Chapter 2: Coordinates and Graphs

2-1: The Coordinate Plane

Plot: - putting points on a grid to form a graph.

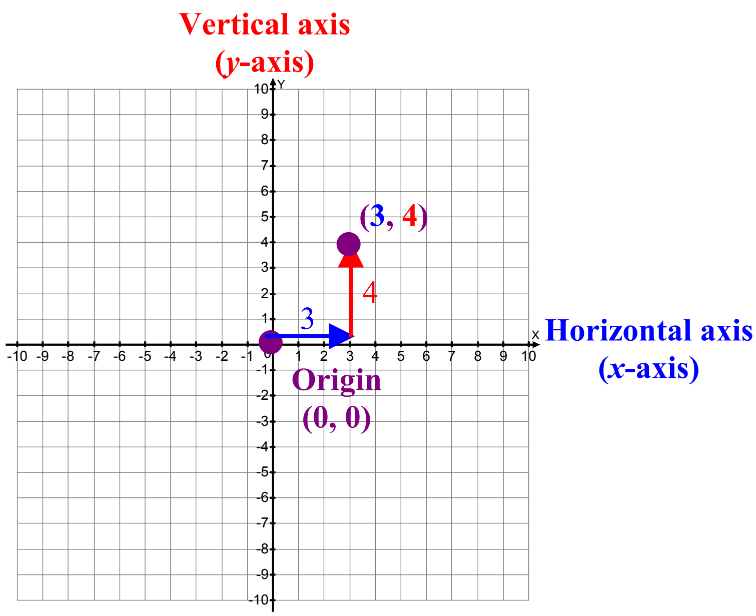
Axis: - the number line on a grid.
- **Horizontal axis**: - also known as the ***x*-axis** that lies horizontally.
- **Vertical axis**: - also known as the ***y*-axis** that lies vertically.

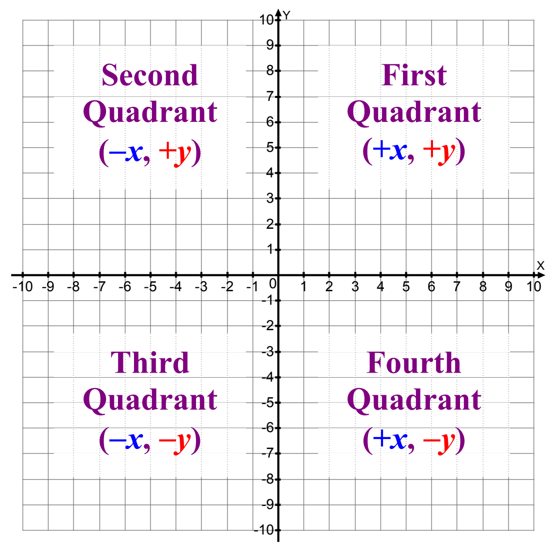
Coordinate Plane: - sometimes refers to as the **Cartesian** Plane.
- a plane consisting of a number line in the ***x*-axis (horizontal number line)** along with the ***y*-axis (vertical number line)**.

Order Pair: - also known as **coordinates** that indicate the location of a point on the grid.
- **Abscissa**: - also called the ***x*-coordinate**. It is written first in the order pair.
- **Ordinate**: - also called the ***y*-coordinate**. It is written last in the order pair.

Origin: - the intersecting point between the horizontal and vertical axis. (0, 0)

Quadrant: - the four areas of the grid as separated by the horizontal and vertical axis.





Graphing Regions in the Coordinate Plane:

1. **Horizontal Line (y = constant)**: - all *y-values are always the same*. Hence, *only x-values changes*.
2. **Vertical Line (x = constant)**: - all *x-values are always the same*. Hence, *only y-values changes*.

