount of time needed to decrease the previous amount by half.

Example 3: A drug is taken every 3 hours. The dosage of this drug is 500 mg. The half-life of this drug is 1.5 hours. Complete the table below and answer the followings

a. What fraction of the initial dosage remains in the body 3 hours later (before the second dose was taken)?
b. Define a recursive sequence to be entered into the calculator by the number of doses.
c. Using the calculator, determine the amount of at the beginning of the second day of treatment.
d. Graph the level of drug in the body versus doses. What is the maintenance level of this drug?
e. Set up an EXCEL spreadsheet that will yield the same result as the calculator.

Time (hours)	Number of Dosage (n)	Amount of Drug Remains from Last Period (mg)	Amount of Drug Excreted (mg)	Amount of Drug Taken (mg)	Amount of Drug Remains for the Period (mg)
0	1	0	0	500	500
1.5	1	500	250	0	250
3.0	2	250	125	500	625
4.5	2	625	**312.5**	**0**	**312.5**
6.0	3	**312.5**	**156.25**	**500**	**656.25**
7.5	3	**656.25**	**328.125**	**0**	**328.125**
9.0	4	**328.125**	**164.0625**	**500**	**664.0625**
10.5	4	**664.0625**	**332.03125**	**0**	**332.03125**
12.0	5	**332.03125**	**166.015625**	**500**	**666.015625**
13.5	5	**666.015625**	**333.0078125**	**0**	**333.0078125**
15.0	6	**333.0078125**	**166.5039063**	**500**	**666.5039063**

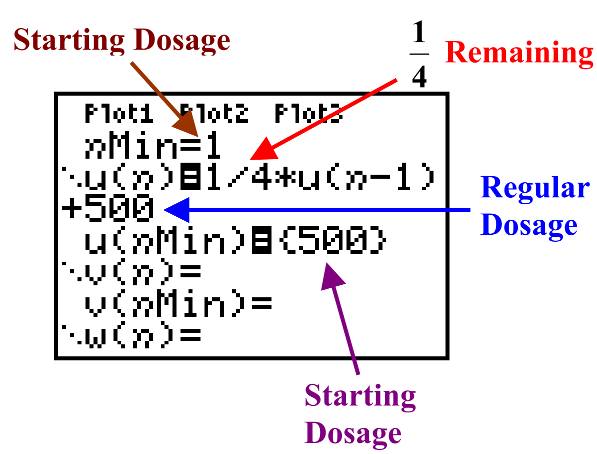
a.

3 hours later (before the second pill was taken), the amount is 250 mg.

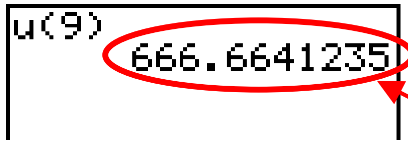
$$\text{Fractions of the Initial Dosage} = \frac{250\text{ mg}}{500\text{ mg}}$$

Fractions of the Initial Dosage = $\frac{1}{4}$

b.

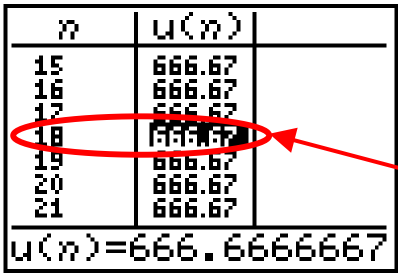


c. At the beginning of the second day, 9 doses would have been taken.

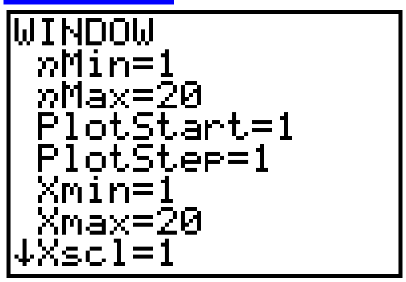


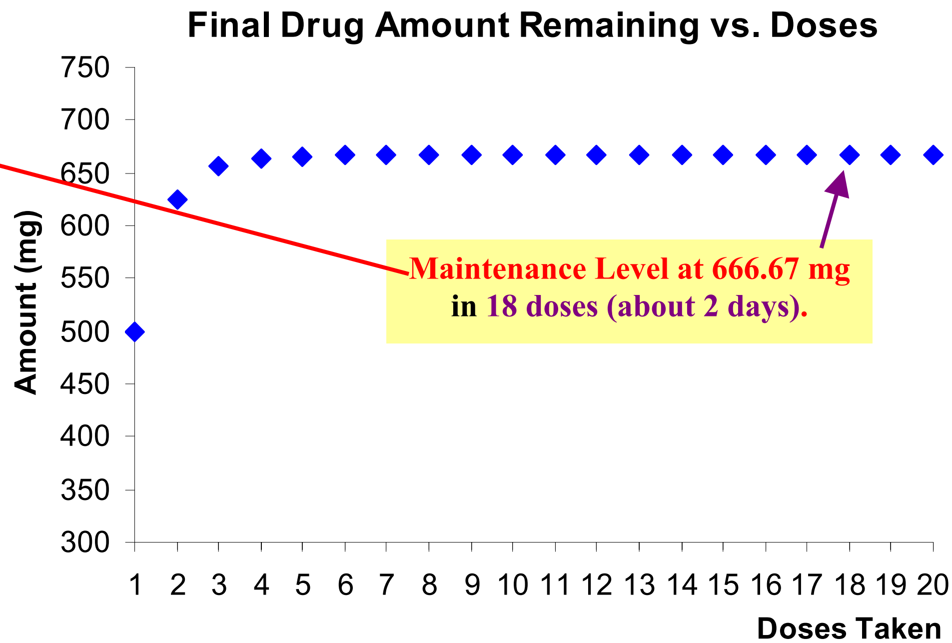
At the beginning of the second day, there will be 666.66 mg in the body.

d. **2nd** **TABLE** **GRAPH**



WINDOW x: [1, 20, 1] y: [300, 750, 50]





e.

	A	B	C	D	E
1	Dosages (n)	Previous Amount Remaining (mg)	Amount Excreted (mg)	Amount of Drugs Taken (mg)	Final Amount Remaining (mg)
2	1	0	=0.25*B2	=500	=B2-C2+D2
3	=A2+1	=E2	=0.75*B3	=500	=B3-C3+D3
4	=A3+1	=E3	=0.75*B4	=500	=B4-C4+D4
5	=A4+1	=E4	=0.75*B5	=500	=B5-C5+D5
6	=A5+1	=E5	=0.75*B6	=500	=B6-C6+D6
7	=A6+1	=E6	=0.75*B7	=500	=B7-C7+D7
8	=A7+1	=E7	=0.75*B8	=500	=B8-C8+D8
9	=A8+1	=E8	=0.75*B9	=500	=B9-C9+D9
10	=A9+1	=E9	=0.75*B10	=500	=B10-C10+D10
11	=A10+1	=E10	=0.75*B11	=500	=B11-C11+D11
12	=A11+1	=E11	=0.75*B12	=500	=B12-C12+D12
13	=A12+1	=E12	=0.75*B13	=500	=B13-C13+D13
14	=A13+1	=E13	=0.75*B14	=500	=B14-C14+D14
15	=A14+1	=E14	=0.75*B15	=500	=B15-C15+D15
16	=A15+1	=E15	=0.75*B16	=500	=B16-C16+D16
17	=A16+1	=E16	=0.75*B17	=500	=B17-C17+D17
18	=A17+1	=E17	=0.75*B18	=500	=B18-C18+D18
19	=A18+1	=E18	=0.75*B19	=500	=B19-C19+D19
20	=A19+1	=E19	=0.75*B20	=500	=B20-C20+D20
21	=A20+1	=E20	=0.75*B21	=500	=B21-C21+D21

