# Chapters 6 & 7: Trigonometric Functions of Angles and Real Numbers

## 6-1 Angle Measures

**Radians**: - a unit **(rad)** to measure the size of an angle.

$$\pi \text{ rad} = 180^\circ \quad \text{OR} \quad \frac{\pi}{180} \text{ rad} = 1^\circ$$

**Divide both Sides by 180**

### Converting Degree to Radian Using Graphing Calculator.

**Step 1: Set Mode to RADIAN.**

**MODE**

```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```

**Step 2: Enter Degree**

```
30
```

**Step 3: Specify Degree Unit and Convert**

**2nd** **ANGLE / APPS**

```
ANGLE
1:°
2:'
3:r
4:▸DMS
5:R▸Pr(
6:R▸Pθ(
7↓P▸Rx(
```

Select Option 1

**ENTER**

**ENTER**

```
30°
      .5235987756
```

### Converting Radian to Degree Using Graphing Calculator.

**Step 1: Set Mode to Degree.**

**MODE**

```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```

**Step 2: Enter Radian**

```
1.25
```

**Step 3: Specify Radian Unit and Convert**

**2nd** **ANGLE / APPS**

```
ANGLE
1:°
2:'
3:r
4:▸DMS
5:R▸Pr(
6:R▸Pθ(
7↓P▸Rx(
```

Select Option 3

**ENTER**

**ENTER**

```
1.25r
      71.61972439
```

**Example 1**: Convert the following into radian.

a. $90^\circ$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$90^\circ = 90 \times \frac{\pi}{180} \text{ rad} = \frac{90\pi}{180} \text{ rad}$$

$$\mathbf{90^\circ = \frac{\pi}{2} \text{ rad} \approx 1.57 \text{ rad}}$$

```
90°
      1.570796327
120°
      2.094395102
```

b. $120^\circ$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$120^\circ = 120 \times \frac{\pi}{180} \text{ rad} = \frac{120\pi}{180} \text{ rad}$$

$$\mathbf{120^\circ = \frac{2\pi}{3} \text{ rad} \approx 2.09 \text{ rad}}$$

c. $135^{\circ}$

$1^{\circ} = \frac{\pi}{180}$ rad

$135^{\circ} = 135 \times \frac{\pi}{180}$ rad $= \frac{135\pi}{180}$ rad

$\mathbf{135^{\circ} = \frac{3\pi}{4}}$ **rad ≈ 2.36 rad**

```
135°
        2.35619449
225°
       3.926990817
```

d. $225^{\circ}$

$1^{\circ} = \frac{\pi}{180}$ rad

$225^{\circ} = 225 \times \frac{\pi}{180}$ rad $= \frac{\mathbf{225\pi}}{\mathbf{180}}$ rad

$\mathbf{225^{\circ} = \frac{5\pi}{4}}$ **rad ≈ 3.93 rad**

e. $240^{\circ}$

$1^{\circ} = \frac{\pi}{180}$ rad

$240^{\circ} = 240 \times \frac{\pi}{180}$ rad $= \frac{240\pi}{180}$ rad

$\mathbf{240^{\circ}} = \frac{4\pi}{3}$ **rad ≈ 4.19 rad**

```
240°
       4.188790205
330°
       5.759586532
```

f. $330^{\circ}$

$1^{\circ} = \frac{\pi}{180}$ rad

$330^{\circ} = 330 \times \frac{\pi}{180}$ rad $= \frac{330\pi}{180}$ rad

$\mathbf{330^{\circ} = \frac{11\pi}{6}}$ **rad ≈ 5.76 rad**

**Example 2**: Convert the following into degree.

a. $2\pi$ rad

$\pi$ rad $= 180^{\circ}$

$2\pi$ rad $= 2(180^{\circ})$

**2π rad = 360°**

```
2πʳ
               360
```

b. $\frac{7\pi}{4}$ rad

$\pi$ rad $= 180^{\circ}$

$\frac{7\pi}{4}$ rad $= \frac{7(180^{\circ})}{4}$

$\mathbf{\frac{7\pi}{4}}$ **rad = 315°**

```
(7π/4)ʳ
               315
```

c. $\frac{3\pi}{2}$ rad

$\pi$ rad $= 180^{\circ}$

$\frac{3\pi}{2}$ rad $= \frac{3(180^{\circ})}{2}$

$\mathbf{\frac{3\pi}{2}}$ **rad = 270°**

```
(3π/2)ʳ
               270
```

d. 3.42 rad

$\pi$ rad $= 180^{\circ}$

3.42 rad $= x$ degrees

$\frac{\pi}{3.42} = \frac{180^{\circ}}{x}$

$x = \frac{3.42 \times 180}{\pi}$

**3.42 rad = 195.95°**

```
3.42ʳ
       195.9515659
```

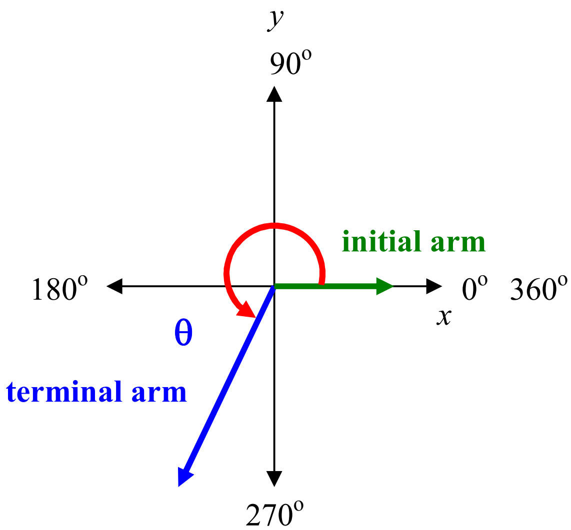
 

**Standard Position Angles**: - angles that can be defined on a coordinate grid.

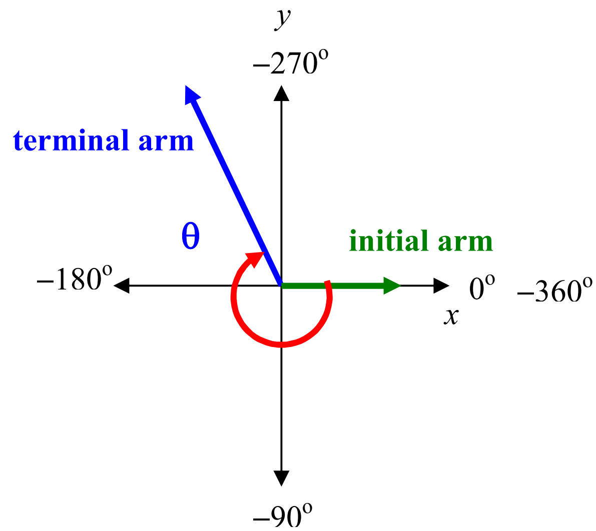
**Initial Arm**: - the beginning ray of the angle, which is fixed on the positive $x$-axis.

**Terminal Arm**: - rotates about the origin (0,0).
- the standard angle ($\theta$) is then measured between the initial arm and terminal arm..

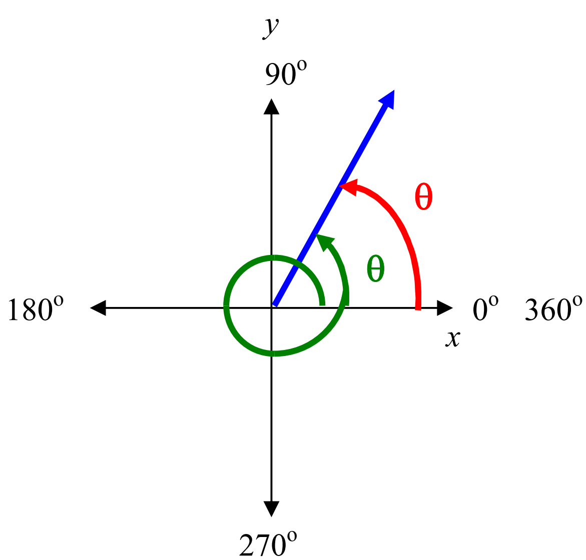
**Positive Angle**: - angle formed by the terminal arm rotated **counter-clockwise**.