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3. Regions Bounded by Inequalities:

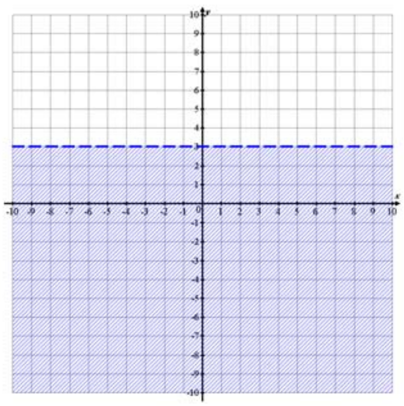
a. **For inequalities with < or >, use BROKEN Line for the graph. For inequalities with ≤ or ≥, use SOLID Line for the graph.**

b. **SHADE in the Proper Region.**

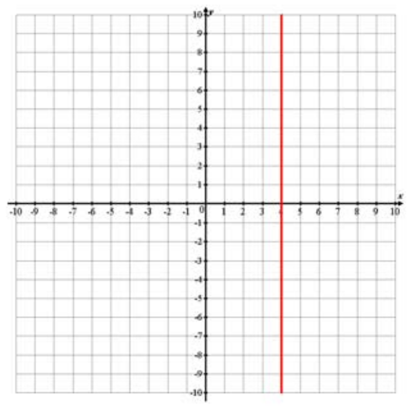
- **i. For < or ≤, shade Below or Left of the Line.**
- **ii. For > or ≥, shade Above or Right of the Line.**
- **iii. For | *x* or *y* | ≤ or < constant, shade Between the two Boundary Lines.**
- **iv. For | *x* or *y* | ≥ or > constant, shade the regions Beyond the two Boundary Lines.**

Example 1: Describe and sketch the regions given by each set.

