Unit 8: Designs

8-1: Reviewing Perimeter, Area, Surface Area and Volume

Perimeter: - the length (one-dimensional) around an object.

Area: - the amount of space (two-dimensional) a flat-object occupies.

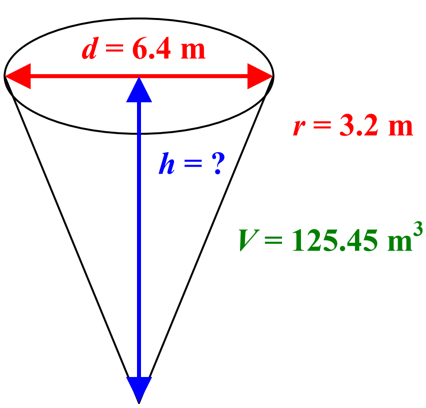
Surface Area: - the amount of surface material (two-dimensional) needed to build a 3-Dimensional object.

Volume: - the amount of space (three-dimensional) an object occupies.

Formulas that might be useful for this unit

Area		***Surface Area***		***Volume***	
Circle	$A = \pi r^2$	Sphere	$SA = 4\pi r^2$	Sphere	$V = \frac{4}{3}\pi r^3$
Triangle	$A = \frac{b \times h}{2}$	Cylinder	$SA = 2\pi r^2 + 2\pi rh$	Cylinder	$V = \pi r^2 h$
Parallelogram	$A = b \times h$	Cone	$SA = \pi r^2 + \pi rs$	Prism	$V = B \times h$, where B is the area of the base
Trapezoid	$A = h\left(\frac{b_1 + b_2}{2}\right)$			Cone	$V = \frac{1}{3}\pi r^2 h$
Circumference (Perimeter of a Circle)	$C = 2\pi r$ $C = \pi d$			Pyramid	$V = \frac{B \times h}{3}$, where B is the area of the base

Example 1: A water-tank in the shape of an inverted cone has a diameter of 6.4 m and can hold 125.45 m^3 of water. To the nearest tenth of a metre, what is the height of this water-tank?



$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{3V}{\pi r^2} = h$$ (Solve for h by manipulating the formula)

$$h = \frac{3(125.45\text{ m}^3)}{\pi(3.2\text{ m})^2}$$

$h = 11.7$ m

Example 2: A spherical rubber balloon has a maximum radius of 40 cm. If on average, a person can exhale 1500 cm^3 of air at one time. How many times an average person need to exhale in order to inflate the rubber balloon to its maximum volume?

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi(40 \text{ cm})^3$$

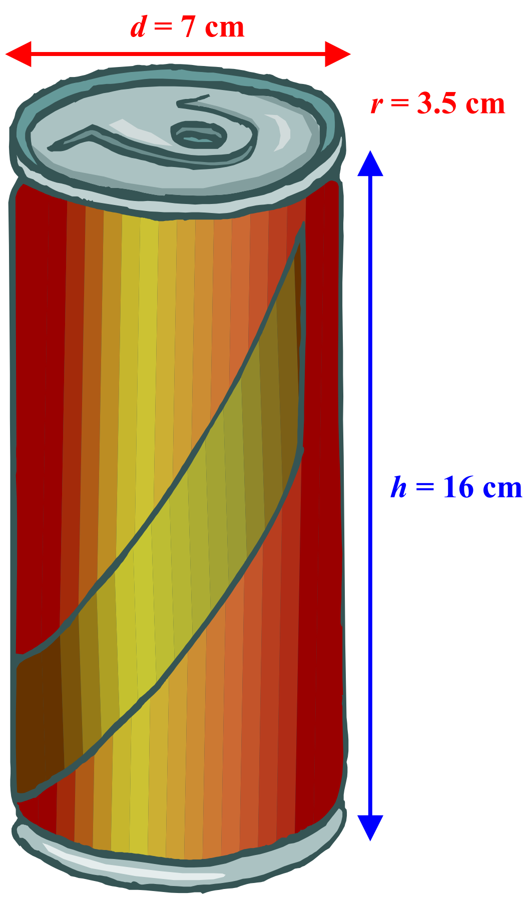
$$\mathbf{V = 268{,}082.5731 \text{ cm}^3}$$

$$\text{Number of times needed to exhale} = \frac{268082.5731 \text{ cm}^3}{1500 \text{ cm}^3 \text{ / exhale}}$$

Need to Exhale 179 times

Example 3: A frozen concentrated juice can is made up of tin for the top and bottom lids with waterproof cardboard paper as the "wall" of the can. Using the diagram below, calculate the cost to produce a can of frozen concentrated juice to the ten-thousandth of a dollar.

Cost of Tin = \$0.0008/cm^2
Cost of Waterproof Cardboard Paper = \$0.0003/cm^3
Cost of Frozen Concentrated Juice = \$0.00035/mL (1 cm^3 = 1 mL)



Volume of the Can = $\pi r^2 h = \pi(3.5 \text{ cm})^2(16 \text{ cm})$
$V = 615.7521601 \text{ cm}^3$
$V = 615.7521601$ mL

Cost of Juice per Can = 615.7521601 mL × \$0.00035/mL
Cost of Frozen Juice per Can = \$0.215513256

Area of the Lids = $2(\pi r^2) = 2\pi(3.5 \text{ cm})^2 = 76.96902001 \text{ cm}^2$

Cost of the Tin Lids per Can = 76.96902001 cm^2 × \$0.0008/cm^2
Cost of the Tin Lids per Can = \$0.061575216

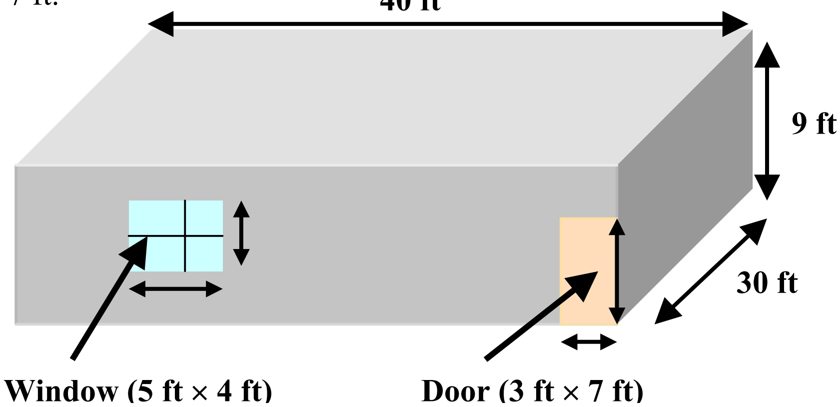
Area of the Cardboard = $2\pi rh = 2\pi(3.5 \text{ cm})(16 \text{ cm})$
Area of the Cardboard = 351.8583772 cm^2

Cost of the Cardboard per Can = 351.8583772 cm^2 × \$0.0003/cm^2
Cost of the Cardboard per Can = \$0.1055575132

Total Cost per Can = Juice Cost + Lids Cost + Cardboard Cost
= \$0.215513256 + \$0.061575216 + \$0.1055575132

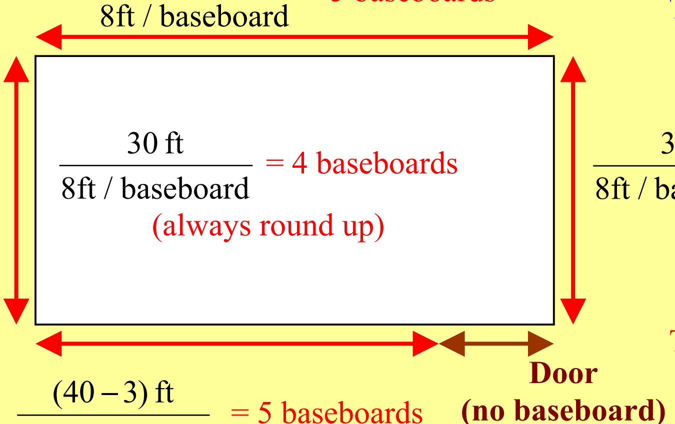
Total Cost per Can = \$0.3826

Example 4: A room, shown below, is measured 40 ft by 30 ft. The height of the room is 9 ft. There is a window in this room with the dimension of 5 ft by 4 ft, and a door with a width of 3 ft along with a height of 7 ft.



a. Baseboards are sold in 8-ft section at \$15. How many 8-ft baseboards are needed to cover the perimeter of the floor? How much would it cost?
b. What is the total wall area of this room?
c. If a one can of paint can covers 380 ft^2, and paint costs \$30.00/can, what is the total cost to paint the walls of this room if two coats of paint are used?

a. $\dfrac{40 \text{ ft}}{8\text{ft / baseboard}}$ = 5 baseboards



$\dfrac{30 \text{ ft}}{8\text{ft / baseboard}}$ = 4 baseboards (always round up)

$\dfrac{30 \text{ ft}}{8\text{ft / baseboard}}$ = 4 baseboards (always round up)

$\dfrac{(40-3) \text{ ft}}{8\text{ft / baseboard}}$ = 5 baseboards (always round up)

Total Number of Baseboards = 5 + 4 + 5 + 4

Total Number of Baseboards = 18

Total Cost = 18 baseboards × \$15 / baseboard

Total Cost = \$270

b. Total Wall Area = Back + Left + Right + Front – Window – Door
= (40 ft × 9 ft) + (30 ft × 9 ft) + (30 ft × 9 ft) + (40 ft × 9 ft) – (5 ft × 4 ft) – (3 ft × 7 ft)
= 360 ft^2 + 270 ft^2 + 270 ft^2 + 360 ft^2 – 20 ft^2 – 21 ft^2

Total Wall Area = 1219 ft^2

c. Number of Cans Needed (Two Coats of Paint) = $\dfrac{1219 \text{ ft}^2}{380 \text{ ft}^2} \times 2$ = 7 cans (always round up)

Total Paint Cost = 7 cans × \$30.00/can

Total Paint Cost = \$210

8-1 Assignment: 8-1 Worksheet

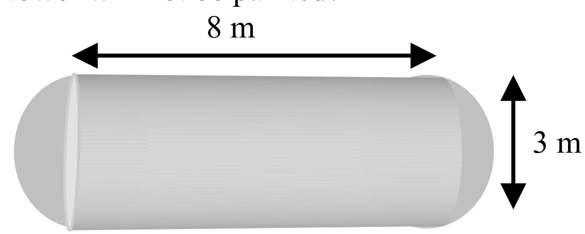
8-1 Worksheet: Reviewing Perimeter, Area, Surface Area and Volume

1. A room is 9.5 m long, 8.8 m wide and 3.5 m high. For health reasons, there must be at least 5 m^3 of air per person in the room. How many students can safely use the room?

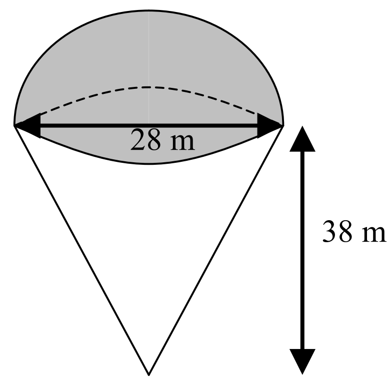
2. A spherical weather balloon 16.2 m in diameter is filled with helium. Each tank of helium provides 30 m^3 of gas and costs \$25.75. What is the cost of filling the balloon with helium?