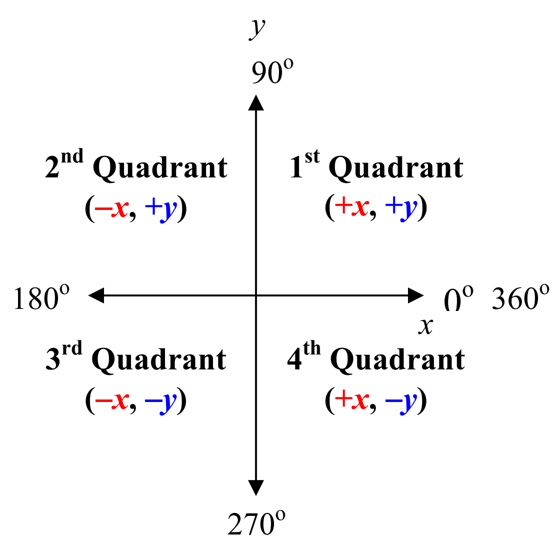

**Negative Angle**: - angle formed by the terminal arm rotated **clockwise**.



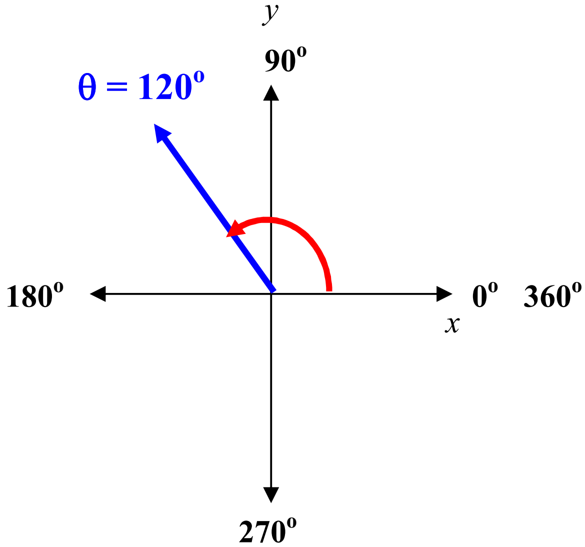

**Coterminal Angles**: - angles form when the terminal arms ends in the same position.

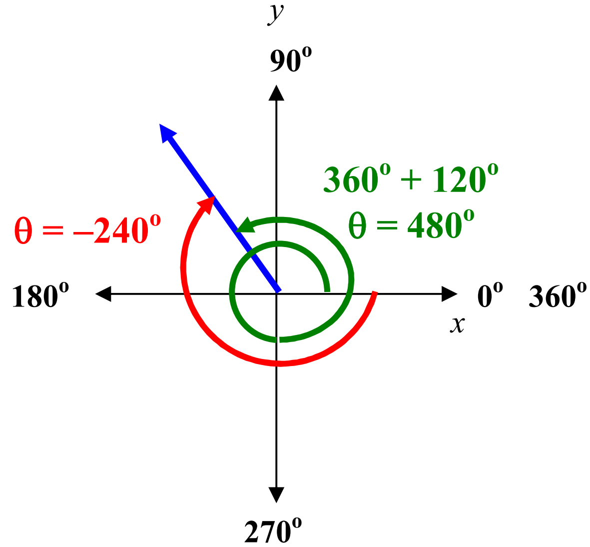



**Quadrants**: - the four parts of the Cartesian Coordinate Grid.



**Example 3**: Given θ = 120°. Draw the angle θ in standard position. Find and draw diagrams for two other angles which are coterminal to θ.

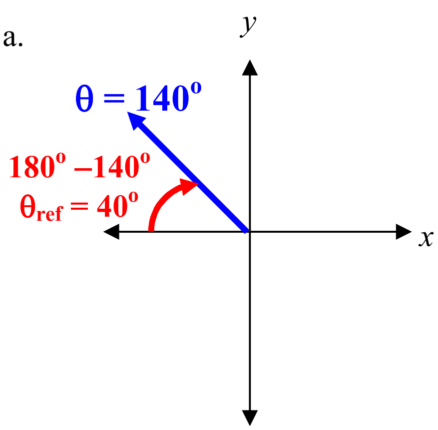


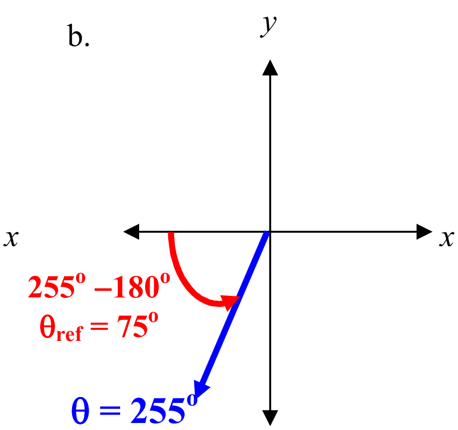

**Reference Angle**: - the acute angle between the terminal arm and the *x*-axis for any standard position angle.

**Example 4**: Find the reference angle for the following angles in standard position.

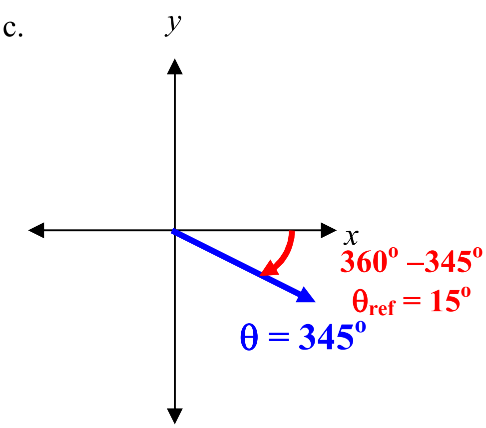
a.




b.




c.




**6-1 Assignment: pg. 454−455 #1, 5, 7, 13, 17, 25, 27, 41, 47**

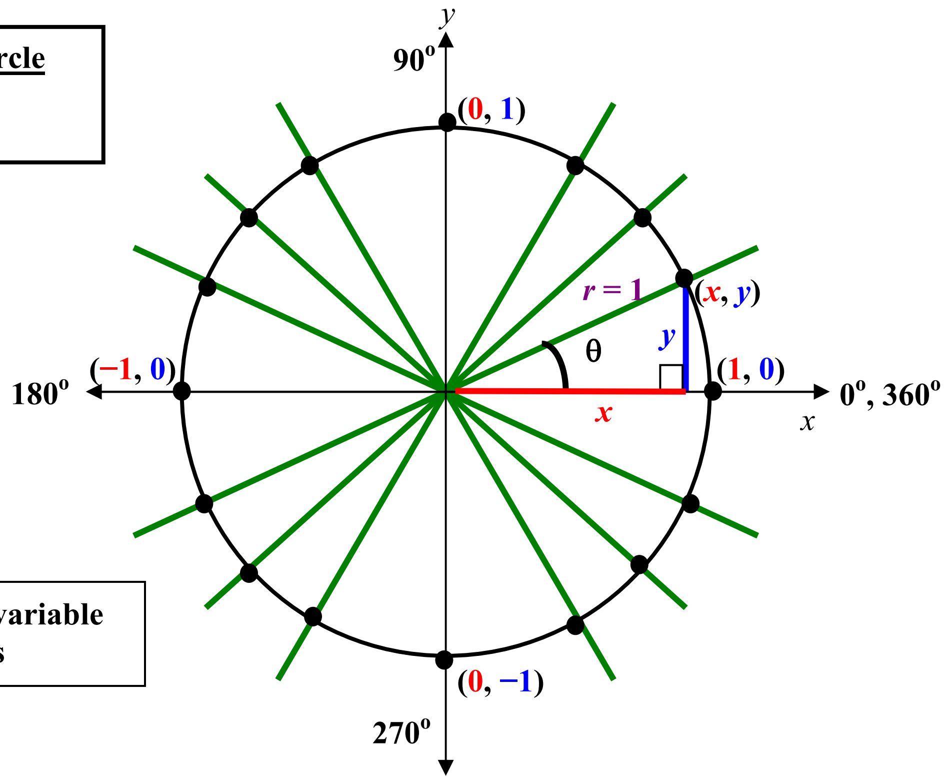
 

## 7-1 The Unit Circle

**Unit Circle**: - a circle with a radius of 1 and centred at (0, 0) that is drawn on a standard Cartesian grid.
- the coordinates of any point of the unit circle can be found using its equation, and they are related to some trigonometric functions such as cosine and sine (more in section 7.2)

**Equation of a Unit Circle**

$$x^2 + y^2 = 1$$

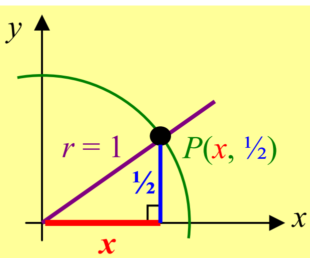



**θ = "theta" – common variable for angles**

**Terminal Point**: - the coordinate of the unit circle of a particular terminal arm's angle ($t$).

**Reference Number**: - also called the reference angle ($\bar{t}$).

**Example 1**: The point $P(x, ½)$ is on the unit circle in the quadrant I, find its $x$-coordinate.



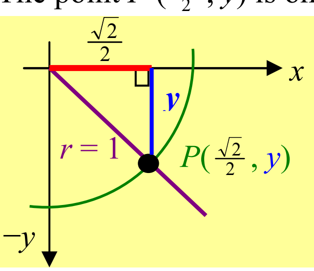

$x^2 + y^2 = 1$
$x^2 + (½)^2 = 1$
$x^2 + ¼ = 1$
$x^2 = 1 - ¼$
$x^2 = ¾$

$x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

In quadrant I, $x > 0$.

$$x = \frac{\sqrt{3}}{2}$$

**Example 2**: The point $P\left(\frac{\sqrt{2}}{2}, y\right)$ is on the unit circle in the quadrant IV, find its $y$-coordinate.