Example 4: In a fish stock recovery program, the government introduced 60 tonnes of fish into Lake Winnipeg every year. However, 35% of Lake Winnipeg's water flows into Lake Winnipegosis. There are presently 360 tonnes of fish in Lake Winnipeg and 95 tonnes of fish in Lake Winnipegosis.

a. Fill in the table below.
b. Define the sequences for the graphing calculator to determine the amount of fish in each lake over any given years.
c. Calculate the amount of fish in each lake 15 years from now.
d. Determine the maintenance levels of each lake and the amount of time needed to reach them.

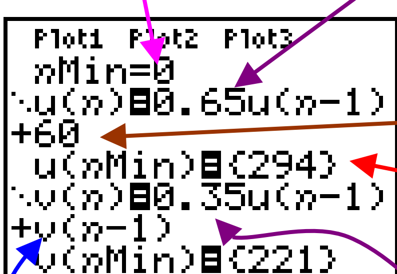
a.	**Lake Winnipeg**				**Lake Winnipegosis**		
Time (years)	**Beginning Amount (Tonne)**	**Amount Lost (Tonne)**	**Amount Added (Tonne)**	**Final Amount (Tonne)**	**Beginning Amount (Tonne)**	**Amount Added (Tonne)**	**Final Amount (Tonne)**
0	360	126	60	294	95	126	221
1	294	**102.9**	**60**	**251.1**	221	**102.9**	**323.9**
2	**251.1**	**87.885**	**60**	**223.215**	**323.9**	**87.885**	**411.785**
3	**223.215**	**78.12525**	**60**	**205.08975**	**411.785**	**78.12525**	**489.91025**
4	**205.08975**	**71.7814125**	**60**	**193.3083375**	**489.91025**	**71.7814125**	**561.6916625**

b. **u(*n*) = Amount at Lake Winnipeg**
v(*n*) = Amount at Lake Winnipegosis

To access v, press **2nd** **v** **8**

Starting Time (year)

(1 – 35%) = 65% Remaining



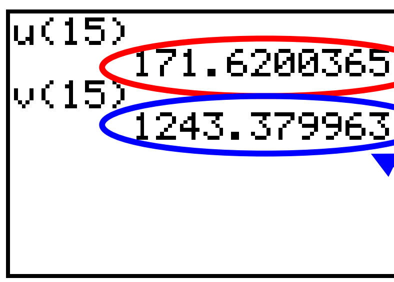
Regular Amount Added

Initial Final Amount

Initial Final Amount

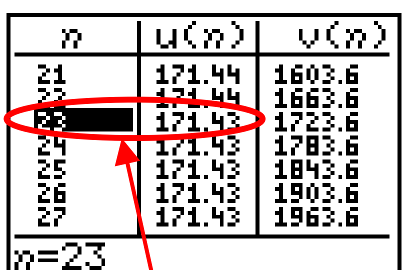
35% Increase with Previous Amount

c.



In 15 years, there will be 172 fish in Lake Winnipeg and 1243 fish in Lake Winnipegosis.

d.



Lake Winnipegosis does NOT have a Maintenance Level because there is no migration of fish out of that lake.

Maintenance Level at 171 fish in Lake Winnipeg in 23 years.

6-2 Assignment: pg. 270 – 273 #1 to 9

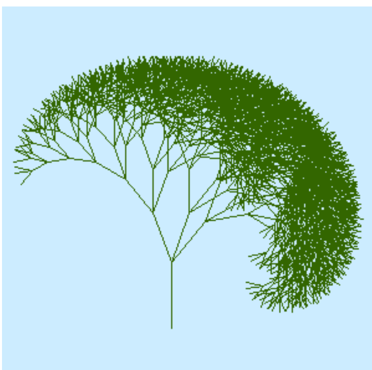
6-3: Introduction to Fractals

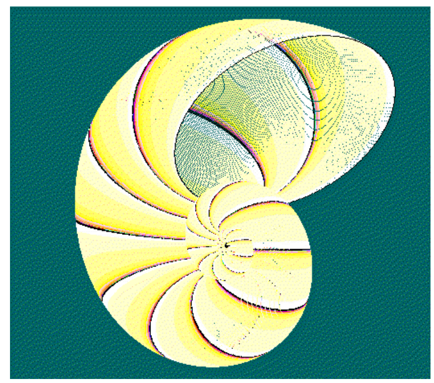
Fractals: - geometric figures that can be generated by **REPEATING the same process many times**.

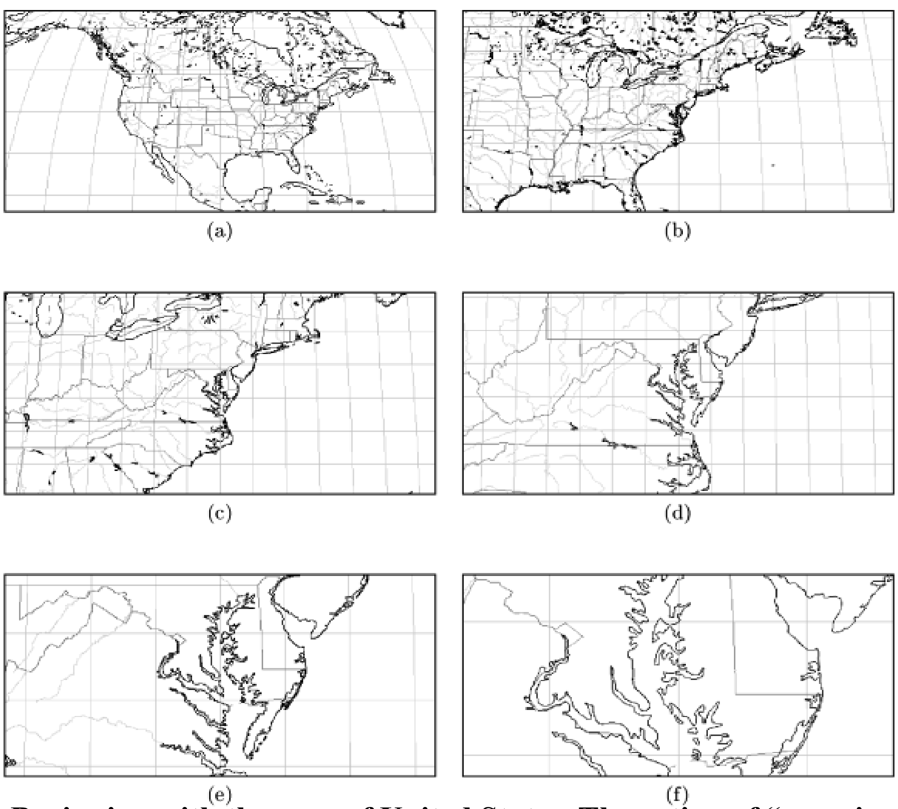
Self-Similarity Fractals: - where a **Part** of a fractal is **Geometrically Similar to the Whole Fractal**.
- this can be applied to simple geometry generations or **ZOOMING in and out of a diagram.**

Iteration: - a step where the sequence pattern repeats itself.