3. John works for Smooth Painting Company. One can of paint costs \$35.00 and covers 25 m^2. Two coats of paint are needed to cover the water tower that has a diameter of 12 m and a height of 30 m. What is John's cost for paint for the tower if the bottom of the tower will not be painted?

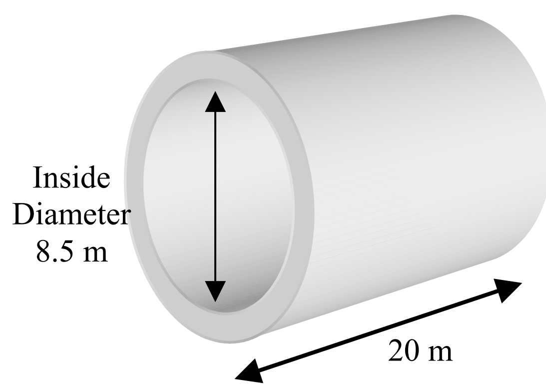
4. A propane storage tank consists of a cylindrical body and hemispherical end sections shown to the right. To the nearest hundredth of a square metre, calculate the area to be tar-coated before the tank is buried in the ground.



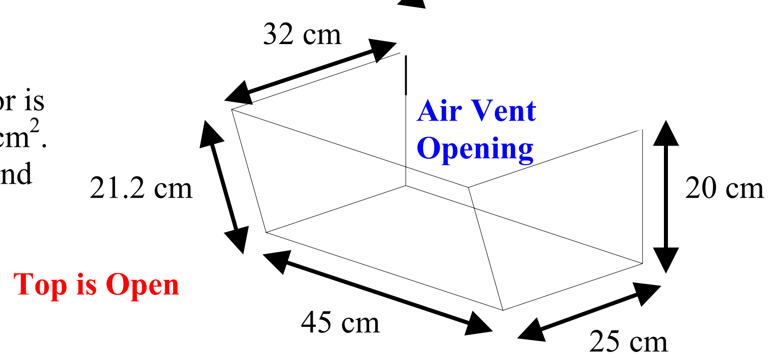
5. The gas contained in the hot air balloon is in the form of a hemisphere on top of a cone. What volume of gas will the balloon hold to the nearest m^3?



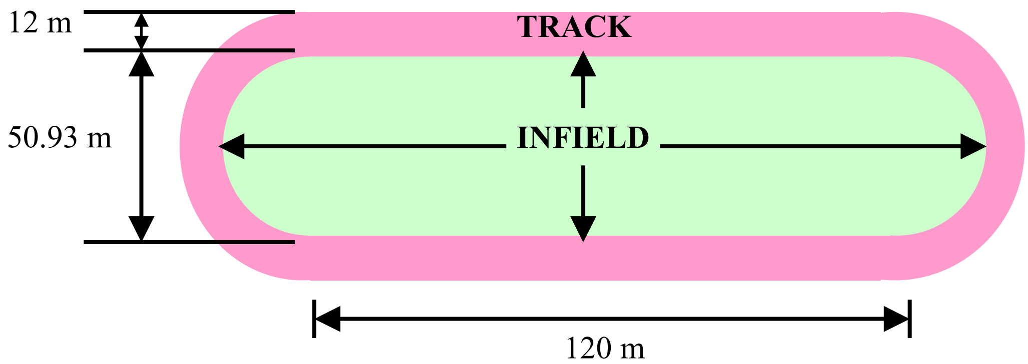
6. The wall of a storm sewage pipe is made out of concrete as shown on the right. If the wall is 0.5 m thick and concrete costs \$85/m^3, calculate the volume of concrete used to make the silo rounded to the nearest m^3 along with the total cost to the nearest dollar.



7. An air deflector is placed over an air vent opening as shown on the right. The deflector is made out of plastic, which costs \$0.00096/cm^2. Calculate the total area of plastic required and the material cost per deflector.

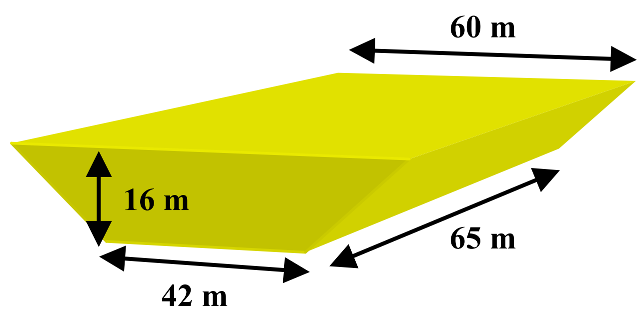


8. A school decided to build a new track area for track and field events as shown below.



a. The inside and outside edges are to be lined with a plastic curbing that cost \$3.95/m. Calculate the total cost of the curbing.
b. The infield area is to be covered with sod at a cost of \$2.50/m^2. Determine the total cost of covering the infield.
c. The base for the track consists of a layer of crushed stone of 30 cm deep. Calculate the cost of the base at \$28/m^3 for crush stone.
d. The track surface will be a composition material costing \$6.55/m^2. What is the total cost of surfacing the track?
e. The contractor of this new track area figures it will take 50 workers over 20 days (at 8 hours per day) to complete the job. If the workers are paid \$15.00 per hour, and the contractor would like to mark up 30% for profit, how much should the contractor quote the school for this new track area?

9. An excavation for the foundation of an office building forms a trapezoidal prism with dimensions shown on the right.
a. Janet operates a front-end loader that holds 2.8 m^3 of earth. What is the least number of loads required to dig the foundation?
b. Dump trucks with a maximum capacity of 11.2 m^3 are used to haul away the earth that Janet removes. If she can load one truck every 7 minutes, how many hours will it take to haul away the earth?



10. A manufacturer produces golf balls with a standard diameter of 5 cm.
a. The golf balls are individually packaged in a cubic paper box. If the paper box has extra overlap material measuring 55 cm^2, what is the total amount of material needed to package one golf ball?
b. Individually packaged golf balls are shipped in a carton box measuring 72 cm by 103 cm by 54 cm. What is the maximum number of golf balls that can be shipped in one carton box?

Answers

1. 58 people (rounded down) **2.** 75 tanks (rounded up); \$1931.25 **3.** 100 cans; \$3500
4. 103.67 m^2 **5.** 13547 m^3 **6.** 283 m^3; \$24,055 **7.** 3219 cm^2; \$3.09
8a. \$3457.83 **8b.** \$20,372.04 **8c.** \$44,120.20 **8d.** \$34,403.25 **8e.** \$289,059.32
9a. 18943 loads **9b.** 552 hours 32 minutes **10a.** 205 cm^2 **10b.** $14 \times 20 \times 10 = 2800$ golf balls

8-2: Using Technologies to Design Minimum Cost

Cost Effectiveness: - when the cost of manufacturing a product or the design of a system is minimized, yet meeting all the requirements of the design.

Highest Cost Effectiveness: - when the minimum cost of the design is achieved.

Example 1: Design a 355 mL pop can with the highest cost effectiveness if the material cost \$0.0005 / cm^2. (1 mL = 1 cm^3)

a. Find the equation that relates height with the radius and the specified volume.

$$V = \pi r^2 h$$
$$355 = \pi r^2 h \qquad \text{(Solving for } h\text{)}$$
$$\frac{355}{\pi r^2} = h$$

$$h = \frac{355}{\pi r^2}$$

b. Create an EXCEL spreadsheet that will calculate the heights, volumes, surface areas and costs for each can with radius 1 cm to 10 cm (by 1 cm interval). Find the dimensions of the can with the least cost.

EXCEL Formulas To enter the exact value of π on EXCEL, type PI()

	A	B	C	D	E
1	**Designing the Highest Cost Effective Pop Can**				
2					
3		**Volume of Pop Can (mL)**		355	
4		**Material Cost ($/square cm)**		0.0005	
5					
6	**Radius (cm)**	**Height (cm)**	**Volume (mL)**	**Surface Area (square cm)**	**Cost ($)**
7	1	=$D$3/(PI()*A7*A7)	=PI()*A7*A7*B7	=2*PI()*A7*A7+2*PI()*A7*B7	=$D$4*D7
8	=A7+1	=$D$3/(PI()*A8*A8)	=PI()*A8*A8*B8	=2*PI()*A8*A8+2*PI()*A8*B8	=$D$4*D8
9	=A8+1	=$D$3/(PI()*A9*A9)	=PI()*A9*A9*B9	=2*PI()*A9*A9+2*PI()*A9*B9	=$D$4*D9
10	=A9+1	=$D$3/(PI()*A10*A10)	=PI()*A10*A10*B10	=2*PI()*A10*A10+2*PI()*A10*B10	=$D$4*D10
11	=A10+1	=$D$3/(PI()*A11*A11)	=PI()*A11*A11*B11	=2*PI()*A11*A11+2*PI()*A11*B11	=$D$4*D11
12	=A11+1	=$D$3/(PI()*A12*A12)	=PI()*A12*A12*B12	=2*PI()*A12*A12+2*PI()*A12*B12	=$D$4*D12
13	=A12+1	=$D$3/(PI()*A13*A13)	=PI()*A13*A13*B13	=2*PI()*A13*A13+2*PI()*A13*B13	=$D$4*D13
14	=A13+1	=$D$3/(PI()*A14*A14)	=PI()*A14*A14*B14	=2*PI()*A14*A14+2*PI()*A14*B14	=$D$4*D14
15	=A14+1	=$D$3/(PI()*A15*A15)	=PI()*A15*A15*B15	=2*PI()*A15*A15+2*PI()*A15*B15	=$D$4*D15
16	=A15+1	=$D$3/(PI()*A16*A16)	=PI()*A16*A16*B16	=2*PI()*A16*A16+2*PI()*A16*B16	=$D$4*D16
17					

Spreadsheet Results

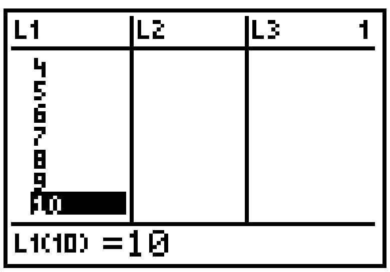
	A	B	C	D	E
1	Designing the Highest Cost Effective Pop Can				
2					
3		Volume of Pop Can (mL)		355	
4		Material Cost ($/square cm)		0.0005	
5					
6	Radius (cm)	Height (cm)	Volume (mL)	Surface Area (square cm)	Cost ($)
7	1	113.000	355	716.283	0.3581
8	2	28.250	355	380.133	0.1901
9	3	12.556	355	293.215	0.1466
10	4	7.063	355	278.031	0.1390
11	5	4.520	355	299.080	0.1495
12	6	3.139	355	344.528	0.1723
13	7	2.306	355	409.305	0.2047
14	8	1.766	355	490.874	0.2454
15	9	1.395	355	587.827	0.2939
16	10	1.130	355	699.319	0.3497

c. Verify that you have the same results as the spreadsheet on the graphing calculator.