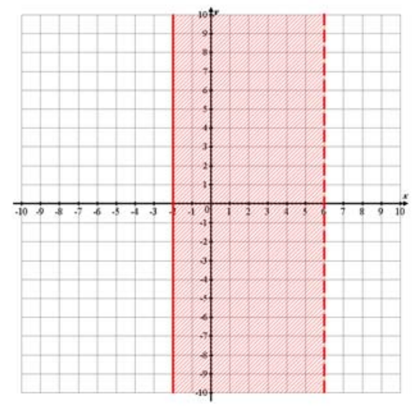
a. $\{(x, y) \mid y < 3\}$



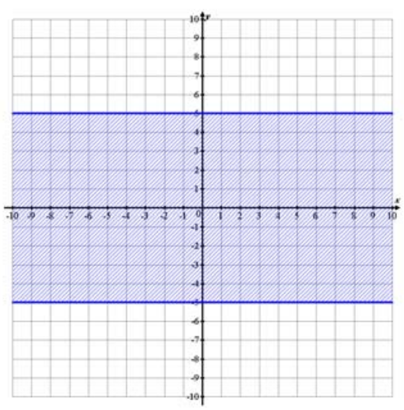
b. $\{(x, y) \mid x = -4\}$



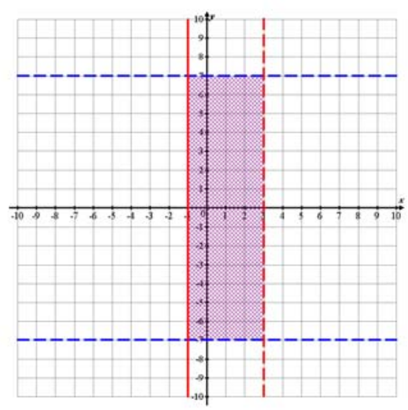
c. $\{(x, y) \mid -2 \le x < 6\}$



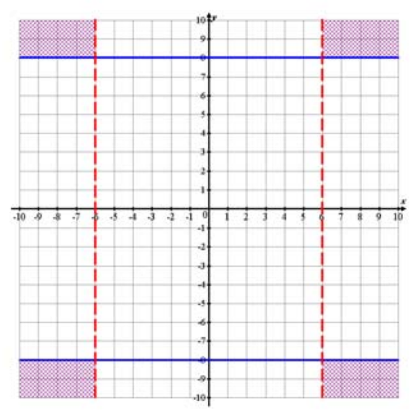
d. $\{(x, y) \mid \ |y| \le 5\ \}$



e. $\{(x, y) \mid -1 \le x < 3 \text{ and } |y| < 7\ \}$



f. $\{(x, y) \mid |x| > 6 \text{ and } |y| \ge 8\ \}$



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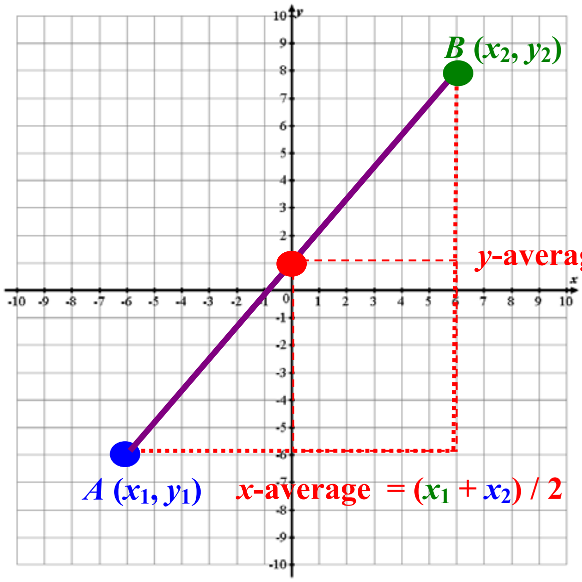


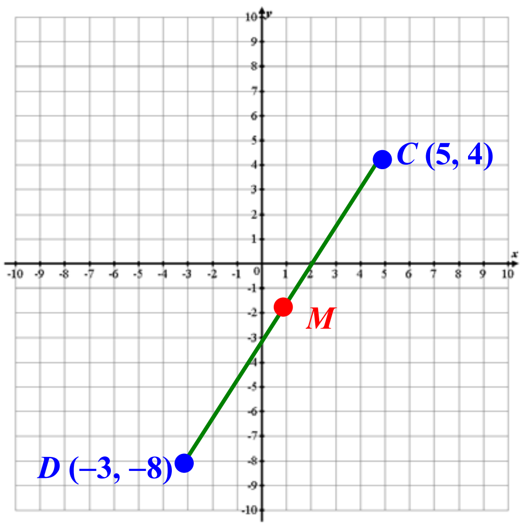
Midpoint: - the location (coordinate) of a point in the middle of a line segment.
- the length of one side of the midpoint is equivalent to the length of the other side.

Midpoint of a Line Segment

M = (average of x-coordinates, average of y-coordinates)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 2: Find the midpoints of the line segments $\overline{CD}$ where C (5, 4) and D (−3, −8).

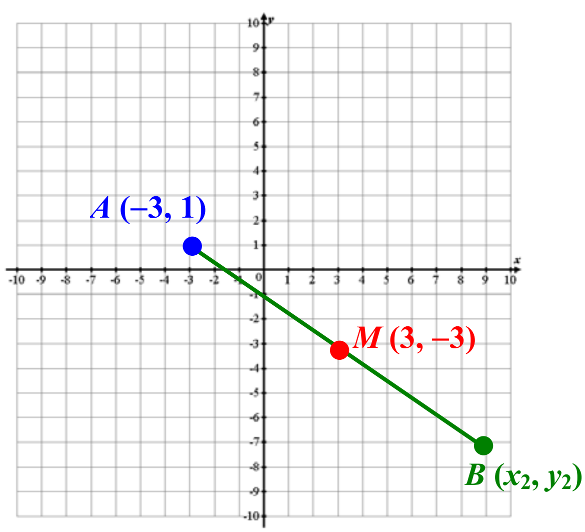


$$M_{\overline{CD}} = \left(\frac{5 + -3}{2}, \frac{4 + -8}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{-4}{2}\right)$$

$$M_{\overline{CD}} = (1, -2)$$

Example 3: Given that the midpoint of $\overline{AB}$ is M (3, −3). If one of the endpoint of $\overline{AB}$ is A (−3, 1), find the coordinate of endpoint B.