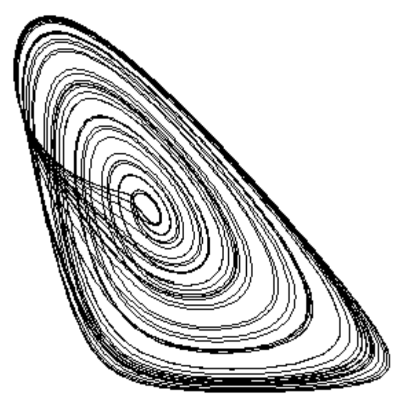
Many natural phenomena that may seem random at first actually have a self-repeating pattern. We can model these patterns using fractals. The following are some beginning iterations and their many iterations after.

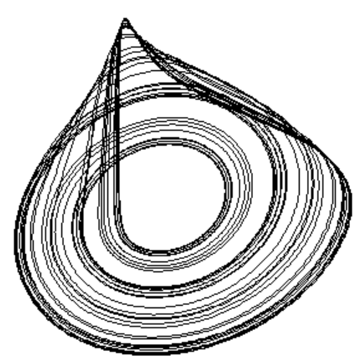


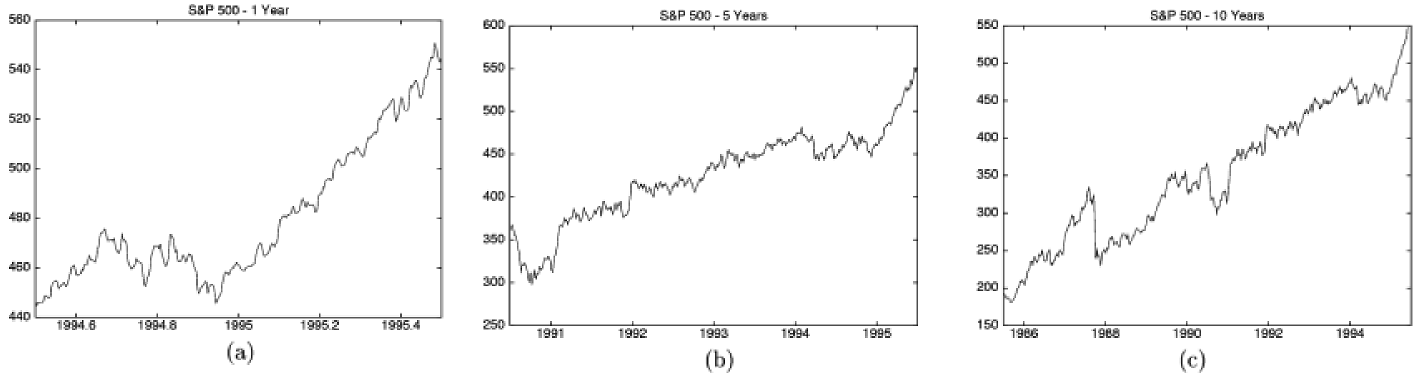




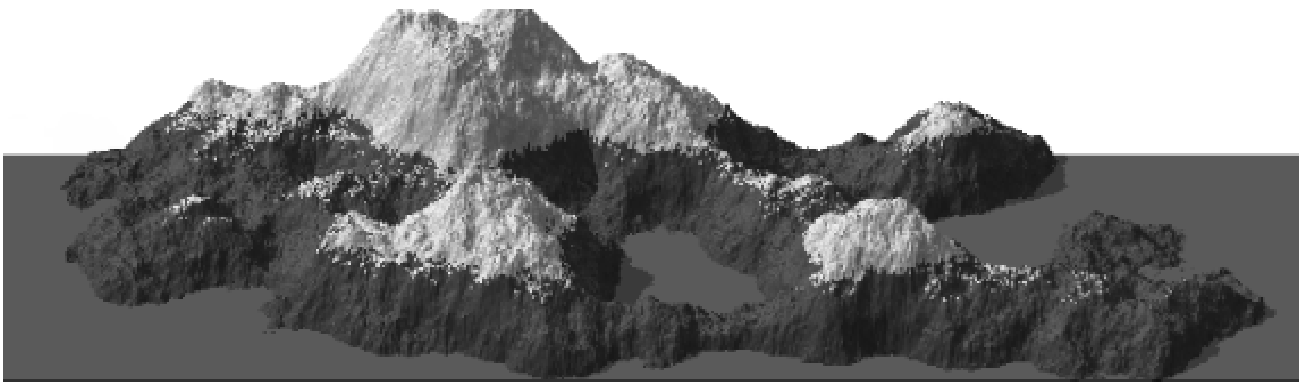
Beginning with the map of United States. The action of "zooming in" results in the last image, which depicts area off Chesapeake Bay, Maryland.



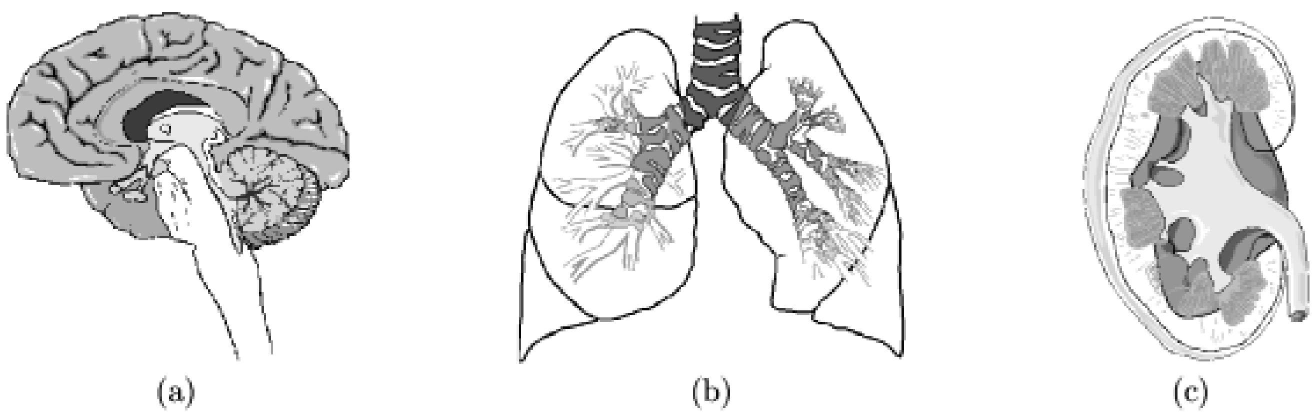




The S&P 500 Index at different time scales: (a) 1 year, (b) 5 years, and (c) 10 years.



Mountains generated by a fractal computer program.