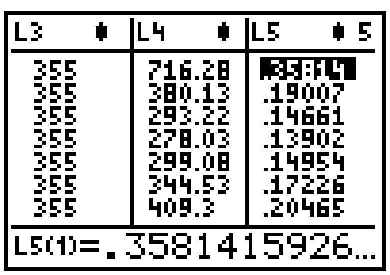
1. Enter 1 to 10 in the L_1 Column of Stats Editor

STAT

ENTER



3. Final Stats Editor Display



2. Enter the following formulas in L_2 to L_5 (All formulas in Stats Editor must begin with quotation mark ").

L_1 = Radius (Enter 1 to 10)

L_2 = Height "$355/(\pi L_1^2)$"

L_3 = Volume "$\pi L_1^2 L_2$"

L_4 = *SA* "$2\pi L_1^2 + 2\pi L_1 L_2$"

L_5 = Cost "$0.0005L_4$"

To access L_1, press 2nd L1 1

To access L_2, press 2nd L2 2

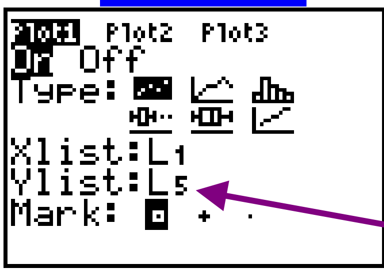
To access L_3, press 2nd L3 3

To access L_4, press 2nd L4 4

d. Graph the Cost versus Radius. Use the appropriate regression to find the radius with the minimum cost.

1. Turn On STATS PLOT

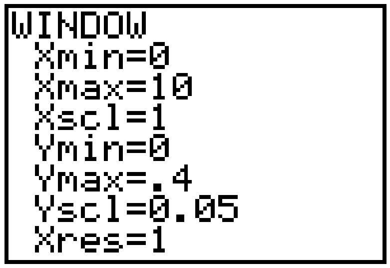
2nd | **STAT PLOT / Y=** | **ENTER**



To access L_5, press **2nd** | **L5 / 5**

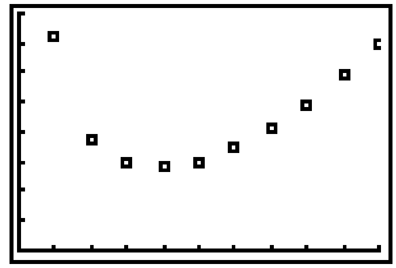
2. Set WINDOW

WINDOW x: [0, 10, 1] y: [0, 0.40, 0.05]



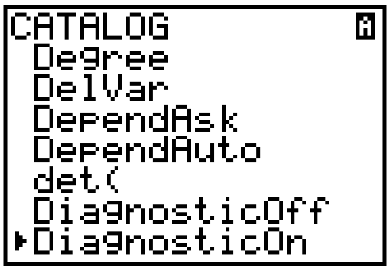
3. Graph

GRAPH



4. Turn Diagnostic On for Correlation Coefficient

2nd | **CATALOG / 0** | ▼

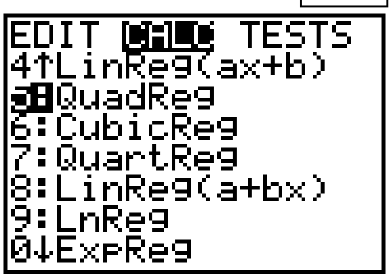


ENTER **ENTER**

5. Decide on the Best Regression Equation

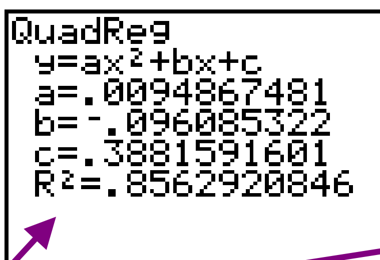
To access L_1, press **2nd** | **L1 / 1**

STAT ▶ ▼



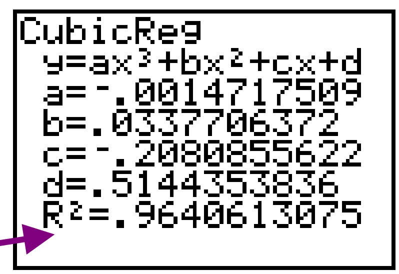
a. Quadratic Regression

QuadReg L_1, L_5



b. Cubic Regression

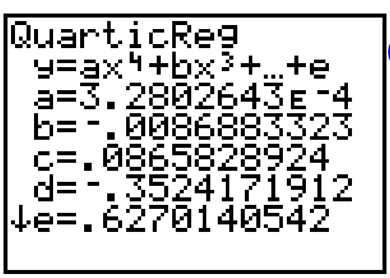
CubicReg L_1, L_5



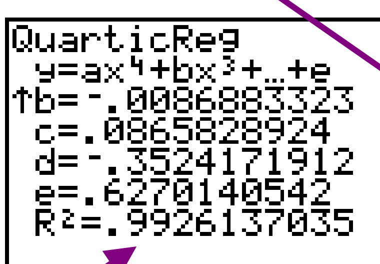
Correlation Coefficients

c. Quartic Regression

QuartReg L_1, L_5

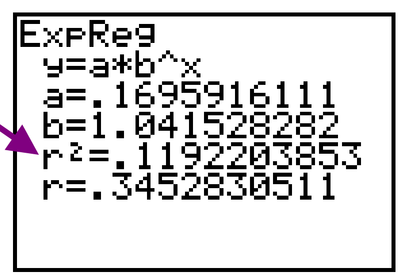


▼



d. Exponential Regression

ExpReg L_1, L_5



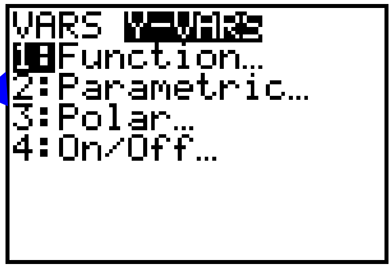
Correlation Coefficient Closest to 1; Best Regression Equation

6. Copy the Best Regression Equation into **Y=** **screen**

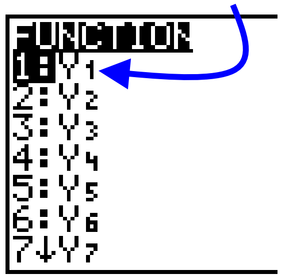
a. QuarticReg L_1, L_5, Y_1

To access Y_1, press **VARS**

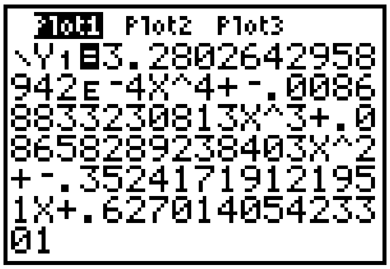
Select Option 1



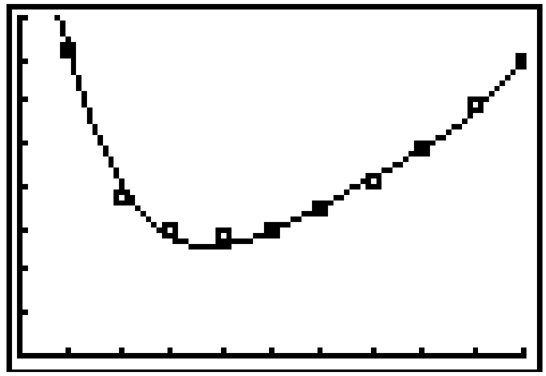
Select Option 1 for Y_1 **ENTER**



Check by pressing **Y=**

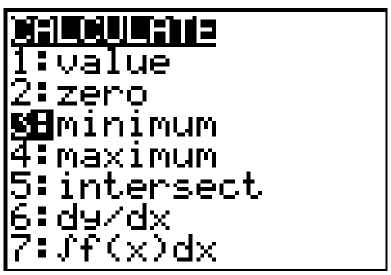


b. Graph **GRAPH**

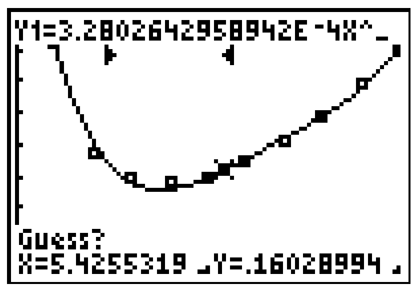


7. Run Minimum to locate the Radius that will give the Minimum Cost

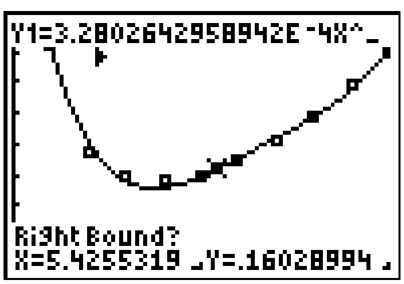
2nd **CALC** **TRACE**



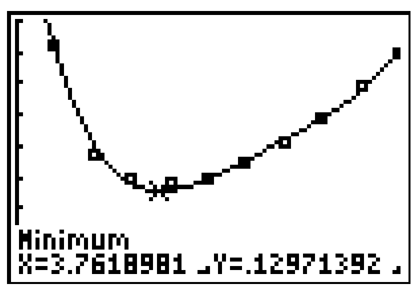
ENTER



ENTER



ENTER



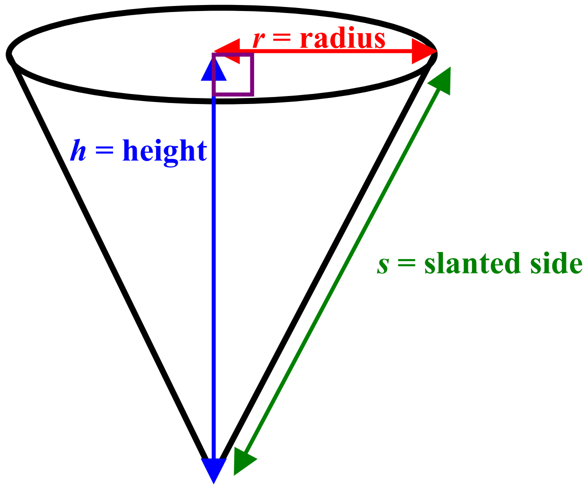
$r = 3.762$ cm $\quad h = \dfrac{V}{\pi r^2} = \dfrac{355}{\pi(3.762)^2}$ $\quad$ **$h = 7.984$ cm**

Therefore, the highest cost effective pop can would have a dimensions of $r = 3.762$ cm and $h = 7.984$ cm with minimum cost of \$0.12/can.

8-2 Assignment: 8-2 Worksheet

8-2 Worksheet: Using Technologies to Design Minimum Cost

1. Design a water tower in the shape of an inverted cone (with an open top) to hold a maximum of 85 m^3 of water. The material and paint cost $70/$m^2$.



Volume of Cone $V = \dfrac{\pi r^2 h}{3}$

Surface Area of Cone (with open top) $SA = \pi r s$

a. Find the equation that relates height (h) with the radius (r) and the specified volume (V).

b. Find the equation that relates slanted side (s) with the radius (r) and the height (h). [In EXCEL, Square Root is typed in as **SQRT(** .]

c. Create an EXCEL spreadsheet that will calculate the heights, volumes, slanted sides, surface areas and costs for each water tower with radius 1 m to 10 m (by 1 m interval). Find the dimensions of the water tower with the least cost.