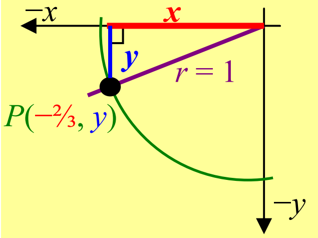

$x^2 + y^2 = 1$
$\left(\frac{\sqrt{2}}{2}\right)^2 + y^2 = 1$
$\left(\frac{2}{4}\right) + y^2 = 1$
$y^2 = 1 - ½$
$y^2 = ½$

$y = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} = \pm\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

In quadrant IV, $y < 0$.

$$y = -\frac{\sqrt{2}}{2}$$

**Example 3**: The point $P\left(-\frac{2}{3}, y\right)$ is on the unit circle in the quadrant III, find its $y$-coordinate.



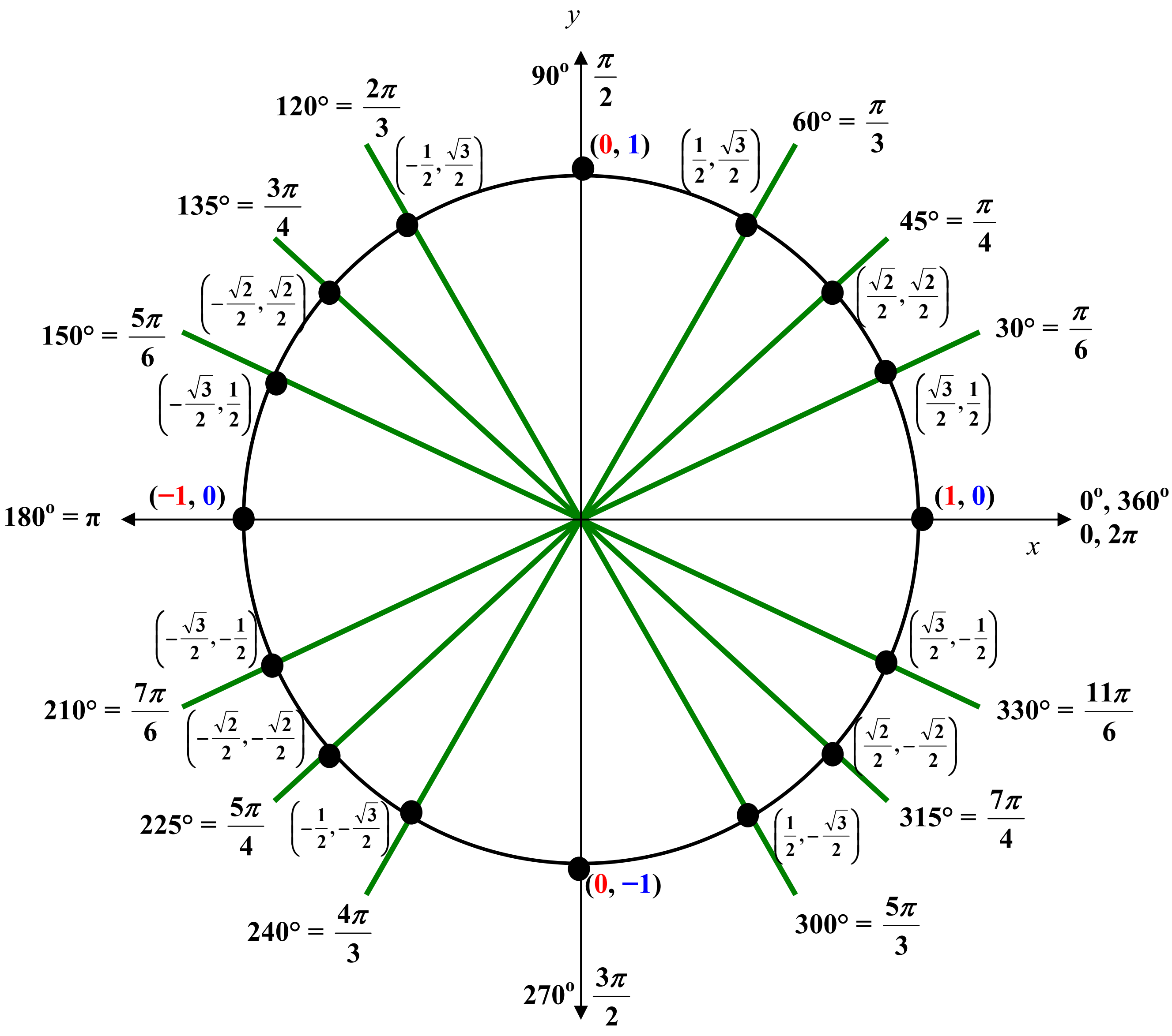

$$\left(-\tfrac{2}{3}\right)^2 + y^2 = 1$$
$$\tfrac{4}{9} + y^2 = 1$$
$$y^2 = 1 - \tfrac{4}{9}$$
$$y^2 = \tfrac{5}{9}$$

$$y = \pm\sqrt{\frac{5}{9}} = \pm\frac{\sqrt{5}}{3}$$

In quadrant III, $y < 0$.

$$y = -\frac{\sqrt{5}}{3}$$

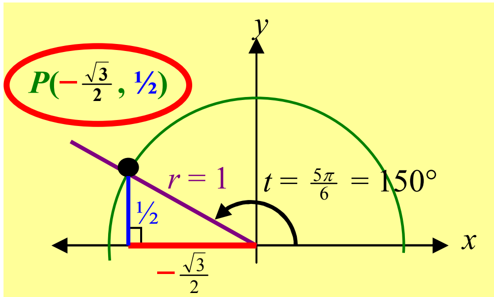
**The Complete Unit Circle**



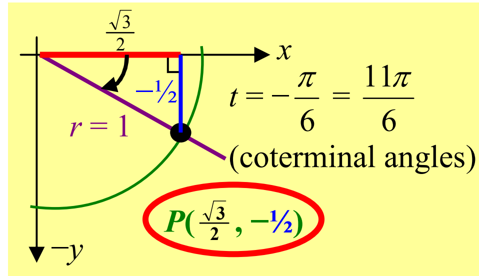

 

**Example 4**: Given the following terminal arm angle, $t$, find the terminal point $P(x, y)$ on the unit circle.

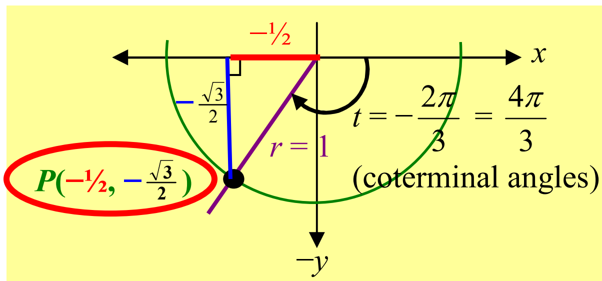
a. $t = \dfrac{5\pi}{6}$



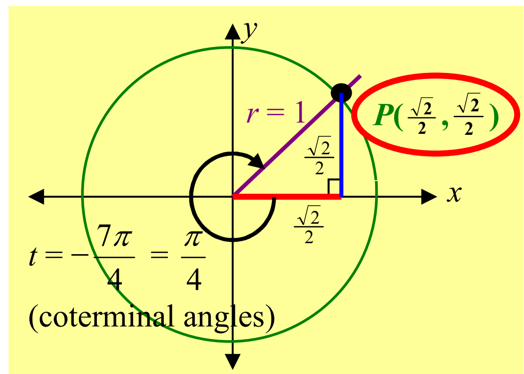

b. $t = -\dfrac{\pi}{6}$




c. $t = -\dfrac{2\pi}{3}$

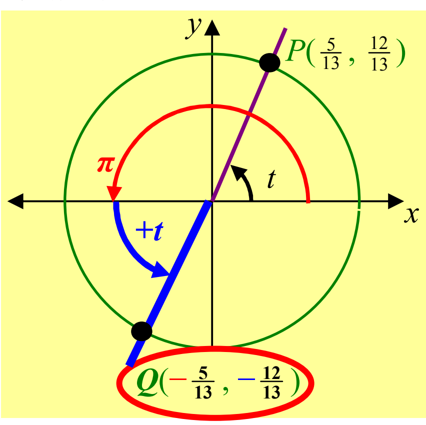



d. $t = -\dfrac{7\pi}{4}$

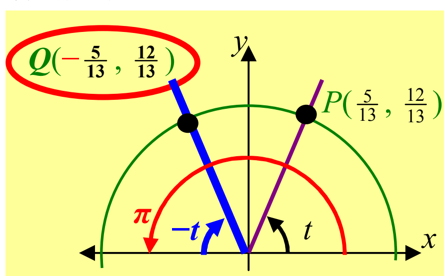



**Example 5**: A terminal point, $\left(\dfrac{5}{13}, \dfrac{12}{13}\right)$, is on an unit circle. Find the terminal point of the following expression if it is on the same unit circle.