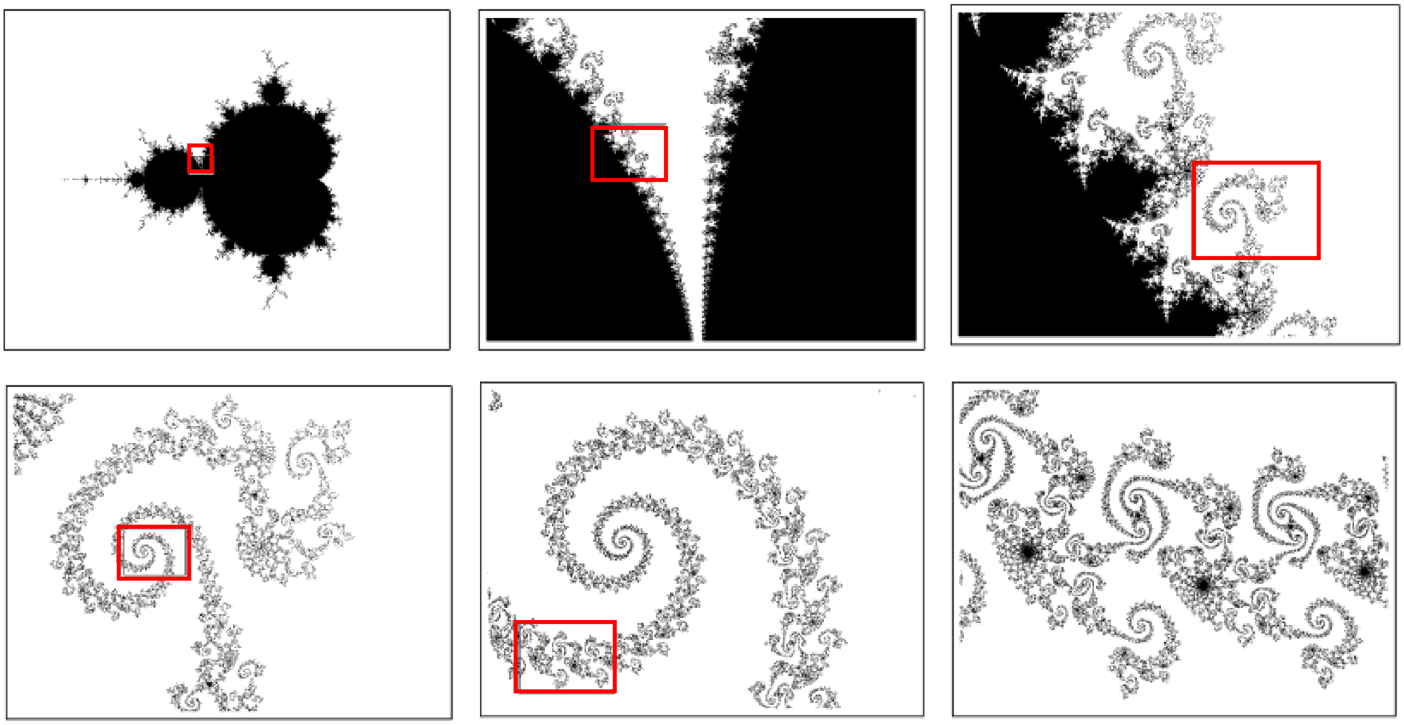


Fractals in the Human Body: (a) Brain, (b) Lungs, and (c) Kidney.

Fractal Generated Cloud

Broccoli, a Natural Fractal

Lightning, Nature's Powerful Fractal



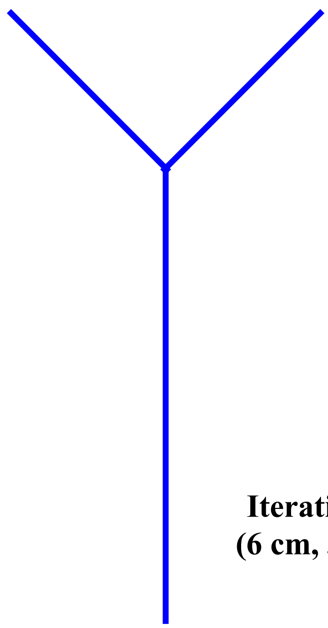
The Mandelbrot Set: Moving from top left to bottom right, each successive diagram illustrates the last image by zooming in a particular area.

Maps showing the Coastlines of the British Isle and the English Channel. The first map (top left corner) is an approximation (original diagram). As the fractal iterations are being generated (moving from top left to bottom right), the coastlines appear more and more detail.

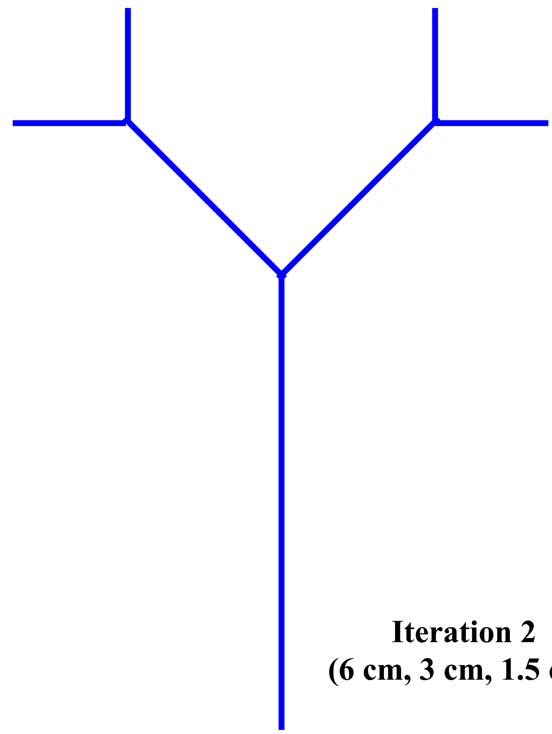
Example 1: Draw the next two iterations of the following fractals and indicate their dimensions.

a. **Simple Tree Fractal**

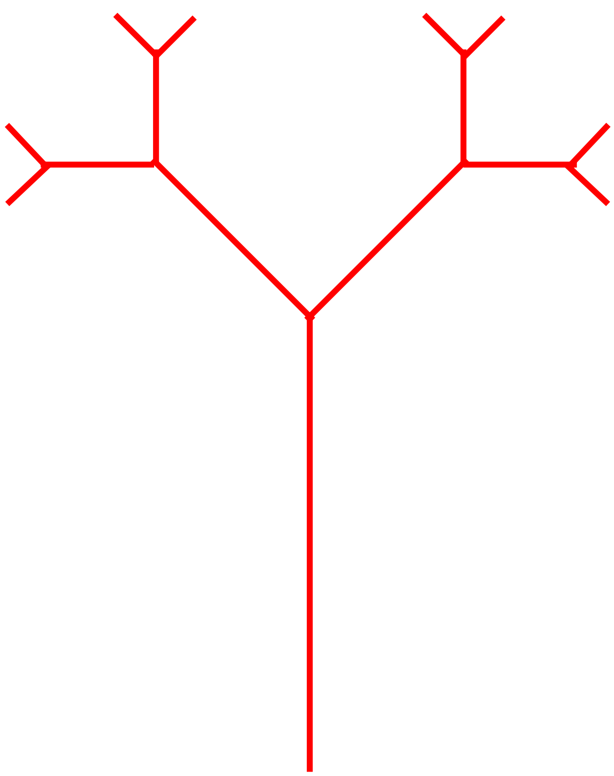
Original
(6 cm)