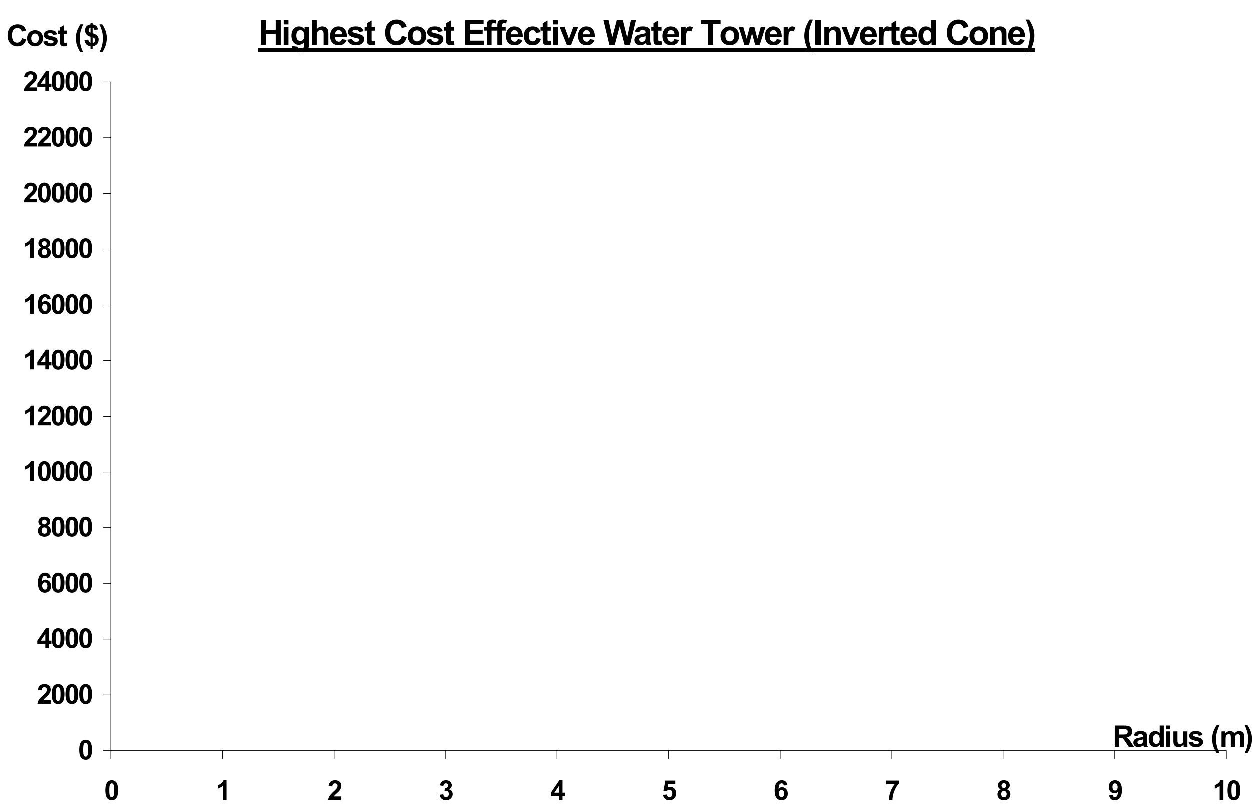
	A	B	C	D	E	F
1	Designing the Highest Cost Effective Water Tower (Inverted Cone)					
2						
3		Volume of Water Tower (cubic metre)			85	
4		Material and Paint Cost ($/square m)			70	
5						
6	Radius (m)	Height (m)	Volume (cubic metre)	Slanted Side (m)	Surface Area (square metre)	Cost ($)
7	1					
8	2					
9	3					
10	4					
11	5					
12	6					
13	7					
14	8					
15	9					
16	10					

d. Verify that you have the same results as the spreadsheet on the graphing calculator.

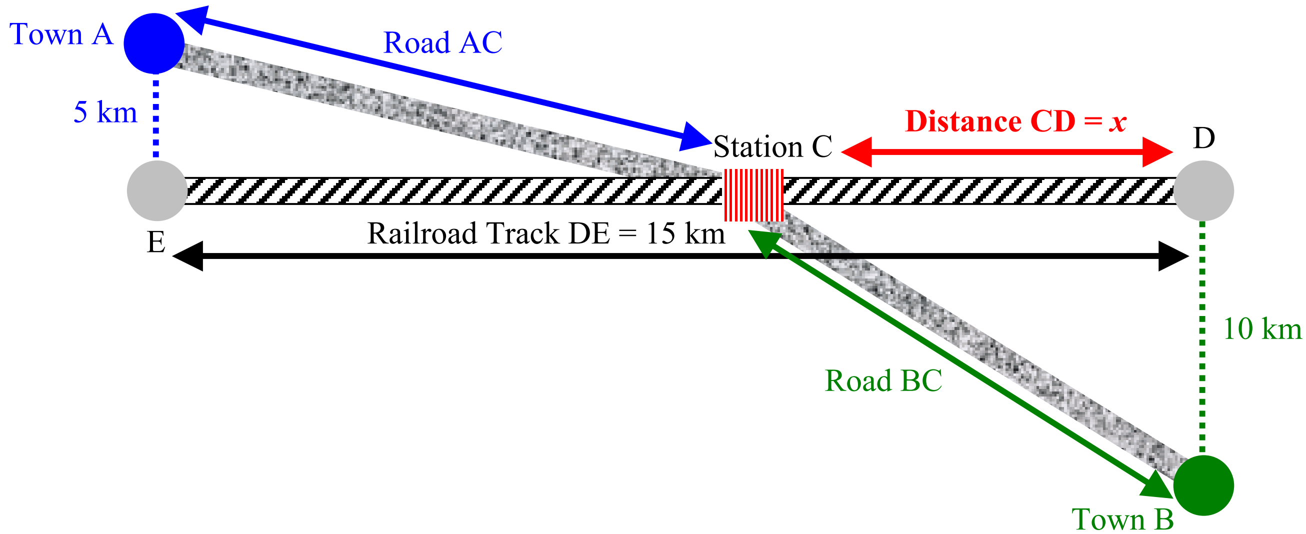
Variable	**STATS Editor Column**	**Formulas to be Entered “ ”**
Radius (r)	L_1	**Enter 1 to 10**
Height (h)	L_2	
Volume (V)	L_3	
Slanted Side (s)	L_4	
Surface Area (SA)	L_5	
Cost (C)	L_6	

e. Graph the Cost versus Radius with Window Setting x:[0, 10, 1] and y:[0, 24000, 2000]. Use the appropriate regression with the best correlation coefficient (R^2) to find the radius with the minimum cost.

Regression Type (L_1, L_6)	**Correlation Coefficient (R^2)**
Linear ($ax + b$)	
Quadratic	
Cubic	
Quartic	
Exponential	



2. As shown below, a stretch of railroad track 15 km between point E and D is selected to put a new train station (C). Town A is 5 km north of point E and town B is 10 km south of Point D. The train station is to be connected to both towns with two roads. The cost of building a road is \$35,000/km.



a. Find the equation that relates the distance of the road BC with the distance of the track CD (x).

b. Find the equation that relates the distance of the road AC with the distance of the track CD (x).

c. Create an EXCEL spreadsheet that will calculate the distances of the roads BC and AC, and the total costs for each station location 0 km to 15 km from point D. Find the distance the station is from point D with the least cost. [Square Root Function on EXCEL is SQRT( ).]

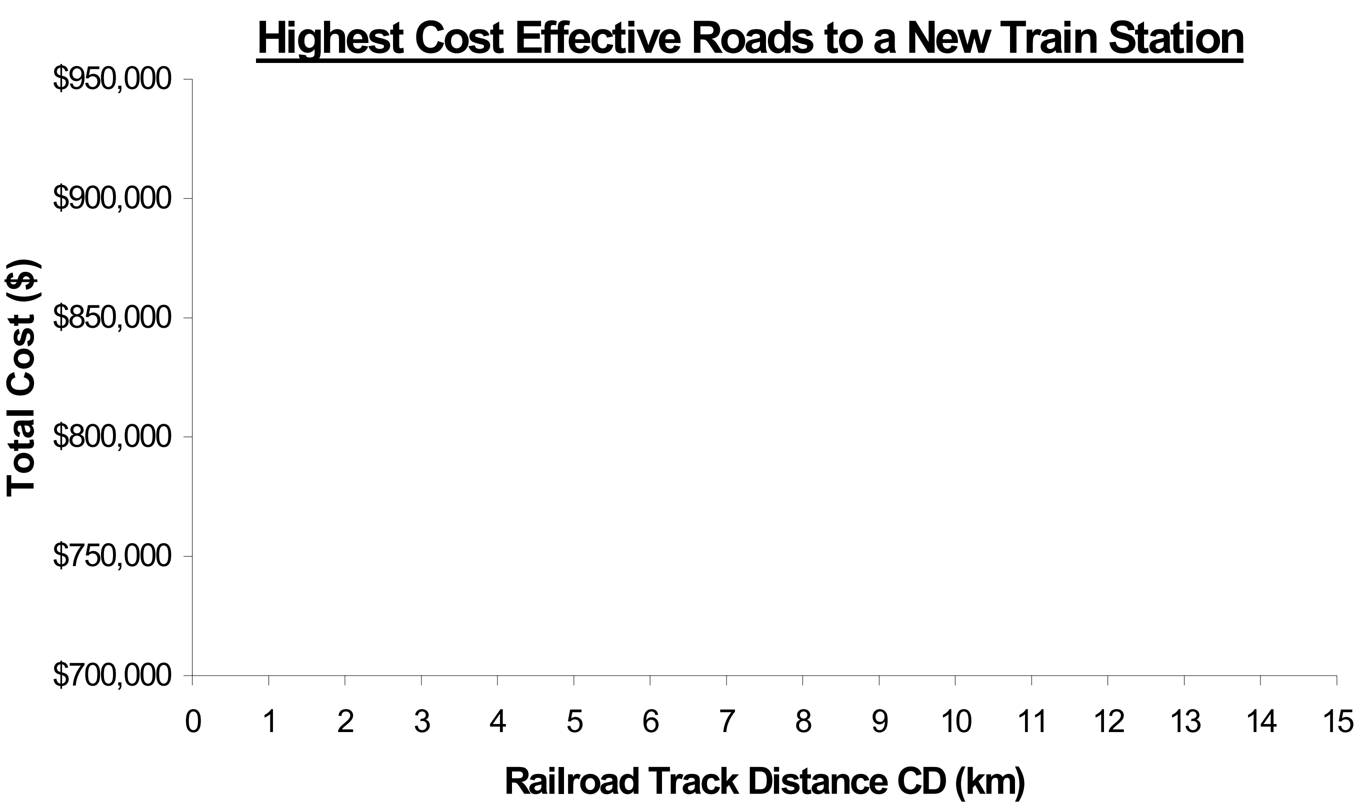
	A	B	C	D
1	**Designing Highest Cost Effective Roads to a New Train Station**			
2				
3		Distance of Point D to Town A (km)		10
4		Distance of Point E to Town A (km)		5
5		Distance of Railroad Tack DE (km)		15
6		Cost of Building Road ($/km)		35000
7				
8	**Railroad Track Distance CD**	**Road Distance BC (km)**	**Road Distance AC (km)**	**Total Cost**
9	0			
10	1			
11	2			
12	3			
13	4			
14	5			
15	6			
16	7			
17	8			
18	9			
19	10			
20	11			
21	12			
22	13			
23	14			
24	15			

d. Verify that you have the same results as the spreadsheet on the graphing calculator.

Variable	STATS Editor Column	Formulas to be Entered “ “
Railroad Track Distance CD (x)	L_1	**Enter 0 to 15**
Road Distance BC	L_2	
Road Distance AC	L_3	
Cost (C)	L_4	

e. Graph the Cost versus Radius with Window Setting x:[0, 10, 1] and y:[0, 24000, 2000]. Use the appropriate regression with the best correlation coefficient (R^2) to find the radius with the minimum cost.