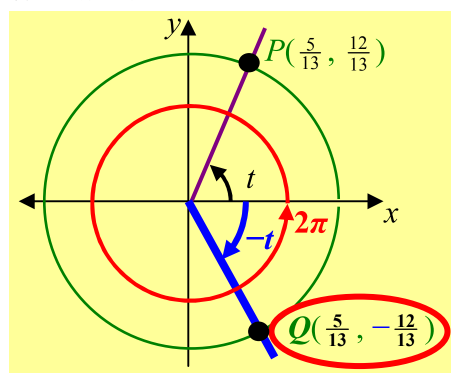
a. $\pi + t$




b. $\pi - t$

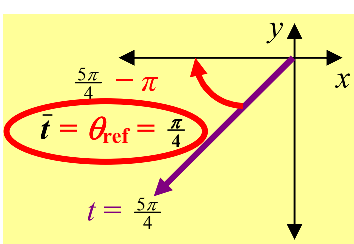



c. $2\pi - t$

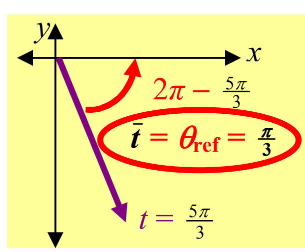



**Example 6**: Find the reference number (reference angle) given the following $t$ below.

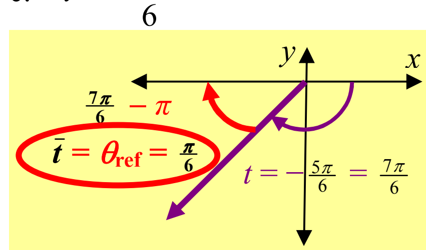
a. $t = \dfrac{5\pi}{4}$




b. $t = \dfrac{5\pi}{3}$

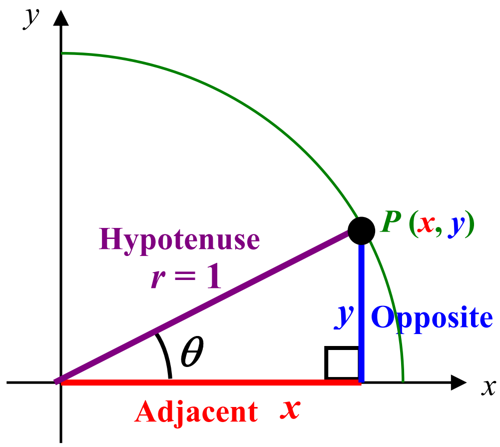



c. $t = -\dfrac{5\pi}{6}$




**7-1 Assignment: pg. 514−516 #7, 11, 13, 17, 19, 25, 27, 29, 31, 33, 41, 51, 53; Honour #55**

## 7-2 Trigonometric Functions of Real Numbers




For any **right angle triangles**, we can use the following simple **trigonometric ratios** or **trigonometric functions**.

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

**SOH** **CAH** **TOA**

Within the unit circle, these trig functions (sometimes called **circular functions**) are reduced to:

$\sin\theta = \dfrac{y}{r} = \dfrac{y}{1} = y$ $\cos\theta = \dfrac{x}{r} = \dfrac{x}{1} = x$ $\tan\theta = \dfrac{y}{x}$

**Note: For tan $\theta$, $x \neq 0$. Hence, tan $\theta$ is undefined at 90° & 270°**

**Reciprocal Trigonometric Function**: - the reciprocal of the regular trig functions.
- **sine (sin)** turns into **cosecant (csc),**
- **cosine (cos)** becomes **secant (sec),**
- and **tangent (tan)** changed to **cotangent (cot).**

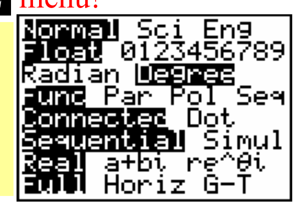
**Reciprocal Trigonometric Functions**

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$ $\sec\theta = \dfrac{\text{hypotenus}}{\text{adjacent}}$ $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

Within the unit circle, these reciprocal trig functions become

$\csc\theta = \dfrac{r}{y} = \dfrac{1}{y}$ $\sec\theta = \dfrac{r}{x} = \dfrac{1}{x}$ $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\left(\frac{y}{x}\right)} = \dfrac{x}{y}$

Depending on the unit of the angle given (degree or radian), be sure that your calculator is set in DEGREE or RADIAN under the settings in your **MODE** menu!




**Note: For cot $\theta$, $y \neq 0$ and $x \neq 0$. Hence, cot $\theta$ is undefined at 0°, 90°, 180°, 270° and 360°.**

**The coordinates ($x$, $y$) are the same as (cos $\theta$, sin $\theta$) of any angle $\theta$ in the unit circle.**

**2nd Quadrant** — **Sin**

sin θ = +
cos θ = –
tan θ = –

**1st Quadrant** — **All**

sin θ = +
cos θ = +
tan θ = +

**Tan**

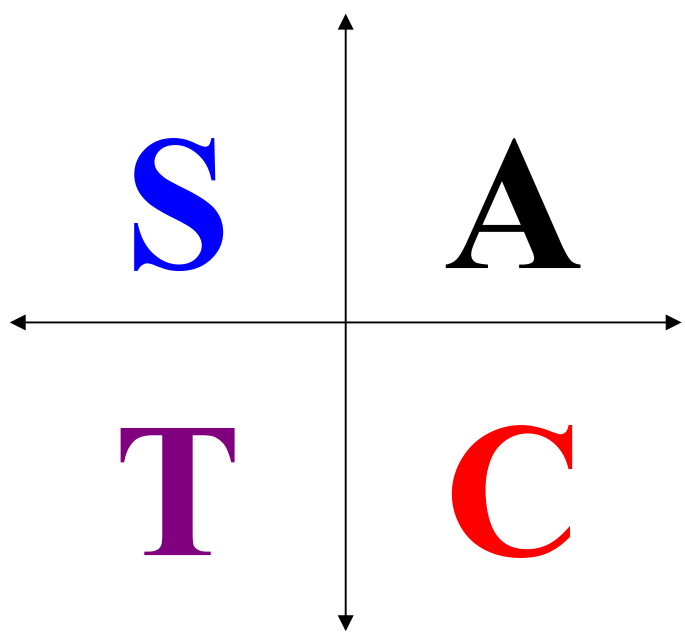
sin θ = –
cos θ = –
tan θ = +

**3rd Quadrant**

**Cos**

sin θ = –
cos θ = +
tan θ = –

**4th Quadrant**



**Example 1**: Using the unit circle, find the exact value of the trigonometric function at the given real number angle.

a. $\sin 30°$ b. $\cos\left(\frac{\pi}{4}\right)$ c. $\tan\left(\frac{\pi}{3}\right)$ d. $\csc 120°$ e. $\cot\left(\frac{7\pi}{6}\right)$

a. At 30°, $P(\frac{\sqrt{3}}{2}, \frac{1}{2})$

$\sin 30° = y$

$\mathbf{\sin 30° = \frac{1}{2}}$

b. At $\frac{\pi}{4}$, $P(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$\cos\left(\frac{\pi}{4}\right) = x$

$\mathbf{\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}}$

c. At $\frac{\pi}{3}$, $P(\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\tan\left(\frac{\pi}{3}\right) = \frac{y}{x} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)}$

$\mathbf{\tan\left(\frac{\pi}{3}\right) = \sqrt{3}}$

d. At 120°, $P(-\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\csc\left(\frac{\pi}{3}\right) = \frac{1}{y} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}$

$\mathbf{\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}}$

e. At $\frac{7\pi}{6}$, $P(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$

$\tan\left(\frac{7\pi}{6}\right) = \frac{x}{y} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)}$

$\mathbf{\tan\left(\frac{7\pi}{6}\right) = \sqrt{3}}$

f. $\sin 300°$ g. $\tan\left(\frac{7\pi}{4}\right)$ h. $\cos\left(-\frac{\pi}{2}\right)$ i. $\sec\left(-\frac{\pi}{6}\right)$ j. $\cot\left(-\frac{3\pi}{2}\right)$

f. At 300°, $P(\frac{1}{2}, -\frac{\sqrt{3}}{2})$

$\sin 300° = y$

$\mathbf{\sin 300° = -\frac{\sqrt{3}}{2}}$

g. At $\frac{7\pi}{4}$, $P(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

$\tan\left(\frac{7\pi}{4}\right) = \frac{y}{x} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)}$

$\mathbf{\tan\left(\frac{7\pi}{4}\right) = -1}$

h. At $-\frac{\pi}{2} = \frac{3\pi}{2}$, $P(0, -1)$

$\cos\left(-\frac{\pi}{2}\right) = x$

$\mathbf{\cos\left(-\frac{\pi}{2}\right) = 0}$

i. At $-\frac{\pi}{6} = \frac{11\pi}{6}$, $P(\frac{\sqrt{3}}{2}, -\frac{1}{2})$

$\sec\left(-\frac{\pi}{6}\right) = \frac{1}{x} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}$

$\mathbf{\sec\left(-\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}}$

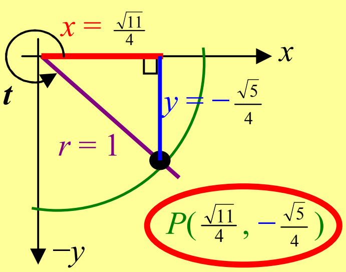
j. At $-\frac{3\pi}{2} = \frac{\pi}{2}$, $P(0, 1)$

$\cot\left(\frac{-3\pi}{2}\right) = \frac{1}{\left(\frac{y}{x}\right)} = \frac{1}{\left(\frac{1}{0}\right)}$

$\mathbf{\cot\left(\frac{-3\pi}{2}\right) = \text{undefined}}$

**Example 2**: Given the terminal point, $P\left(\frac{\sqrt{11}}{4}, -\frac{\sqrt{5}}{4}\right)$, find the values of the trigonometric functions.