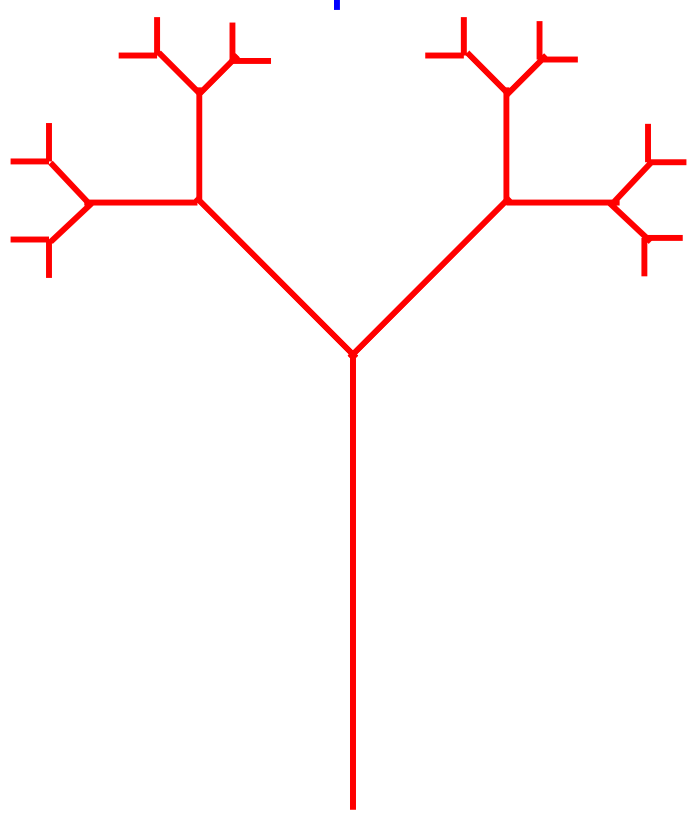
Iteration 1
(6 cm, 3 cm)



Iteration 2
(6 cm, 3 cm, 1.5 cm)

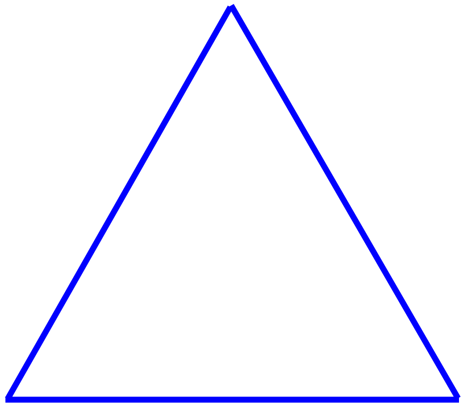


Iteration 3
(6 cm, 3 cm, 1.5 cm, 0.75 cm)

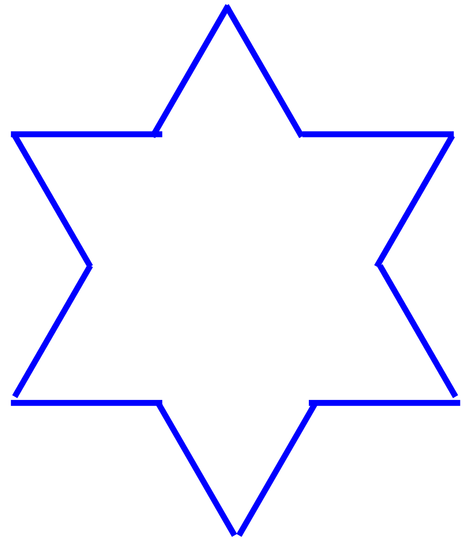


Iteration 4
(6 cm, 3 cm, 1.5 cm, 0.75 cm, 0.375 cm)

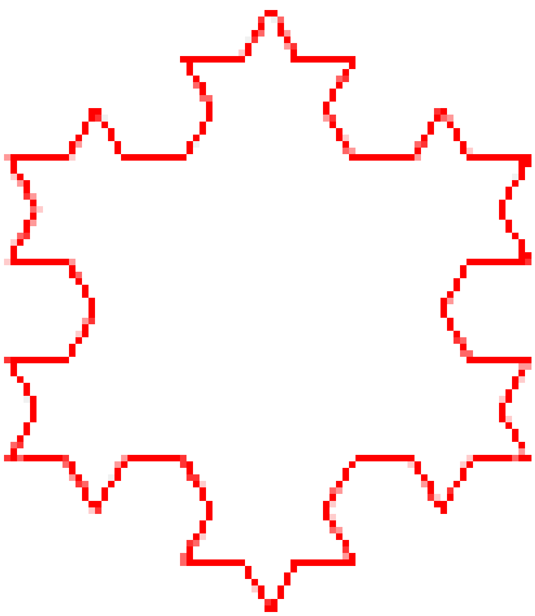
b. **Koch Snowflake**



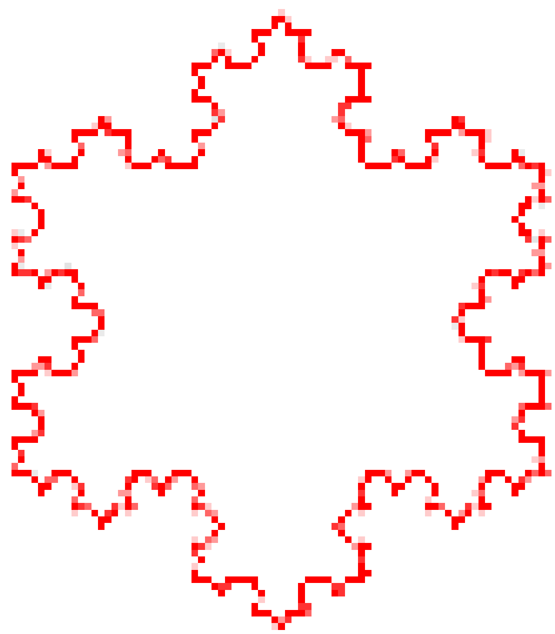
Original ($s = 6$ cm)



Iteration 1 ($s = 2$ cm)

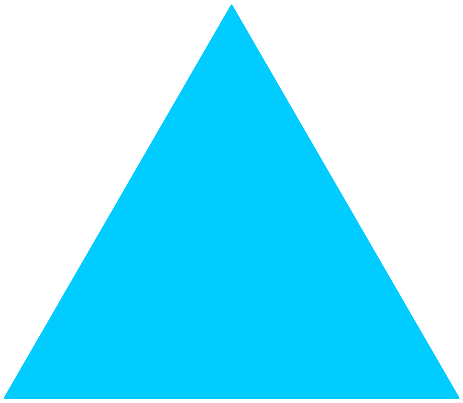


Iteration 2 ($s = \frac{2}{3}$ cm)