Regression Type (L_1, L_6)	Correlation Coefficient (R^2)
Linear (ax + b)	
Quadratic	
Cubic	
Quartic	
Exponential	



Answers

1a. $h = \dfrac{3V}{\pi r^2} = \dfrac{3(85)}{\pi r^2}$ $\quad h = \dfrac{255}{\pi r^2}$

1b. $s^2 = r^2 + h^2$ (Pythagoras Theorem) $\quad s = \sqrt{r^2 + h^2}$

1c. EXCEL Formulas

	A	B	C	D	E	F
1	Designing the Highest Cost Effective Water Tower (Inverted Cone)					
2						
3		Volume of Water Tower (cubic metre)			85	
4		Material and Paint Cost ($/square m)			70	
5						
6	Radius (m)	Height (m)	Volume (cubic metre)	Slanted Side (m)	Surface Area (square metre)	Cost ($)
7	1	=3*$E$3/(PI()*A7*A7)	=PI()*A7*A7*B7/3	=SQRT(A7^2+B7^2)	=PI()*A7*D7	=$E$4*E7
8	=A7+1	=3*$E$3/(PI()*A8*A8)	=PI()*A8*A8*B8/3	=SQRT(A8^2+B8^2)	=PI()*A8*D8	=$E$4*E8
9	=A8+1	=3*$E$3/(PI()*A9*A9)	=PI()*A9*A9*B9/3	=SQRT(A9^2+B9^2)	=PI()*A9*D9	=$E$4*E9
10	=A9+1	=3*$E$3/(PI()*A10*A10)	=PI()*A10*A10*B10/3	=SQRT(A10^2+B10^2)	=PI()*A10*D10	=$E$4*E10
11	=A10+1	=3*$E$3/(PI()*A11*A11)	=PI()*A11*A11*B11/3	=SQRT(A11^2+B11^2)	=PI()*A11*D11	=$E$4*E11
12	=A11+1	=3*$E$3/(PI()*A12*A12)	=PI()*A12*A12*B12/3	=SQRT(A12^2+B12^2)	=PI()*A12*D12	=$E$4*E12
13	=A12+1	=3*$E$3/(PI()*A13*A13)	=PI()*A13*A13*B13/3	=SQRT(A13^2+B13^2)	=PI()*A13*D13	=$E$4*E13
14	=A13+1	=3*$E$3/(PI()*A14*A14)	=PI()*A14*A14*B14/3	=SQRT(A14^2+B14^2)	=PI()*A14*D14	=$E$4*E14
15	=A14+1	=3*$E$3/(PI()*A15*A15)	=PI()*A15*A15*B15/3	=SQRT(A15^2+B15^2)	=PI()*A15*D15	=$E$4*E15
16	=A15+1	=3*$E$3/(PI()*A16*A16)	=PI()*A16*A16*B16/3	=SQRT(A16^2+B16^2)	=PI()*A16*D16	=$E$4*E16

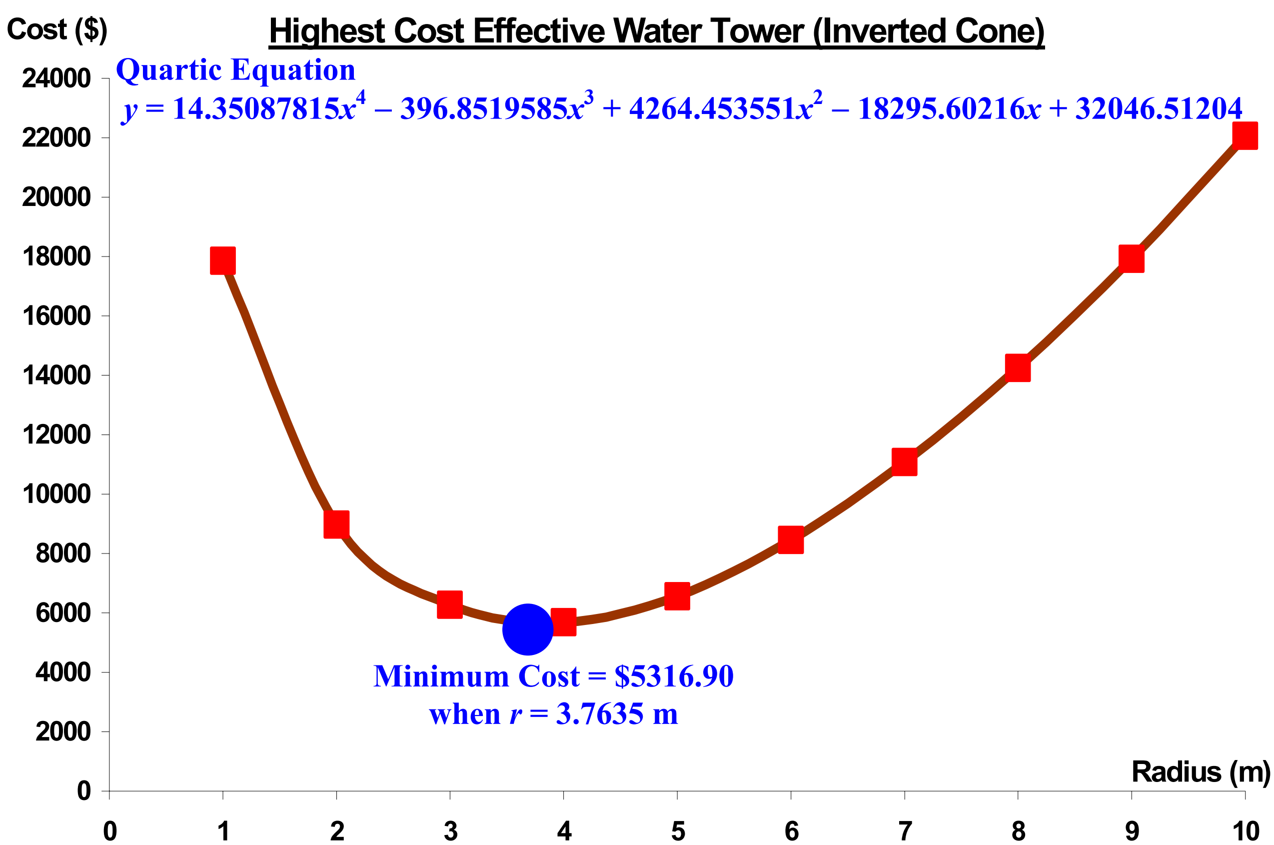
Spreadsheet Results **(Least Cost at radius of 4 m)**

	A	B	C	D	E	F
1	Designing the Highest Cost Effective Water Tower (Inverted Cone)					
2						
3		Volume of Water Tower (cubic metre)			85	
4		Material and Paint Cost ($/square m)			70	
5						
6	Radius (m)	Height (m)	Volume (cubic metre)	Slanted Side (m)	Surface Area (square metre)	Cost ($)
7	1	81.169	85	81.175	255.019	$ 17,851.35
8	2	20.292	85	20.391	128.118	$ 8,968.24
9	3	9.019	85	9.505	89.579	$ 6,270.55
10	4	5.073	85	6.460	81.183	$ 5,682.81
11	5	3.247	85	5.962	93.646	$ 6,555.19
12	6	2.255	85	6.410	120.819	$ 8,457.34
13	7	1.657	85	7.193	158.190	$ 11,073.27
14	8	1.268	85	8.100	203.573	$ 14,250.10
15	9	1.002	85	9.056	256.042	$ 17,922.91
16	10	0.812	85	10.033	315.192	$ 22,063.47

Variable	STATS Editor Column	Formulas to be Entered
Radius (r)	L_1	**Enter 1 to 10**
Height (h)	L_2	**"$255/(\pi L_1^2)$"**
Volume (V)	L_3	**"$\pi L_1^2 L_2/3$"**
Slanted Side (s)	L_4	**"$\sqrt{(L_1^2+L_2^2)}$"**
Surface Area (SA)	L_5	**"$\pi L_1 L_4$"**
Cost (C)	L_6	**"$70L_5$"**

1e. Regression Type and Correlation Coefficients (R^2)

Regression Type (L_1, L_6)	Correlation Coefficient (R^2)
Linear (ax + b)	**0.253497663**
Quadratic	**0.9185341169**
Cubic	**0.9861826646**
Quartic	**0.9974708192** **(Closest to 1)**
Exponential	**0.2426556321**



2a. Using Pythagoras Theorem

Distance of Road BC = $\sqrt{\overline{CD}^2 + 10^2}$

Distance of Road BC = $\sqrt{x^2 + 10^2}$

2b. Using Pythagoras Theorem

Distance of Road AC = $\sqrt{\left(15 - \overline{CD}\right)^2 + 5^2}$

Distance of Road AC = $\sqrt{(15 - x)^2 + 5^2}$

2c. EXCEL Formulas

	A	B	C	D
1	**Designing Highest Cost Effective Roads to a New Train Station**			
2				
3		**Distance of Point D to Town A (km)**		10
4		**Distance of Point E to Town A (km)**		5
5		**Distance of Railroad Tack DE (km)**		15
6		**Cost of Building Road ($/km)**		35000
7				
8	**Railroad Track Distance CD**	**Road Distance BC (km)**	**Road Distance AC (km)**	**Total Cost**
9	0	=SQRT(A9^2+$D$3^2)	=SQRT(($D$5-A9)^2+$D$4^2)	=$D$6*(B9+C9)
10	=A9+1	=SQRT(A10^2+$D$3^2)	=SQRT(($D$5-A10)^2+$D$4^2)	=$D$6*(B10+C10)
11	=A10+1	=SQRT(A11^2+$D$3^2)	=SQRT(($D$5-A11)^2+$D$4^2)	=$D$6*(B11+C11)
12	=A11+1	=SQRT(A12^2+$D$3^2)	=SQRT(($D$5-A12)^2+$D$4^2)	=$D$6*(B12+C12)
13	=A12+1	=SQRT(A13^2+$D$3^2)	=SQRT(($D$5-A13)^2+$D$4^2)	=$D$6*(B13+C13)
14	=A13+1	=SQRT(A14^2+$D$3^2)	=SQRT(($D$5-A14)^2+$D$4^2)	=$D$6*(B14+C14)
15	=A14+1	=SQRT(A15^2+$D$3^2)	=SQRT(($D$5-A15)^2+$D$4^2)	=$D$6*(B15+C15)
16	=A15+1	=SQRT(A16^2+$D$3^2)	=SQRT(($D$5-A16)^2+$D$4^2)	=$D$6*(B16+C16)
17	=A16+1	=SQRT(A17^2+$D$3^2)	=SQRT(($D$5-A17)^2+$D$4^2)	=$D$6*(B17+C17)
18	=A17+1	=SQRT(A18^2+$D$3^2)	=SQRT(($D$5-A18)^2+$D$4^2)	=$D$6*(B18+C18)
19	=A18+1	=SQRT(A19^2+$D$3^2)	=SQRT(($D$5-A19)^2+$D$4^2)	=$D$6*(B19+C19)
20	=A19+1	=SQRT(A20^2+$D$3^2)	=SQRT(($D$5-A20)^2+$D$4^2)	=$D$6*(B20+C20)
21	=A20+1	=SQRT(A21^2+$D$3^2)	=SQRT(($D$5-A21)^2+$D$4^2)	=$D$6*(B21+C21)
22	=A21+1	=SQRT(A22^2+$D$3^2)	=SQRT(($D$5-A22)^2+$D$4^2)	=$D$6*(B22+C22)
23	=A22+1	=SQRT(A23^2+$D$3^2)	=SQRT(($D$5-A23)^2+$D$4^2)	=$D$6*(B23+C23)
24	=A23+1	=SQRT(A24^2+$D$3^2)	=SQRT(($D$5-A24)^2+$D$4^2)	=$D$6*(B24+C24)