$x > 0$ and $y < 0$ means $P$ is at quadrant IV




$\sin t = y \rightarrow \mathbf{\sin t = -\frac{\sqrt{5}}{4}}$

$\cos t = x \rightarrow \mathbf{\cos t = \frac{\sqrt{11}}{4}}$

$\tan t = \frac{y}{x} = \frac{\left(-\frac{\sqrt{5}}{4}\right)}{\left(\frac{\sqrt{11}}{4}\right)} \rightarrow \mathbf{\tan t = -\frac{\sqrt{5}}{\sqrt{11}}}$

$\csc t = \frac{1}{y} = \frac{1}{\left(-\frac{\sqrt{5}}{4}\right)} \rightarrow \mathbf{\csc t = -\frac{4}{\sqrt{5}}}$

$\sec t = \frac{1}{x} = \frac{1}{\left(\frac{\sqrt{11}}{4}\right)} \rightarrow \mathbf{\sec t = \frac{4}{\sqrt{11}}}$

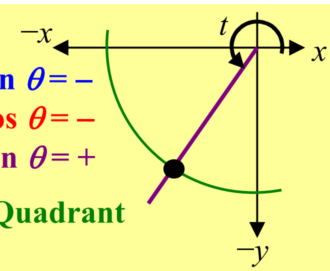
$\cot t = \frac{x}{y} = \frac{\left(\frac{\sqrt{11}}{4}\right)}{\left(-\frac{\sqrt{5}}{4}\right)} \rightarrow \mathbf{\cot t = -\frac{\sqrt{11}}{\sqrt{5}}}$

**Example 3**: From the information given below, determine which quadrant the terminal point has to be at.

a. $\sin t < 0$ and $\cos t < 0$

$\sin t < 0$ means $y < 0$ ;
$\cos t < 0$ means $x < 0$
Since both $x$ and $y$ is negative in the quadrant III,

***t* must be in quadrant III.**




**sin θ = −**
**cos θ = −**
**tan θ = +**

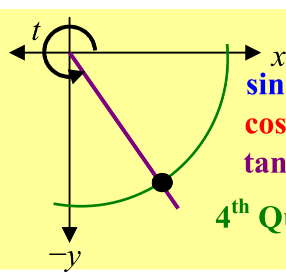
**3rd Quadrant**

b. $\cot t < 0$ and $\sec t > 0$

$\cot t < 0$ means $\tan t < 0$ (Quadrants II and **IV**) ;

$\sec t > 0$ means $\cos t > 0$ (Quadrants I and **IV**)

***t* must be in quadrant IV.**




**sin θ = −**
**cos θ = +**
**tan θ = −**

**4th Quadrant**

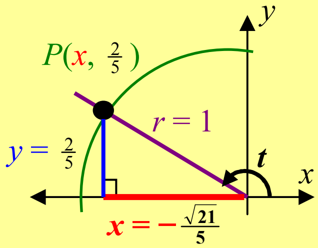
**Example 4**: Find the values of the trigonometric functions of $t$ from the given information.

a. $\sin t = \dfrac{2}{5}$ and $\tan t < 0$

$\sin t > 0$ means $y > 0$ (Quadrants I and **II**) ;

$\tan t < 0$ (Quadrants **II** and IV)

***t* is in quadrant II.**




$$x^2 + y^2 = 1$$
$$x^2 + \left(\tfrac{2}{5}\right)^2 = 1$$
$$x^2 + \tfrac{4}{25} = 1$$
$$x^2 = 1 - \tfrac{4}{25}$$
$$x^2 = \tfrac{21}{25}$$

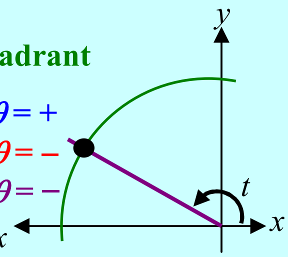
$$x = \pm\sqrt{\frac{21}{25}} = \pm\frac{\sqrt{21}}{5}$$

In quadrant II, $x < 0$.

$$\boldsymbol{x = -\frac{\sqrt{21}}{5}}$$

**2nd Quadrant**

**$\sin\theta = +$**
**$\cos\theta = -$**
**$\tan\theta = -$**




$\sin t = y \rightarrow$ $\boldsymbol{\sin t = \frac{2}{5}}$

$\cos t = x \rightarrow$ $\boldsymbol{\cos t = -\frac{\sqrt{21}}{5}}$

$\tan t = \dfrac{y}{x} = \dfrac{\left(\frac{2}{5}\right)}{\left(-\frac{\sqrt{21}}{5}\right)} \rightarrow$ $\boldsymbol{\tan t = -\frac{2}{\sqrt{21}}}$

$\csc t = \dfrac{1}{y} = \dfrac{1}{\left(\frac{2}{5}\right)} \rightarrow$ $\boldsymbol{\csc t = \frac{5}{2}}$

$\sec t = \dfrac{1}{x} = \dfrac{1}{\left(-\frac{\sqrt{21}}{5}\right)} \rightarrow$ $\boldsymbol{\sec t = -\frac{5}{\sqrt{21}}}$

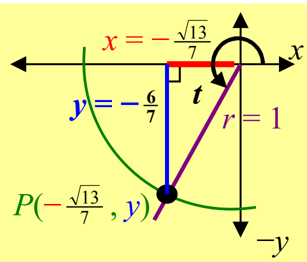
$\cot t = \dfrac{x}{y} = \dfrac{\left(-\frac{\sqrt{21}}{5}\right)}{\left(\frac{2}{5}\right)} \rightarrow$ $\boldsymbol{\cot t = -\frac{\sqrt{21}}{2}}$

b. $\cos t = -\dfrac{\sqrt{13}}{7}$ and $\sin t < 0$

$\cos t < 0$ means $x < 0$ (Quadrants II and **III**) ;

$\sin t < 0$ means $y < 0$ (Quadrants **III** and IV)

***t* is in quadrant III.**



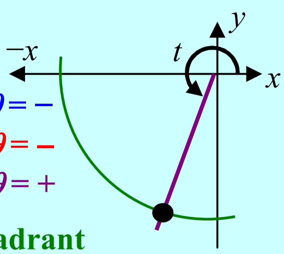

$$x^2 + y^2 = 1$$
$$\left(-\tfrac{\sqrt{13}}{7}\right)^2 + y^2 = 1$$
$$\tfrac{13}{49} + y^2 = 1$$
$$y^2 = 1 - \tfrac{13}{49}$$
$$y^2 = \tfrac{36}{49}$$

$$y = \pm\sqrt{\frac{36}{49}} = \pm\frac{6}{7}$$

In quadrant III, $y < 0$.

$$\boldsymbol{y = -\frac{6}{7}}$$

**$\sin\theta = -$**
**$\cos\theta = -$**
**$\tan\theta = +$**




**3rd Quadrant**

$\sin t = y \rightarrow$ $\boldsymbol{\sin t = -\frac{6}{7}}$

$\cos t = x \rightarrow$ $\boldsymbol{\cos t = -\frac{\sqrt{13}}{7}}$

$\tan t = \dfrac{y}{x} = \dfrac{\left(-\frac{6}{7}\right)}{\left(-\frac{\sqrt{13}}{7}\right)} \rightarrow$ $\boldsymbol{\tan t = \frac{6}{\sqrt{13}}}$

$\csc t = \dfrac{1}{y} = \dfrac{1}{\left(-\frac{6}{7}\right)} \rightarrow$ $\boldsymbol{\csc t = -\frac{7}{6}}$

$\sec t = \dfrac{1}{x} = \dfrac{1}{\left(-\frac{\sqrt{13}}{7}\right)} \rightarrow$ $\boldsymbol{\sec t = -\frac{7}{\sqrt{13}}}$