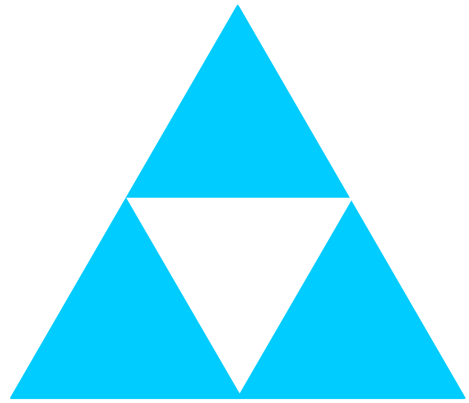


Iteration 3 ($s = \frac{2}{9}$ cm)

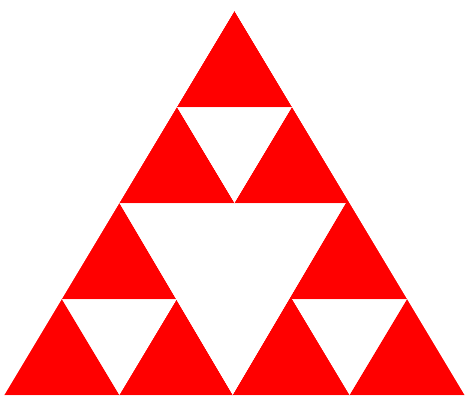
c. **Sierpinski Gasket**



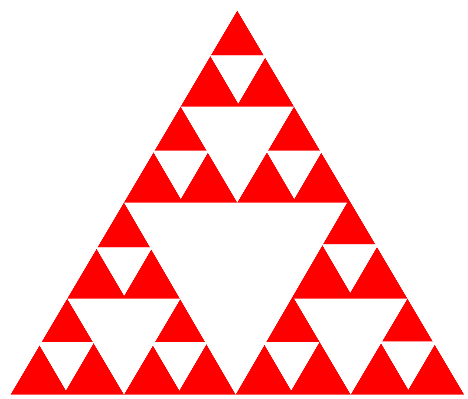
Original (*s* = 6 cm)



Iteration 1 (*s* = 3 cm)



Iteration 2 (*s* = 1.5 cm)



Iteration 3 (*s* = 0.75 cm)

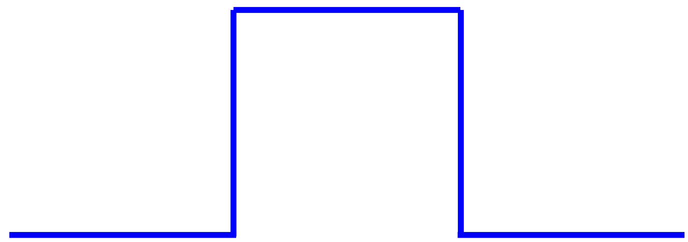
6-3 Assignment: **6-3 Worksheet: Drawing Fractals**

6-3 Worksheet: Drawing Fractals

1. Draw the next two iterations of each of the following fractal. Label their dimensions

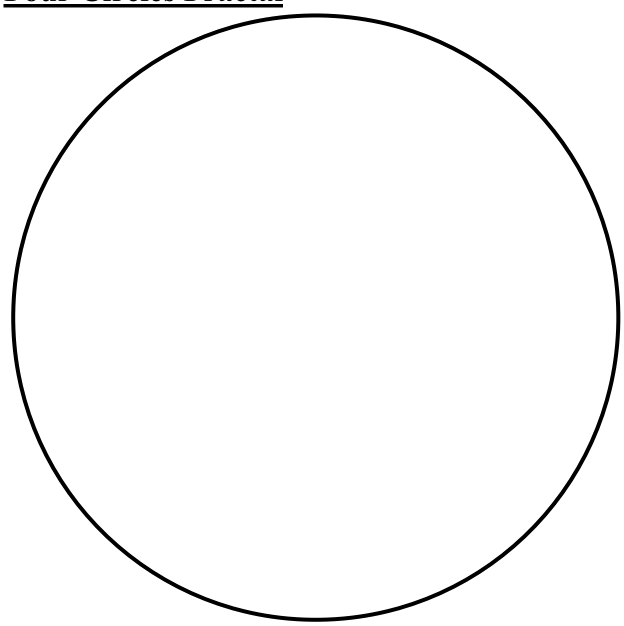
a. Hat Fractal

Original ($s = 9$ cm)

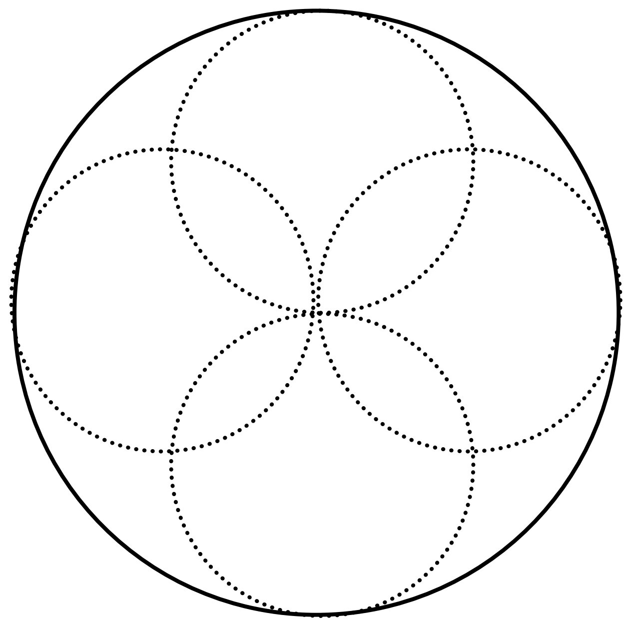


Iteration 1 ($s = 3$ cm)

b. Four Circles Fractal

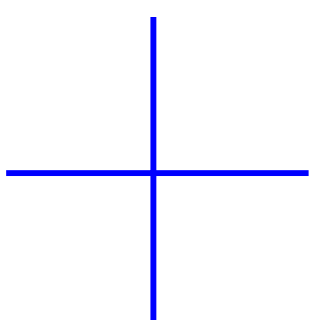


Original ($r = 4$ cm)