Spreadsheet Results

	A	B	C	D	E
1	**Designing Highest Cost Effective Roads to a New Train Station**				
2					
3		Distance of Point D to Town A (km)		10	
4		Distance of Point E to Town A (km)		5	
5		Distance of Railroad Tack DE (km)		15	
6		Cost of Building Road ($/km)		35000	
7					
8	**Railroad Track Distance CD**	**Road Distance BC (km)**	**Road Distance AC (km)**	**Total Cost**	
9	0	10.00000	15.81139	$ 903,398.59	
10	1	10.04988	14.86607	$ 872,058.05	
11	2	10.19804	13.92839	$ 844,424.96	
12	3	10.44031	13.00000	$ 820,410.73	
13	4	10.77033	12.08305	$ 799,868.15	
14	5	11.18034	11.18034	$ 782,623.79	
15	6	11.66190	10.29563	$ 768,513.69	
16	7	12.20656	9.43398	$ 757,418.79	
17	8	12.80625	8.60233	$ 749,300.08	
18	9	13.45362	7.81025	$ 744,235.58	
19	10	14.14214	7.07107	$ 742,462.12	
20	11	14.86607	6.40312	$ 744,421.75	
21	12	15.62050	5.83095	$ 750,800.79	
22	13	16.40122	5.38516	$ 762,523.45	
23	14	17.20465	5.09902	$ 780,628.45	
24	15	18.02776	5.00000	$ 805,971.47	

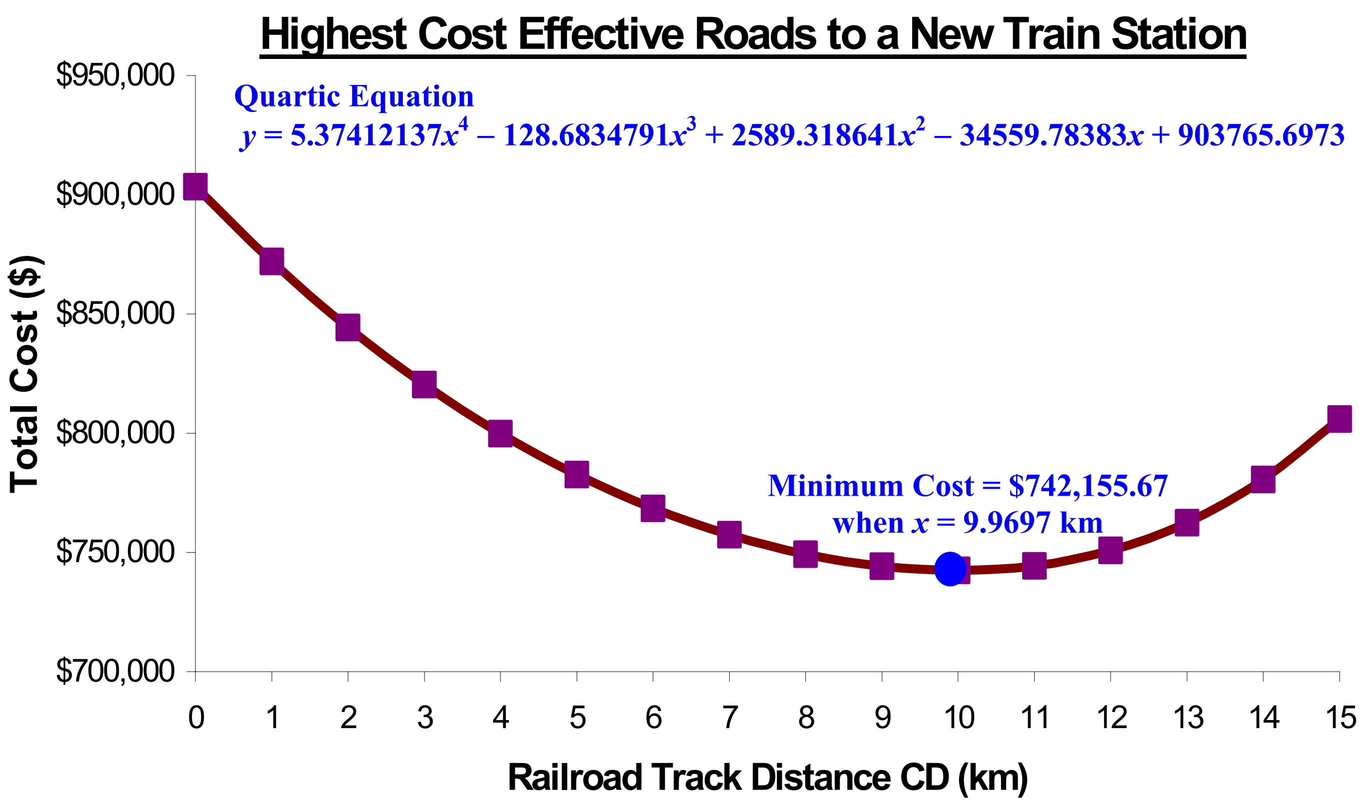
Least Cost at Railroad Track CD Distance of 10 m

2d. Graphing Calculator Formulas

Variable	STATS Editor Column	Formulas to be Entered
Railroad Track Distance CD (x)	L_1	**Enter 1 to 15**
Road Distance BC	L_2	**"$\sqrt{(L_1^2+10^2)}$"**
Road Distance AC	L_3	**"$\sqrt{((15-L_1)^2+5^2)}$"**
Cost (C)	L_4	**"$35000(L_2+L_3)$"**

2e. Regression Type and Correlation Coefficients (R^2)

Regression Type (L_1, L_6)	Correlation Coefficient (R^2)
Linear ($ax + b$)	**0.4816484651**
Quadratic	**0.9961779882**
Cubic	**0.9988553759**
Quartic	**0.9999681768** **(Closest to 1)**
Exponential	**0.4775725146**



8-3: Using Exponential Regression

When a set of data appears to have an exponential relation, we can determine the average rate of growth or decline by running the exponential regression function from the graphing calculator.

Exponential Regression

$$y = ab^x$$

y* = final amount at time *x
***a* = initial amount at time 0**
***b* = overall rate**
***x* = time**

Compound Interest Formula

$$A = P(1 + r)^n$$

A* = final amount at time *n
***P* = initial amount at time 0**
***r* = rate of increase or decrease ($r > 0$ increase ; $r < 0$ decrease)**
***n* = time**

As we can see, $y = A$, $x = n$, $a = P$, and $b = (1 + r)$ between these two formulas. When we find ***b***, the overall rate, we can find the rate of increase or decrease, ***r***.

Example 1: The average total income of Canadian families is shown in the table below.

Year	Number of Year Since 1990	Average Canadian Family Total Income
1993	3	$57,605
1994	4	$58,666
1995	5	$58,592
1996	6	$59,451
1997	7	$60,772
1998	8	$63,247
1999	9	$63,818

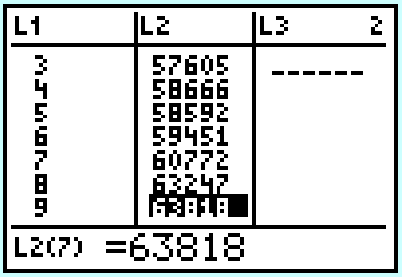
a. Enter the above table in the Stats Editor of the graphing calculator and determine the equation of the average Canadian family total income versus the number of years since 1990 using exponential regression.
b. What is the average rate of increase for the average Canadian family total income between the years 1993 to 1999?
c. Graph the average Canadian family total income versus the number of years since 1990 for 25 years (Scatter Plot and Exponential Regression Equation).
d. Using the equation obtained in part b.; predict the average Canadian family total income in the year 2010.
e. Find the year when the average Canadian family total income will exceed $70,000.

a. Obtaining Exponential Equation

1. Enter first and second column into L_1 and L_2.

STAT

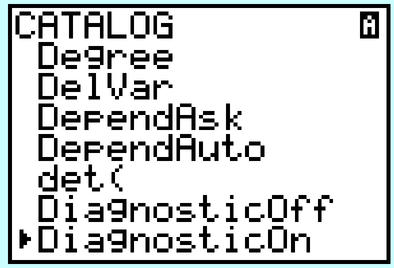
ENTER



2. Turn Diagnostic On for Correlation Coefficient

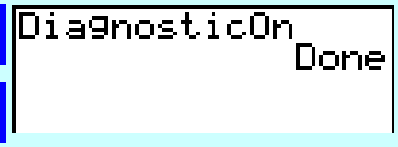
2nd

CATALOG
0



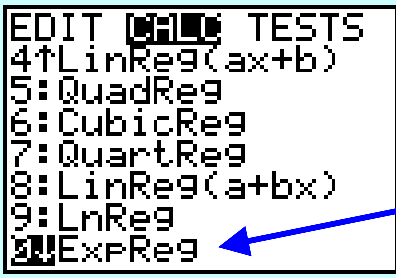
ENTER

ENTER



3. Run Exponential Regression and Copy to Y_1.

STAT

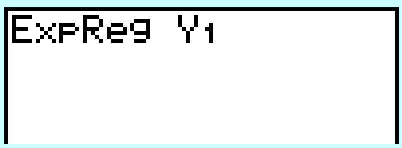


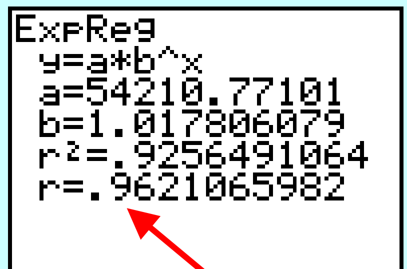
Select Option 0

VARS

ENTER

ENTER





Correlation is good
(*r* is close to 1)

$y = 54210.77\,(1.017806079)^x$

OR

$A = 54210.77\,(1 + 0.017806079)^n$

b. $r = 0.017806079$

Average Rate of Increase = 1.78%

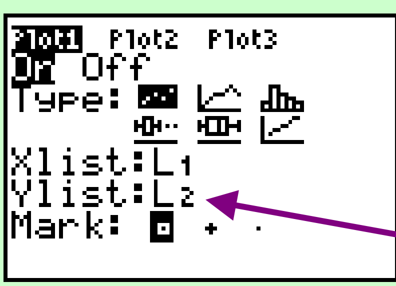
c. Graphing Scatter Plot and Exponential Regression Equation