$\cot t = \dfrac{x}{y} = \dfrac{\left(-\frac{\sqrt{13}}{7}\right)}{\left(-\frac{6}{7}\right)} \rightarrow$ $\boldsymbol{\cot t = \frac{\sqrt{13}}{6}}$

**7-2 Assignment: pg. 524−526 #5, 9, 11, 17, 33, 37, 41, 43, 51, 63, 67, 82;**
**Honour #59, 73, 77, 83**

## 7-3 Trigonometric Graphs

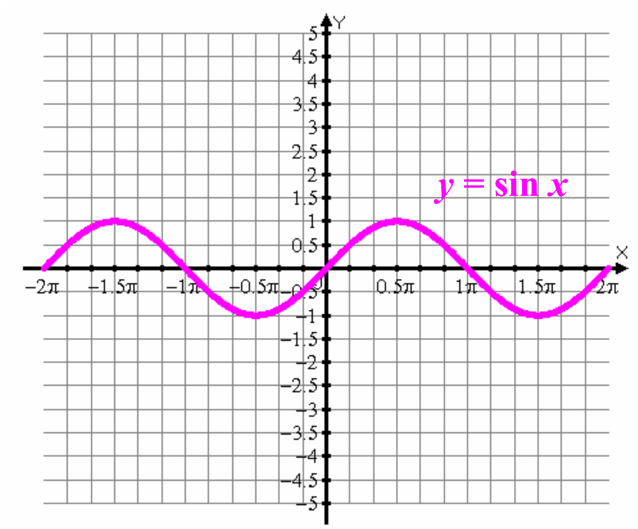
$y = a \sin k(x + b) + c$ $y = a \cos k(x + b) + c$

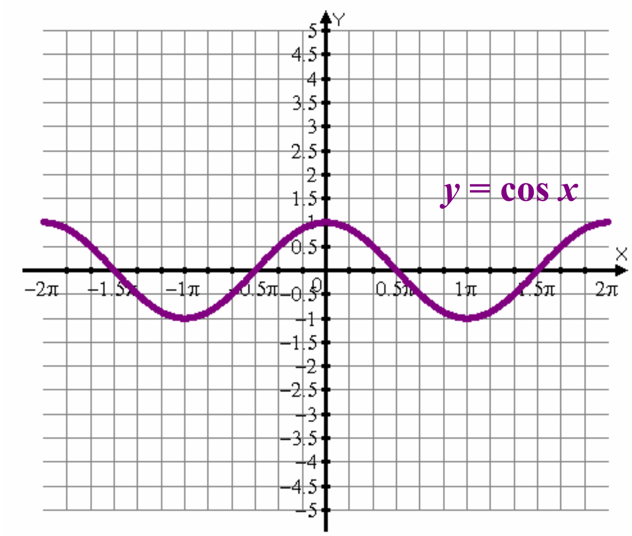
$|a|$ = Amplitude $c$ = Vertical Displacement (how far away from the $x$–axis)

$b$ = Horizontal Displacement (Phase Shift) $b > 0$ (shifted left) $b < 0$ (shifted right)

$k$ = number of complete cycles in $2\pi$ Period = $\frac{2\pi}{k} = \frac{360^\circ}{k}$

Range = Minimum $\leq y \leq$ Maximum



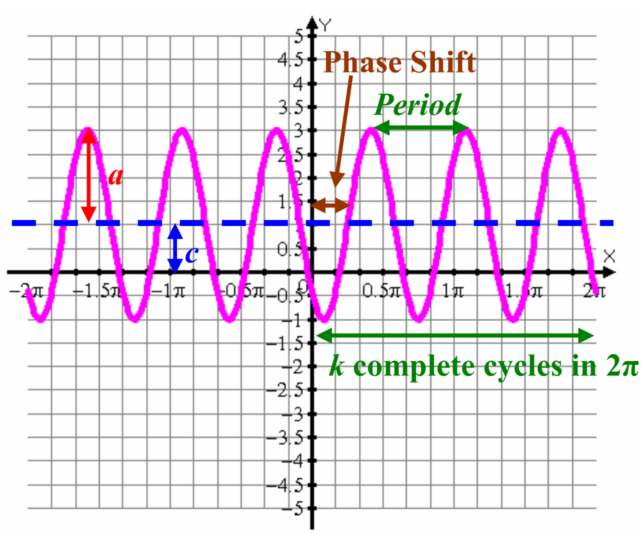

Examples: $y = 2 \sin 3(x - \frac{\pi}{4}) + 1$ $y = 3 \cos 2(x - \frac{7\pi}{12}) - 2$

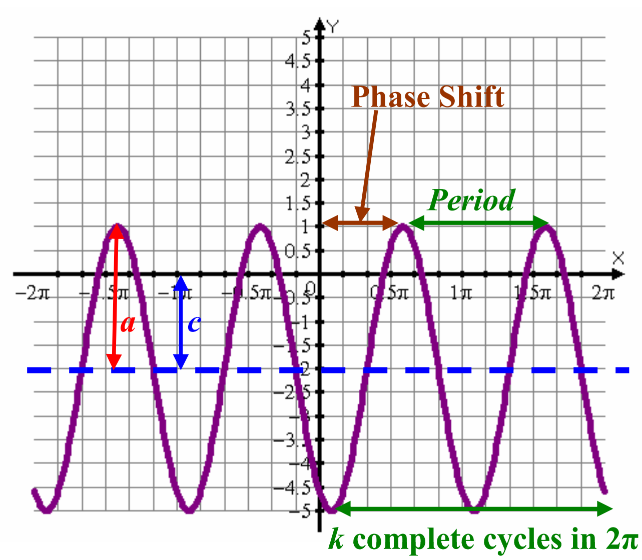
$a = 2$ $k = 3$ cycles in $2\pi$ $b = \frac{\pi}{4}$ right

$c = 1$ Period = $\frac{2\pi}{3}$ Range: $-1 \leq y \leq 3$

$a = 3$ $k = 2$ cycles in $2\pi$ $b = \frac{7\pi}{12}$

$d = -2$ Period = $\pi$ Range: $-5 \leq y \leq 1$



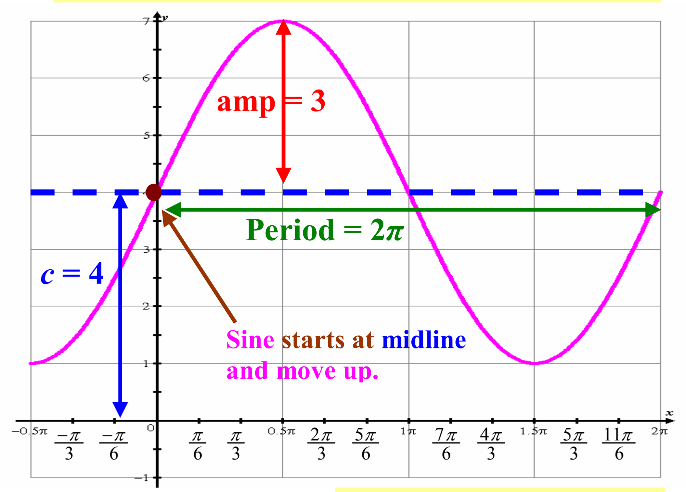
## Graphing Trigonometric Functions

1. Identify the **amplitude**, **phase shift**, **number of complete cycles in $2\pi$**, and **vertical displacement**.
2. Calculate the **period** and the **range**.
3. From the **period** and **phase shift**, **determine the interval needed along the $x$-axis**.
4. From the **vertical displacement** and the **range**, **determine the interval needed on the $y$-axis**.
5. **Divide each period into four sections, use some fix points from the original sine and cosine graph** (such as $x = 0$, $\frac{\pi}{2}$, $\pi$, $\frac{3\pi}{2}$, and $2\pi$) **along with the range to plot some points**. Then connect the dots.

**Example 1**: Find the amplitude, period, phase shift, and vertical displacement of the function and sketch its graph over at least one period.