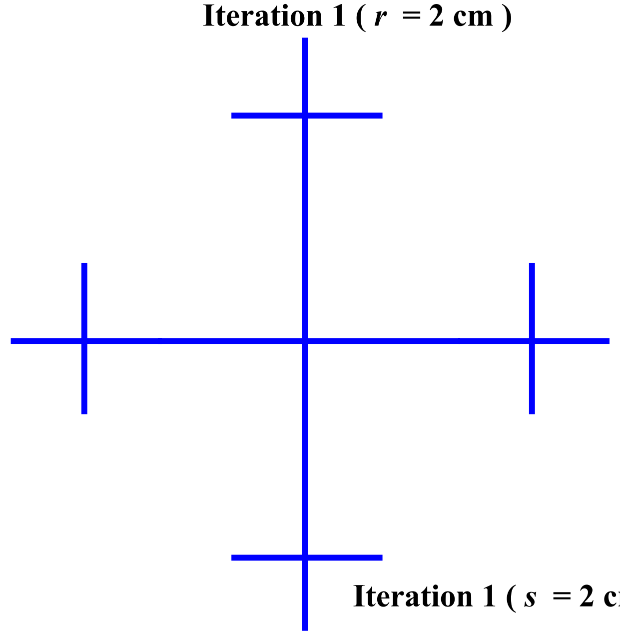


Iteration 1 ($r = 2$ cm)

c. Crosses



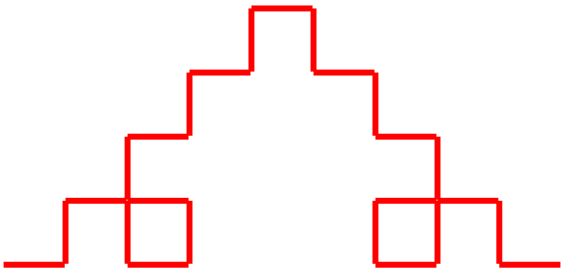
Original ($s = 4$ cm)



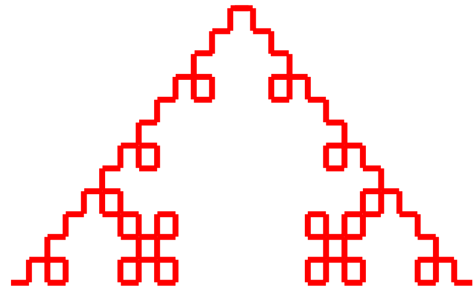
Iteration 1 ($s = 2$ cm)

Answers

1a. **Hat Fractals**

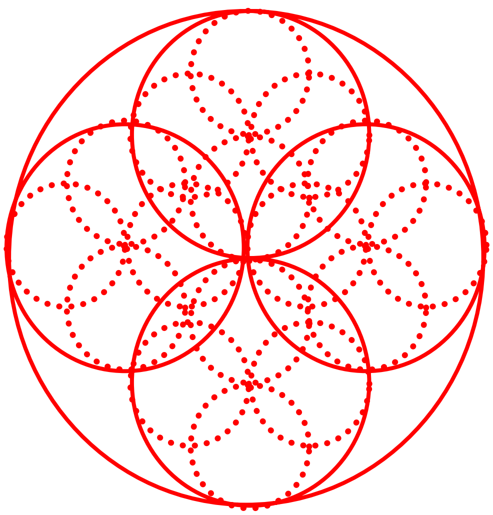


Iteration 2 (s = 1 cm)

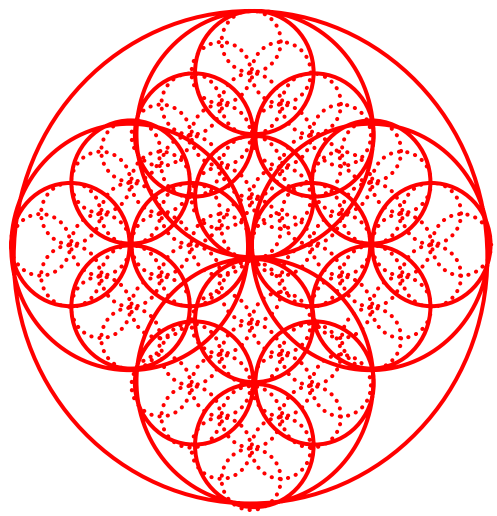


Iteration 3 ($s = \frac{1}{3}$ cm)

1b. **Four-Circles Fractals**

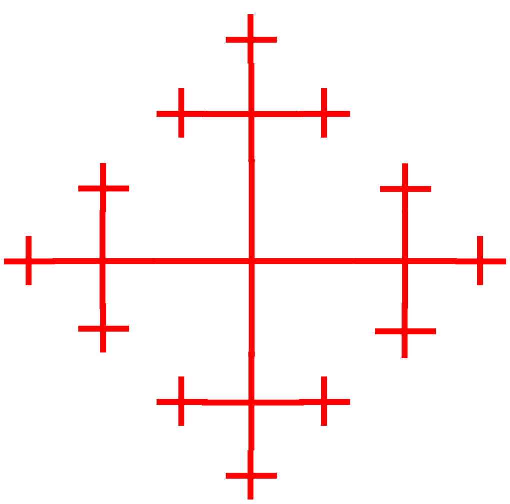


Iteration 2 (r = 1 cm)

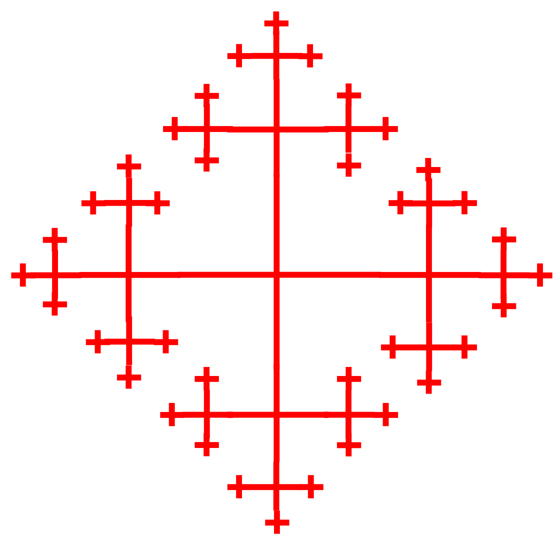


Iteration 3 (r = 0.5 cm)

1b. **Crosses**



Iteration 2 (s = 1 cm)

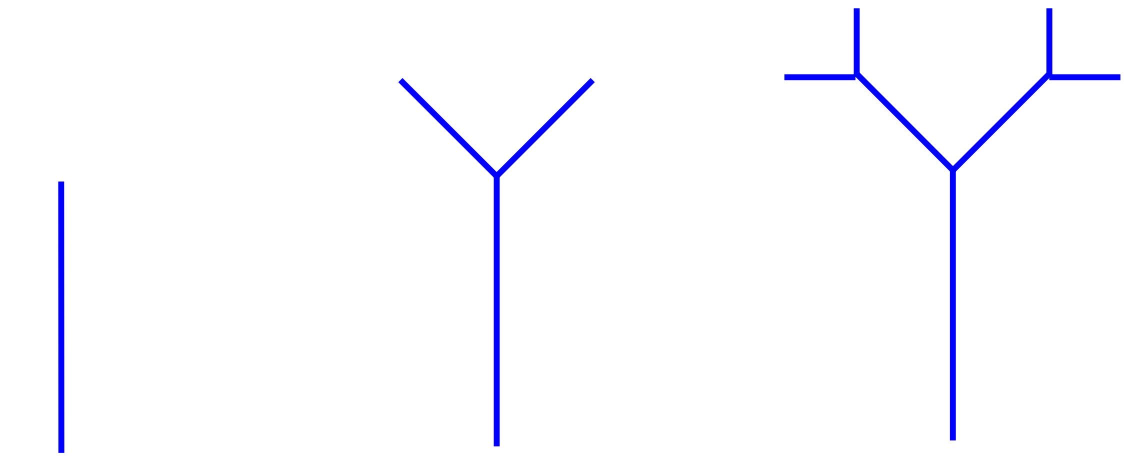


Iteration 3 (s = 0.5 cm)

6-4: Some Properties of Fractals

After a few iterations from the original, we can determine the pattern of the fractals. Thereby, predicting the properties of the new iterations.