1. Turn On STATS PLOT

2nd

STAT PLOT
Y=

ENTER



To access L_2, press

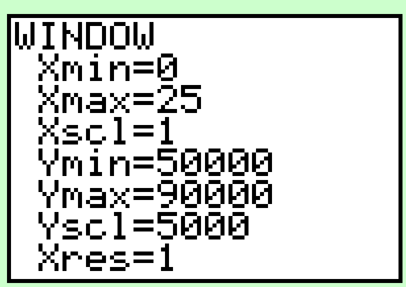
2nd

L2
2

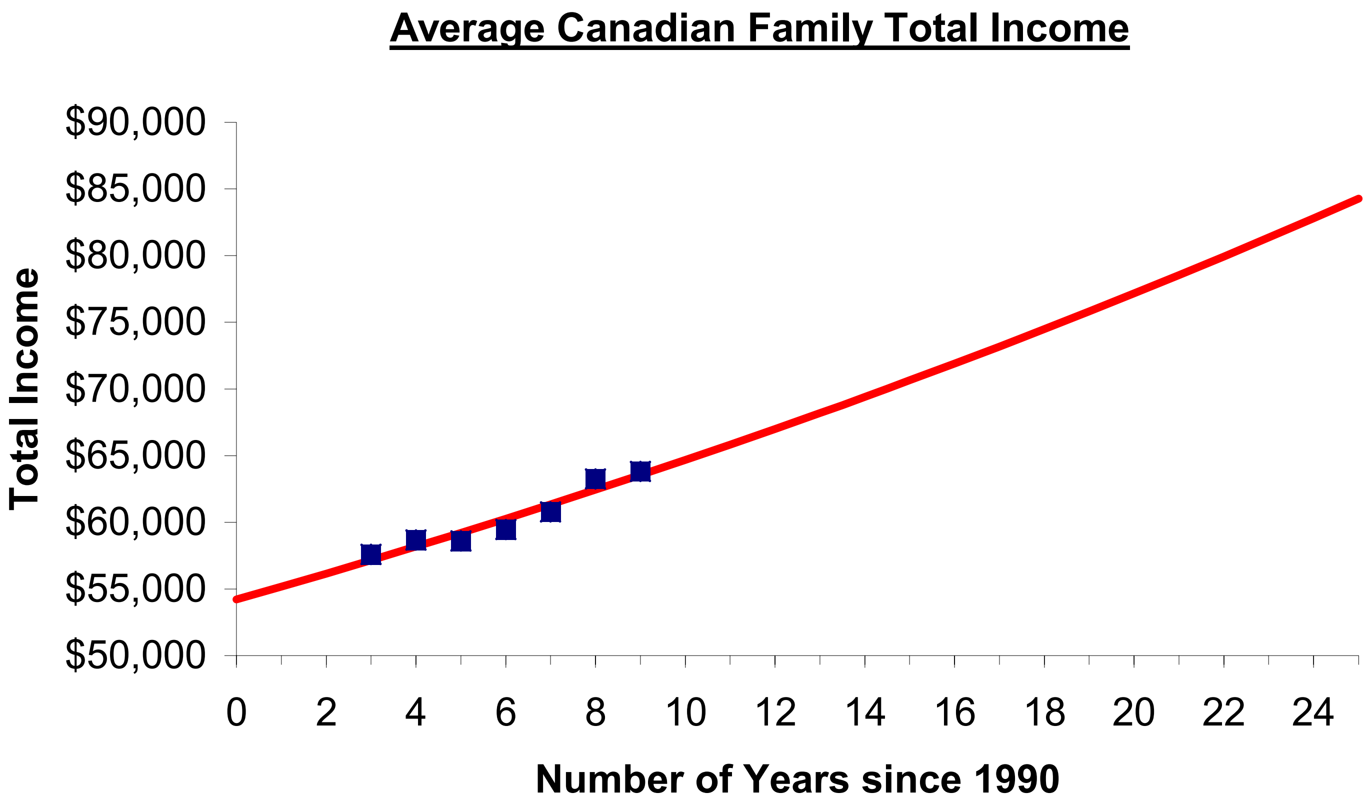
2. Set WINDOW

WINDOW

x: [0, 25, 1] y: [50000, 90000 5000]



3. Graph GRAPH



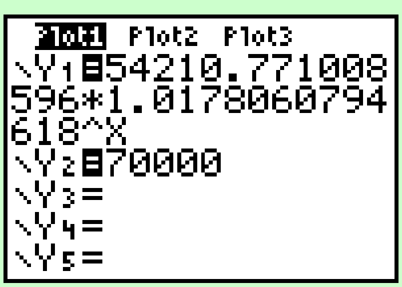
d. In the year 2010, $x = 2010 - 1990 = 20$. Using the Regression Equation, $y = 54210.77\,(1.017806079)^x$,

$y = 54210.77\,(1.017806079)^{20}$
$y = \$77158.96$

The average Canadian family total income in 2010 is $77158.96

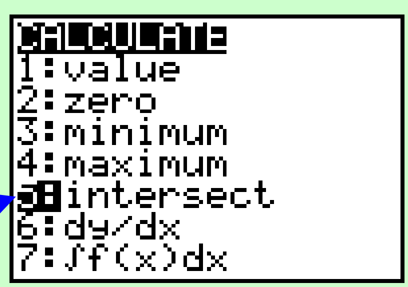
e. **The year at which the average Canadian family total income exceeds $70,000.**

1. Set $Y_2 = 70000$ Y=



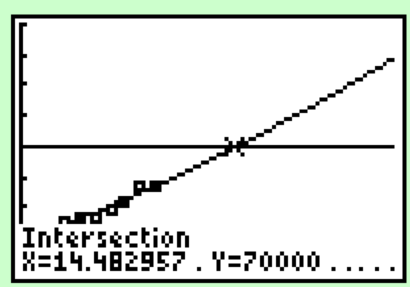
2. Find Intersecting point between Y_1 and Y_2.

2nd TRACE



Select Option 5

ENTER ENTER ENTER



$x = 14.482957$ (round up)
$x = 15$ years

15 years since 1990
Year 2005

Example 2: The number of Canadian retail businesses that have filed for bankruptcies is shown below.

Year	1997	1998	1999	2000	2001
Number of Year Since 1997	0	1	2	3	4
Number of Bankruptcies filed by Canadian Retail Businesses	2579	2266	1964	1821	1814

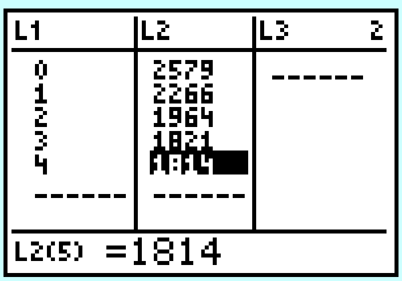
a. Enter the above table in the Stats Editor of the graphing calculator and determine the equation of the number of bankruptcies filed by Canadian retail businesses versus the number of years since 1997 using exponential regression.
b. What is the average rate of decrease for the number of bankruptcies filed by Canadian retail businesses between the years 1997 to 2001?
c. Graph the number of bankruptcies filed by Canadian retail businesses versus the number of years since 1997 for 25 years (Scatter Plot and Exponential Regression Equation).
d. Using the equation obtained in part b.; predict the number of bankruptcies filed by Canadian retail businesses in the year 2015.
e. Find the year when there will be less than 1000 Canadian retail businesses file for bankruptcies.

a. Obtaining Exponential Equation

1. Enter first and second column into L_1 and L_2.

STAT

ENTER



2. Turn Diagnostic On for Correlation Coefficient

2nd

CATALOG
0

▼

ENTER

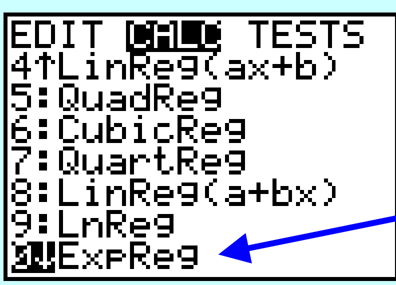
ENTER

3. Run Exponential Regression and Copy to Y_1.

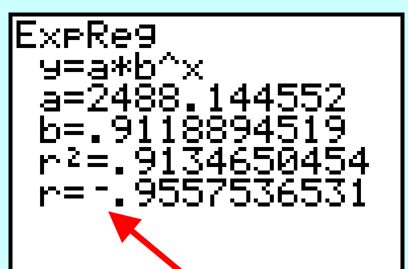
STAT

▶

▼



Select Option 0

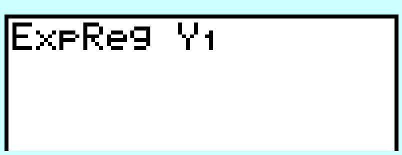


VARS

▶

ENTER

ENTER



Correlation is good (r is close to –1)