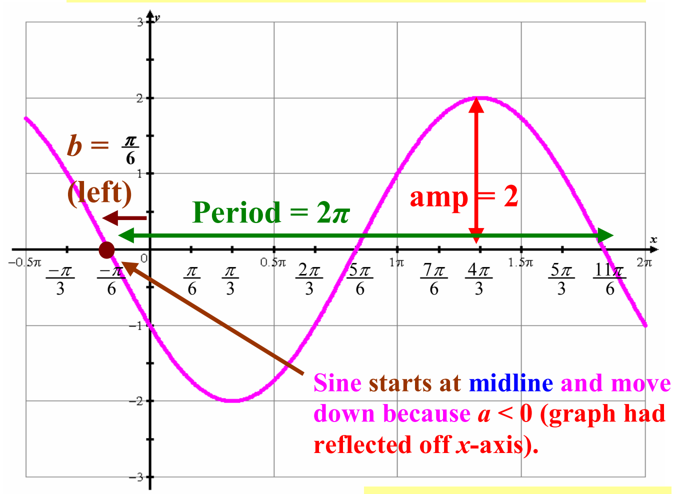
a. $y = 3\sin x + 4$

**amp = 3** **$c$ = 4 up**
**Range: $1 \le y \le 7$**
**$b$ = 0 (no phase shift)** **$k$ = 1 cycle in $2\pi$**
Period = $\frac{2\pi}{k} = \frac{2\pi}{1}$ **Period = $2\pi$**



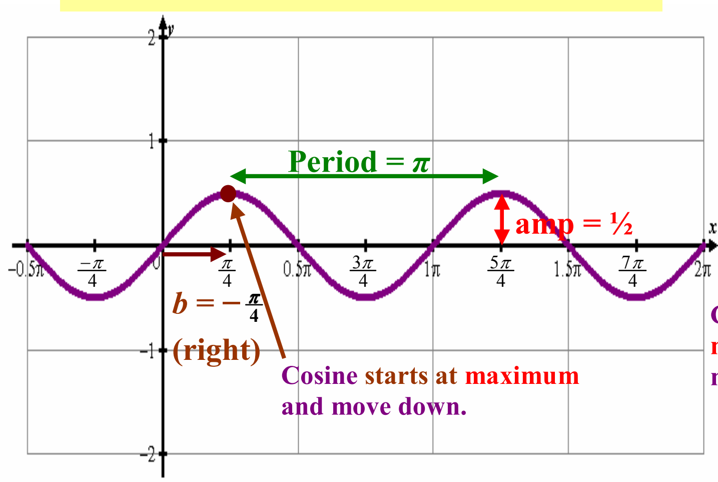

b. $y = -2\sin\left(x + \frac{\pi}{6}\right)$

**amp = 2 (|−2| = 2)**
**$c$ = 0 (no vert. disp.)**
**Range: $-2 \le y \le 2$**
**$b = \frac{\pi}{6}$ to the left** **$k$ = 1 cycle in $2\pi$**
Period = $\frac{2\pi}{k} = \frac{2\pi}{1}$ **Period = $2\pi$**




c. $y = ½\cos 2\left(x - \frac{\pi}{4}\right)$

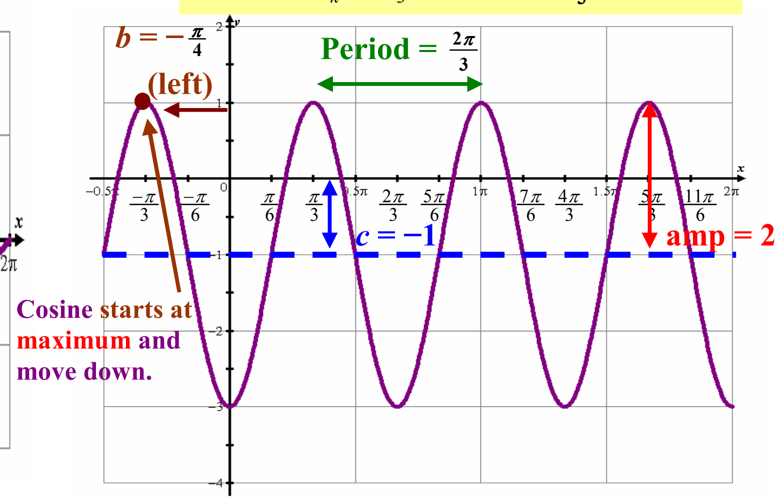
**amplitude = ½**
**$c$ = 0 (no vert. disp.)**
**Range: $-½ \le y \le ½$**
**$b = -\frac{\pi}{4}$ (right)** **$k$ = 2 cycles in $2\pi$**
Period = $\frac{2\pi}{k} = \frac{2\pi}{2}$ **Period = $\pi$**




d. $y = 2\cos(3x + \pi) - 1$

$y = 2\cos 3\left(x + \frac{\pi}{3}\right) - 1$

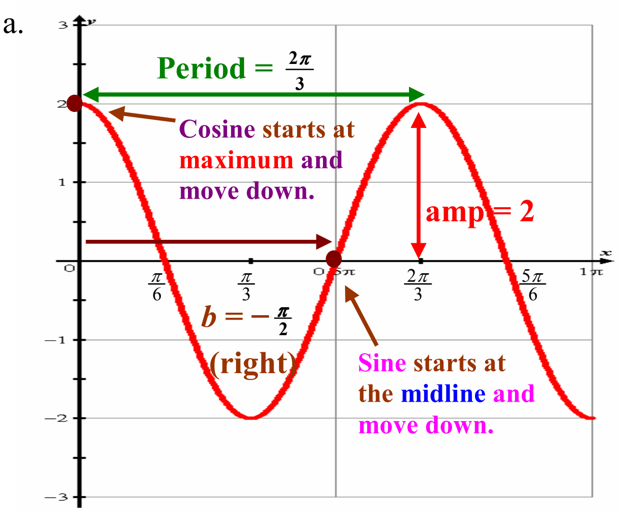
**amplitude = 2**
**$c$ = −1 (1 down)**
**Range: $-3 \le y \le 1$**
**$b = \frac{\pi}{3}$ (left)** **$k$ = 3 cycles in $2\pi$**
Period = $\frac{2\pi}{k} = \frac{2\pi}{3}$ **Period = $\frac{2\pi}{3}$**



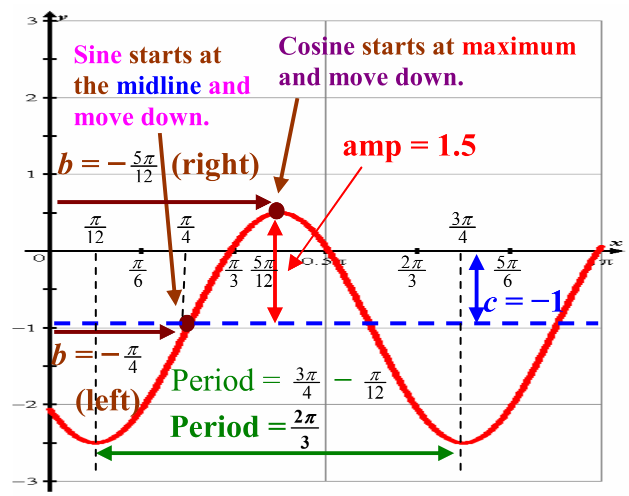

 

**Example 2**: The graph of one complete period of a sine or cosine curve is given. Find the amplitude, period, phase shift, and vertical displacement. Write an equation that represent the curve in the form of sine and cosine functions.

a.




b.




**a.** Period = $\frac{2\pi}{3}$  amp. = 2 → $a = 2$  $c = 0$ (no vert. disp.)

Period = $\frac{2\pi}{k}$  **For cosine, $b = 0$.**  **For sine, $b = -\frac{\pi}{4}$.**

$\frac{2\pi}{3} = \frac{2\pi}{k}$

$k = 3$  $y = 2\cos(3x)$  $y = 2\sin 3\left(x - \frac{\pi}{4}\right)$

**b.** Period = $\frac{2\pi}{3}$  amp. = 1.5 → $a = 1.5$  $c = -1$

Period = $\frac{2\pi}{k}$  **For cosine, $b = -\frac{5\pi}{12}$.**

$\frac{2\pi}{3} = \frac{2\pi}{k}$

$k = 3$  $y = 1.5\cos\left[3\left(x - \frac{5\pi}{12}\right)\right] - 1$

**For sine, $b = -\frac{\pi}{4}$.**

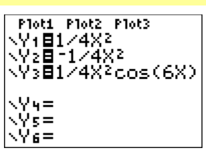
$y = 1.5\sin\left[3\left(x - \frac{\pi}{4}\right)\right] - 1$

**To Graph Trig Functions on the Graphing Calculator:**

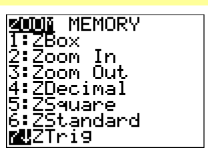
1. Set calculator to **Radian** in the MODE screen.
2. Enter the equation in the Y= screen.
3. Press ZOOM and select **ZTrig** (option 7).

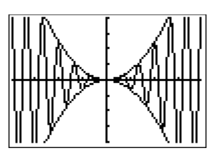
**Example 3**: Graph $y = \frac{1}{4}x^2$, $y = -\frac{1}{4}x^2$ and $y = \frac{1}{4}x^2 \cos 6x$. How are the graphs related?