Example 1: Using the tree fractal underneath, fill out the table below.

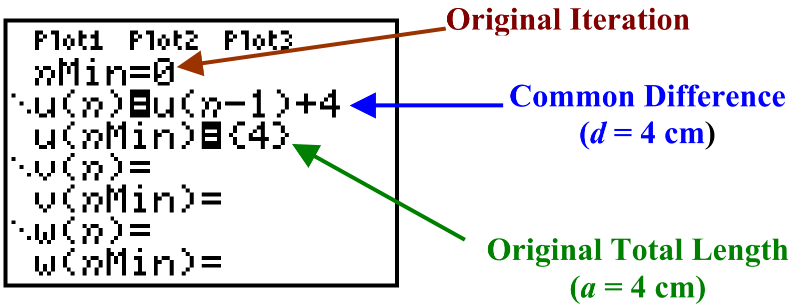


Original **Iteration 1** **Iteration 2**

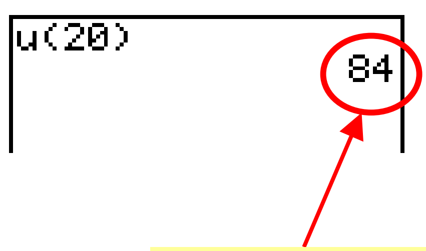
	Number of New Branches	Length of the New Branch (cm)	Total New Length Added (cm)	Total Length (cm)
Original	1	4	4	4
Iteration 1	2	2	**4**	**8**
Iteration 2	4	1	**4**	**12**
Iteration 3	**8**	**0.5**	**4**	**16**

a. Write a sequence that will relate the total length of the fractals at any iterations (n).

The total lengths form an **arithmetic sequence** **starting at a = 4 cm** with a **common difference, d = 4 cm**.

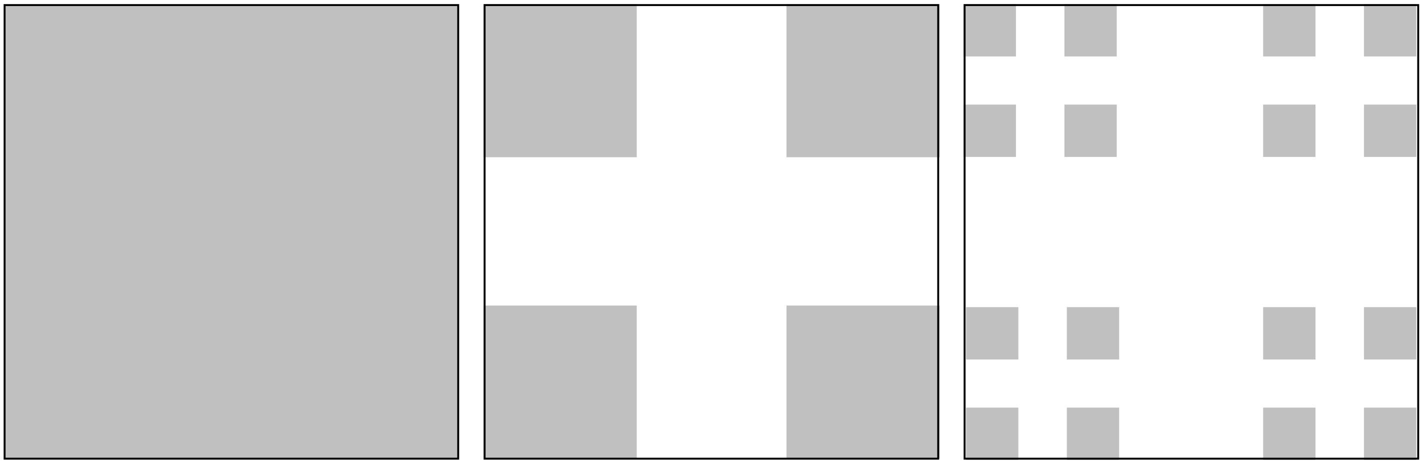


b. Find the total length at iteration 20.



The total length at Iteration 20 is 84 cm.

Example 2: Using the fractal underneath, fill out the table below.



Original **Iteration 1** **Iteration 2**

	Number of Shaded Squares	Length of each Shaded Square (cm)	Area of each Shaded Square (cm^2)	Total Shaded Area (cm^2)
Original	1	6	36	36
Iteration 1	$\times 4$ → 4	2	$\div 9$ → **4**	$\times \frac{4}{9}$ → **16**
Iteration 2	$\times 4$ → 16	$\frac{2}{3}$ = **0.66…**	$\div 9$ → $\frac{4}{9}$ = **0.44…**	$\times \frac{4}{9}$ → $\frac{64}{9}$ = **7.11…**
Iteration 3	$\times 4$ → **64**	$\frac{2}{9}$ = **0.22…**	$\div 9$ → $\frac{4}{81}$ = **0.04938272...**	$\times \frac{4}{9}$ → $\frac{256}{9}$ = **3.1604938…**

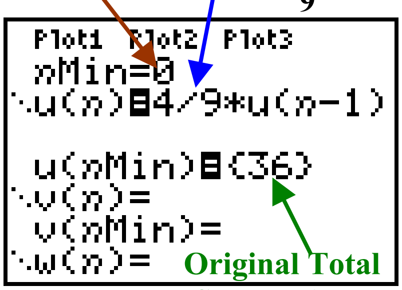
a. Write a sequence that will relate the total shaded area of the fractals at any iterations (n).

The total shaded area forms a geometric sequence.

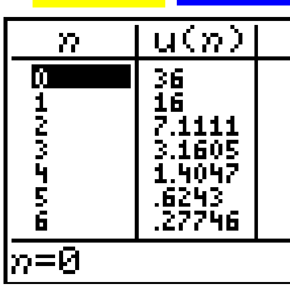
Common Ratio, $r = \frac{4}{9}$

$r = \frac{16}{36} = 0.44... = \frac{4}{9}$ $\quad r = \frac{7.11...}{16} = 0.44... = \frac{4}{9}$

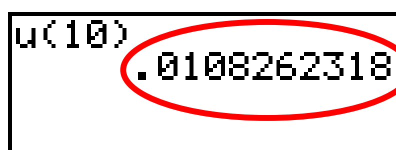
Original Iteration **Common Ratio** ($r = \frac{4}{9}$) **Verify with** **2nd** **TABLE** **GRAPH**



Original Total Shaded Area



b. Find the total shaded area at iteration 10.



The total length at Iteration 10 is 0.0108 cm^2.

c. What is the limit of the total shaded area?

As the diagrams above would indicate, **the total shaded area will become smaller and smaller until it is close to 0 cm^2.**

d. What is the limit of the total **non-shaded** area?

On the other hand, **the total non-shaded area will become bigger and bigger until it is close to 36 cm^2 (the shaded area in the original diagram).**

6-4 Assignment
pg. 289 – 291
#1 to 4