$y = 2488.144552\,(0.9118894519)^x$ OR $A = 2488.144552\,(1 - 0.0881105481)^n$

b. $r = 1 - b$

$r = 1 - 0.9118894519$

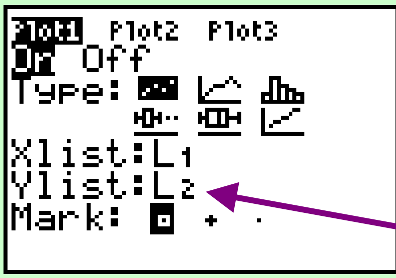
$r = 0.0881105481$

Average Rate of Decrease = 8.811%

c. Graphing Scatter Plot and Exponential Regression Equation

1. Turn On STATS PLOT

2nd **STAT PLOT / Y=** **ENTER**

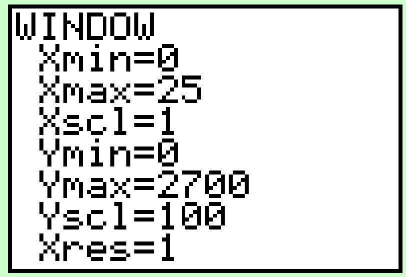


To access L_2, press **2nd** **L2 / 2**

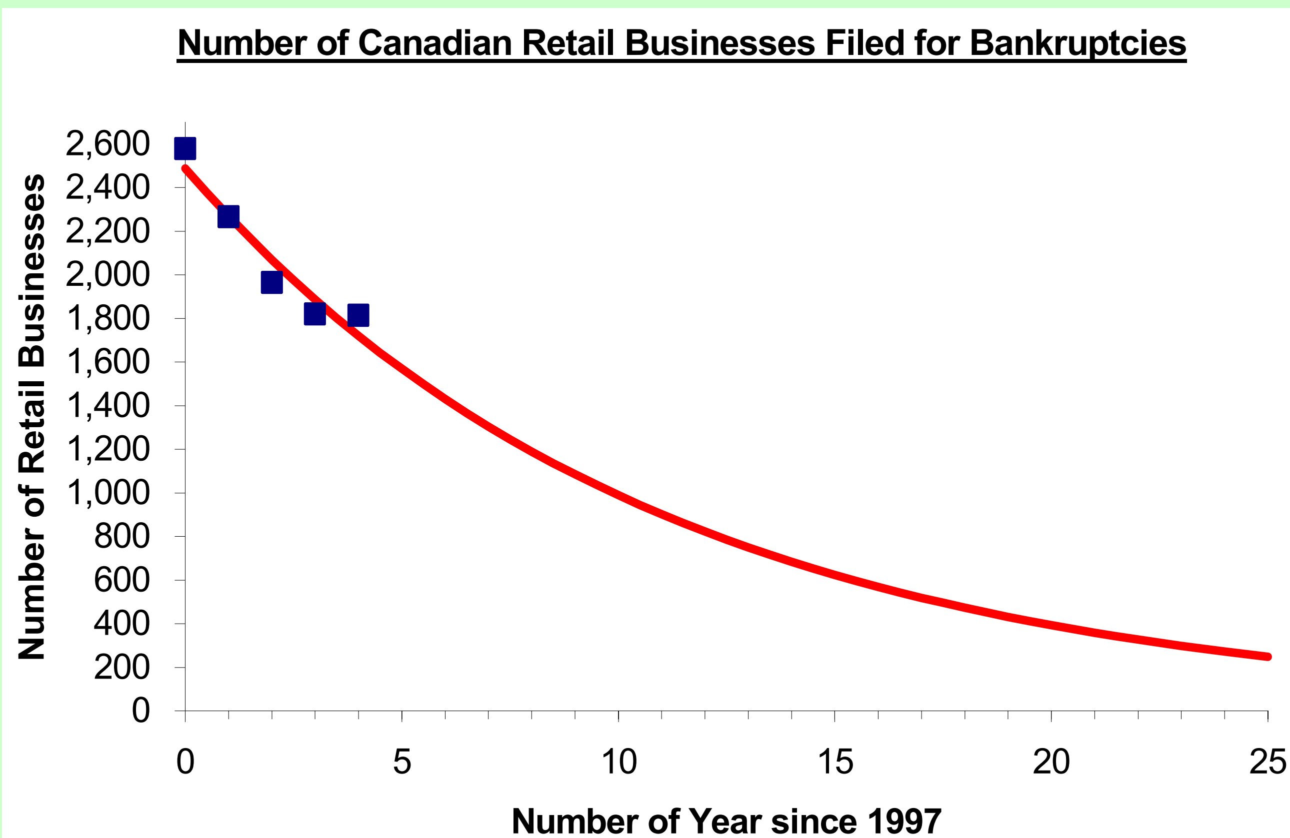
2. Set WINDOW

WINDOW

x: **[0, 25, 1]** y: **[0, 2700 100]**



3. Graph **GRAPH**



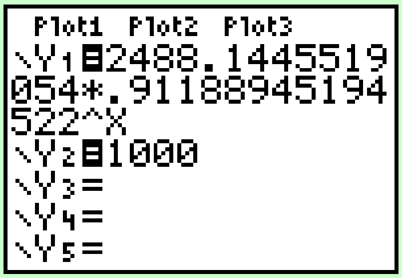
d. In 2015, $x = 2015 - 1997 = 18$. Using the Regression Equation, $y = 2488.144552\,(0.9118894519)^x$,

$y = 2488.144552\,(0.9118894519)^{18}$
$y = 472.9716021$

In 2015, there will be 473 Canadian retail businesses that will file for bankruptcies.

e. The year at which there will be less than 1000 Canadian retail businesses that will file for bankruptcies.

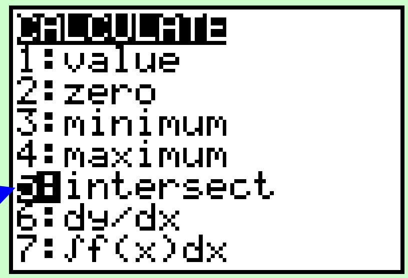
1. Set $Y_2 = 1000$ **Y=**



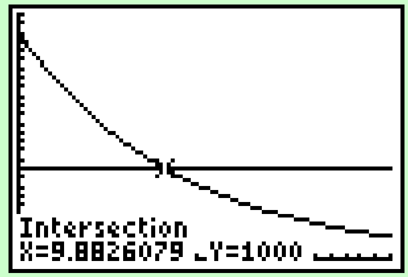
Select Option 5

2. Find Intersecting point between Y_1 and Y_2.

2nd **TRACE**



ENTER **ENTER** **ENTER**



$x = 9.8826079$ (round up)
$x = 10$ years

10 years since 1997
Year 2007

8-3 Assignment: 8-3 Worksheet

8-3 Worksheet: Using Exponential Regression

1. The world oil production measured in millions of barrels (Mbl) over time is shown in the table below.

Year	Number of Year since 1880	World Oil Production (Mbl)	Year	Number of Year Since 1880	World Oil Production (Mbl)
1880	0	30	1962	82	8882
1890	10	77	1964	84	10310
1900	20	149	1966	86	12016
1905	25	215	1968	88	14104
1910	30	328	1970	90	16690
1915	35	432	1972	92	18584
1920	40	689	1974	94	20389
1925	45	1069	1976	96	20188
1930	50	1412	1978	98	21922
1935	55	1655	1980	100	21722
1940	60	2150	1982	102	19411
1945	65	2595	1984	104	19837
1950	70	3803	1986	106	20246
1955	75	5626	1988	108	21338
1960	80	7674			

a. Enter the above table in the Stats Editor of the graphing calculator and determine the equation of the amount of world oil production in Mbl versus the number of years since 1880 using exponential regression.
b. What is the average rate of increase for the world oil production between the years 1880 to 1988?
c. Graph the world oil production versus the number of years since 1880 for 150 years (Scatter Plot and Exponential Regression Equation). ***x*:[0, 150, 10] *y*:[0, 150000, 10000]**
d. Using the equation obtained in part a.; predict the world oil production in the year 2010.
e. Find the year when the world oil production will exceed 120,000 Mbl.

2. A new $30,000 vehicle and its depreciation over time is as follows

Year	0	1	2	3	4
Value of the Vehicle	$30,000	$25,500	$21,675	$19,500	$17,500

a. Enter the above table in the Stats Editor of the graphing calculator and determine the equation of the value of the vehicle versus time using exponential regression.
b. What is the average rate of decrease for the value of the vehicle?
c. Graph the value of the vehicle versus time for 10 years (Scatter Plot and Exponential Regression Equation). ***x*:[0, 15, 1] *y*:[0, 35000, 5000]**
d. Using the equation obtained in part a.; predict value of the vehicle in its 9^{th} year to the nearest dollar.
e. Find the year when the value of the vehicle will be below $5000.

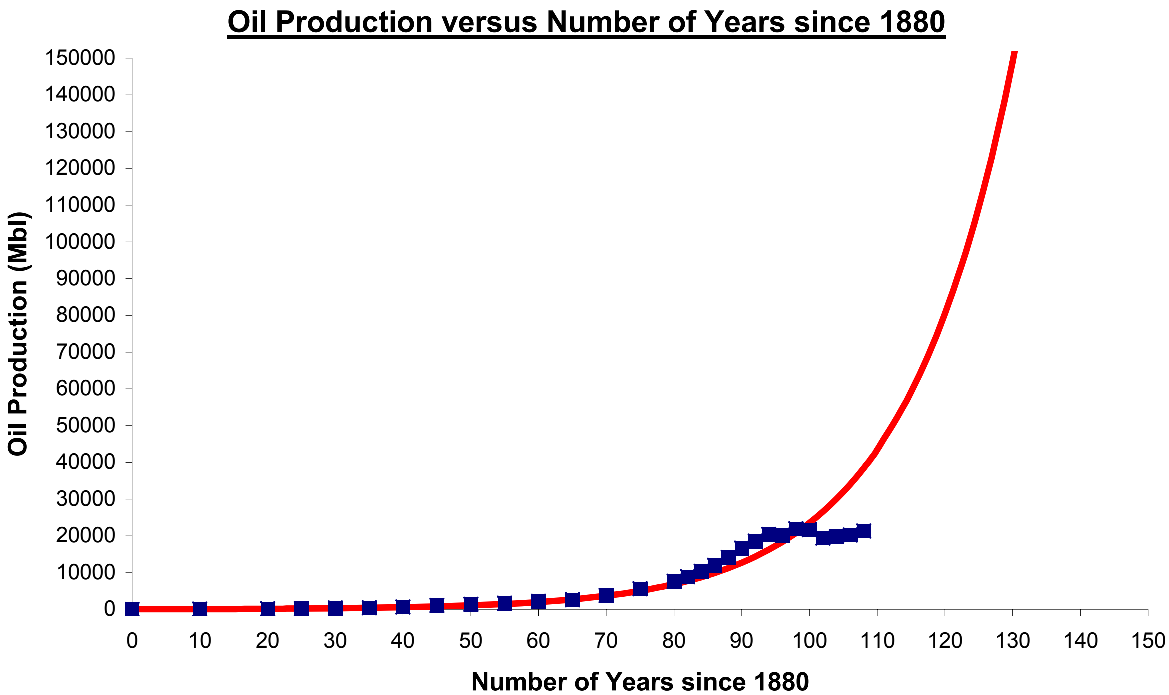
Answers

1a. $y = 49.6789623\ (1.063521516)^x$ OR $A = 49.6789623\ (1 + 0.063521516)^n$

1b. Average Rate of Increase = 6.3522%

1d. In 2010, x = 130 years, y = 149,000.92 Mbl

1c.



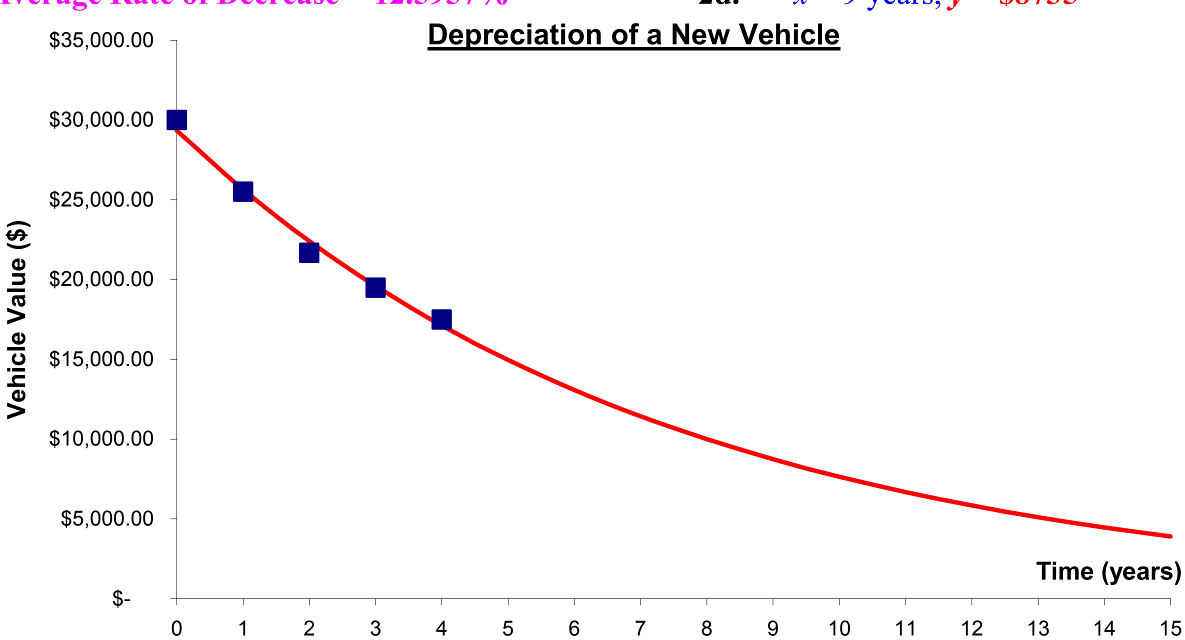
1e. y = 120,000 Mbl ; x = 127 (rounded up) In the year 2007

2a. $y = 29340.87333\ (0.8740430058)^x$ OR $A = 29340.87333\ (1 - 0.1259569942)^n$

2b. Average Rate of Decrease = 12.5957%

2d. x = 9 years, y = $8735

2c.



2e. y = $5000 ; x = 14th year (rounded up)