In Radian Mode, enter equations in Y=




ZOOM Select **ZTrig**

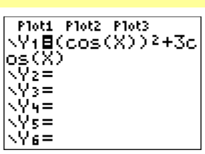


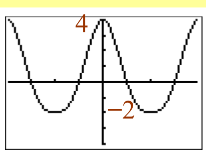
**The Trig equation ($Y_3$) is bounded by the quadratic equations ($Y_1$ & $Y_2$). This is because the Trig equation consists of both a quadratic function ($\frac{1}{4}x^2$) and a trig function ($\cos(6x)$).**

**Example 4**: Find the range of $y = \cos^2 x + 3\cos x$.

Enter equation in Y=




ZOOM Select **ZTrig**

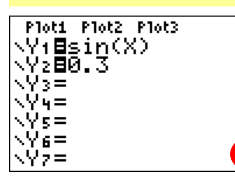



**From the graph, the Range is**

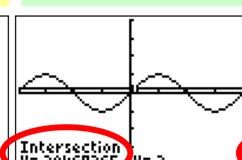
$-2 \le y \le 4$

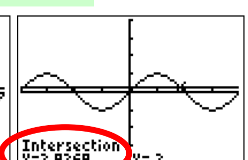
**Example 5**: Find all solutions of $\sin x = 0.3$ for $[0, \pi]$.

Enter equation in Y=




**Run Intersect twice in 2nd TRACE**




**7-3 Assignment: pg. 537−539 #17, 21, 25, 29, 33, 37, 41, 43, 45, 47, 67, 69, 75, 77; Honour #59, 82**

## 7-5 Modeling Harmonic Motion

Sometimes, a description of the periodic pattern (**harmonic motion** if the pattern applies to a moving object) is given. In such case, it is very important to **determine the features of the graph** (**amplitude, period, horizontal displacement, and vertical displacement**). They will be used to generate the parameters needed for the basic trigonometric function, $y = a \sin[\omega(t + b)] + c$, or $y = a \cos[\omega(t + b)] + c$.
**(Note: $k$ – the number of complete cycles in $2\pi$ is now replaced by $\omega$.)**

**Period**: - the amount of time needed to complete one cycle.

**Frequency**: - the number of cycles per unit of time. (The longer is the period; the smaller the frequency.)

$$y = a \sin[\omega(t + b)] + c \qquad y = a \cos[\omega(t + b)] + c$$

**$|a|$ = Amplitude** **$c$ = Vertical Displacement (distance between *mid-line* and *t*-axis)**

**$b$ = Horizontal Displacement (Phase Shift)** **$b > 0$ (shifted left)** **$b < 0$ (shifted right)**

**$\omega$ = number of complete cycles in $2\pi$** **Period = $\dfrac{2\pi}{\omega}$** **Frequency = $\dfrac{\omega}{2\pi}$**

**Range = Minimum $\leq y \leq$ Maximum**

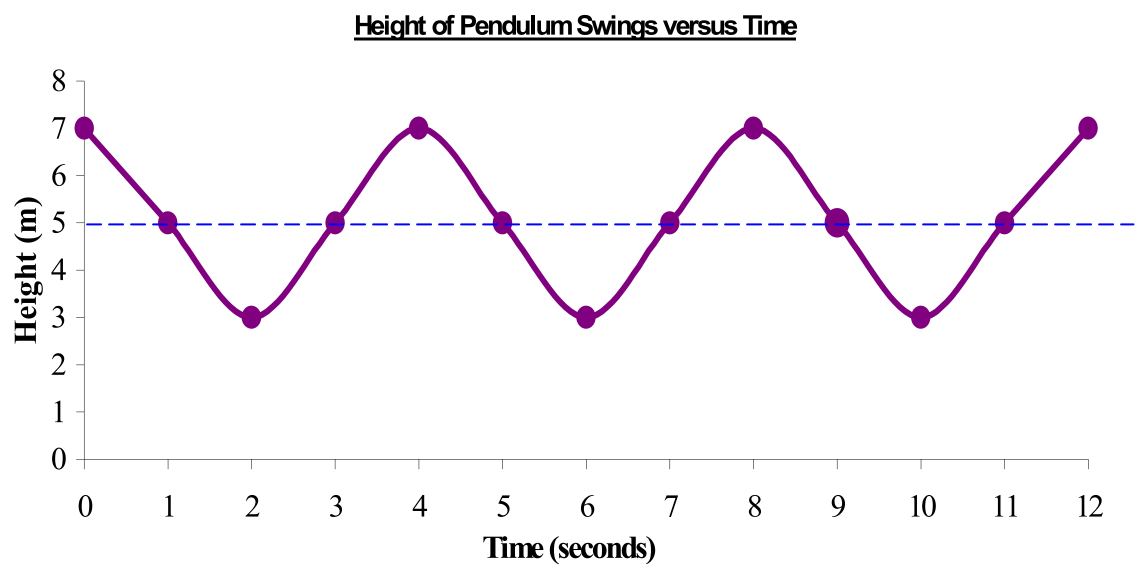
**Example 1**: A mechanical pendulum has a height of 3 m off the ground. When it swings to the highest point, its height is 7 m off the ground. It makes 15 complete swings per minute, and the starting point is on the right side of the rest position.

a. What is the period of the pendulum?
b. Draw a graph to describe the height of the pendulum versus time for 3 complete cycles.
c. Explain all the features of the graph and determine the equation of height in terms of time.
d. Find the height of the pendulum at 10.3 seconds.
e. At what time(s) will the height of the pendulum be at 5.5 m during the first complete cycle?

**a.** Frequency = 15 swings / min = $\dfrac{15 \text{ swings}}{60 \text{ seconds}}$ **Frequency = ¼ swing/ sec**

Period = $\dfrac{\text{time in seconds}}{1 \text{ cycle (or swing)}} = \dfrac{4 \text{ seconds}}{1 \text{ swing}}$ **Period = 4 sec / cycle**

**b.**



c. **Characteristics of the Graph**

**Amplitude = | $a$ | = 2 m (how far the height is varied from one side of the swing to the rest position)**

**Vertical Displacement = $c$ = 5 m (the average height of the pendulum)**

**Range: 3 m ≤ $h$ ≤ 7 m (the min and max heights of the pendulum)**

**Period = 4 sec (time to complete one full swing)** $\text{Period} = \frac{2\pi}{\omega}$ $\omega = \frac{2\pi \text{ rad}}{\text{Period}} = \frac{2\pi}{4}$ $\omega = \frac{\pi}{2}$

**For cosine function, Horizontal Translation $b$ = 0**

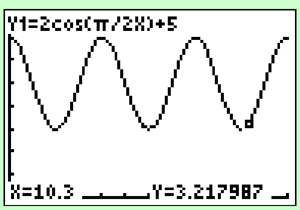
**For sine function, Horizontal Translation $b$ = –3 second (right)**

| $a$ | $\omega$ | $b$ | $c$ |
|---|---|---|---|
| 2 | $\frac{\pi}{2}$ | 0 (for cos)<br>−3 (for sin) | 5 |

$h = 2\cos\left(\frac{\pi}{2}t\right) + 5$ $h = 2\sin\left[\frac{\pi}{2}(t - 3)\right] + 5$

d. Height at 10.3 seconds

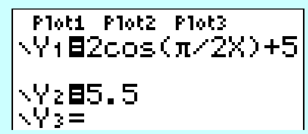
1. **Enter equation in Radian Mode**
2. **Run TRACE**
3. **Window Settings:**
   **$x$: [0, 12, 1] and $y$: [ 0, 8, 1]**



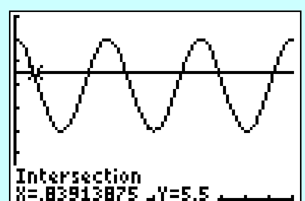

**Height = 3.218 m**

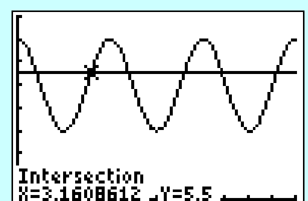
e. When will the pendulum reach 5.5 m during the first complete cycle?

1. **Enter $Y_2$ equation as 5.5**




2. **Run Intersect twice on the first cycle.**




**$t$ = 0.839 seconds and 3.161 seconds**

**Example 2**: The London Eye is one of the largest ferris wheels. It has a diameter of 135 m and the bottom of the wheel passes 1 m above ground. A complete revolution takes 30 minutes and the visitors are treated with an uninterrupted view of the city as far out as 40 km (25 miles). Determine the equation of the visitor's height as a function of time starting at the lowest point of the wheel.

Amplitude = Half the diameter = $\frac{135\text{ m}}{2}$ **| $a$ | = 67.5 m**

Vert. Disp. = Height between ground & mid-line **$c$ = 67.5 m + 1 m**

Period = 30 min = $\frac{2\pi}{\omega}$ $\omega = \frac{2\pi}{30}$ $\omega = \frac{\pi}{15}$

**For cosine function, 15 mins to get to highest point** **$b$ = −15**

**For sine function, 7.5 mins to be half way up** **$b$ = –7.5**

$h = 67.5\cos\left[\frac{\pi}{15}(t - 15)\right] + 68.5$ $h = 67.5\sin\left[\frac{\pi}{15}(t - 7.5)\right] + 68.5$

**7-5 Assignment: pg. 559−561 #25, 27, 29, 31, 34, 40, 41; Honour #35**