$$M_{\overline{AB}} = \left(\frac{x_2 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(3, -3) = \left(\frac{-3 + x_2}{2}, \frac{1 + y_2}{2}\right)$$

$$3 = \frac{-3 + x_2}{2}$$
$$3 \times 2 = -3 + x_2$$
$$6 = -3 + x_2$$
$$6 + 3 = x_2$$
$$x_2 = 9$$

$$-3 = \frac{1 + y_2}{2}$$
$$-3 \times 2 = 1 + y_2$$
$$-6 = 1 + y_2$$
$$-6 - 1 = y_2$$
$$y_2 = -7$$

$$B = (9, -7)$$

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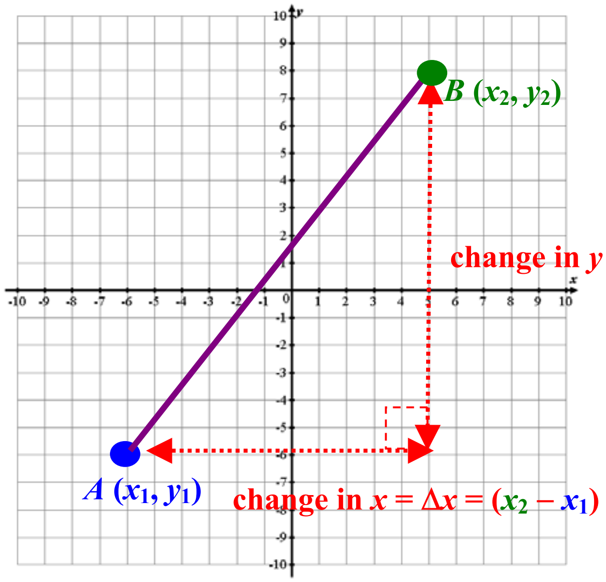


Distance Between Two Points

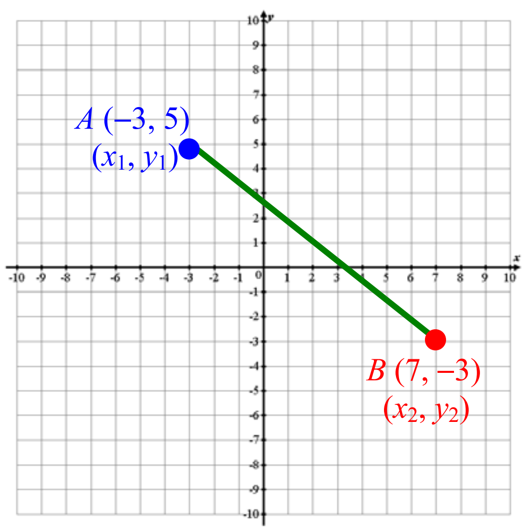
d^2 = (change in x)^2 + (change in y)^2

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Distance Formula

The distance d between points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example 4: Find the distance (in exact value) between A (−3, 5) and B (7, −3).



$$d_{\overline{AB}} = \sqrt{(7 - -3)^2 + (-3 - 5)^2}$$
$$= \sqrt{(10)^2 + (-8)^2}$$
$$= \sqrt{100 + 64}$$
$$= \sqrt{164}$$
$$d_{\overline{AB}} = 2\sqrt{41}$$

Example 5: A circle has a diameter with endpoints (−4, −1) and (2, 6). Find the exact length of the radius.

Let A (x_1, y_1) = (−4, −1) and B (x_2, y_2) = (2, 6)

Length of Diameter $\overline{AB}$

$$d_{\overline{AB}} = \sqrt{(2 - -4)^2 + (6 - -1)^2}$$
$$= \sqrt{6^2 + 7^2} = \sqrt{36 + 49}$$
$$d_{\overline{AB}} = \sqrt{85}$$

Length of the Radius = $\dfrac{\text{diameter } \overline{AB}}{2}$

Length of the Radius = $\dfrac{\sqrt{85}}{2}$

2-1 Assignment: pg. 150–152 #9, 13, 17, 23, 25, 27, 31, 35, 43, 53, 55; pg. 102 #13, 19, 21 Honours: pg. 151 #48, 49

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2-2: Graphs of Equations in Two Variables (Part 1)

Graphs of an Equation: - is a set of order pairs (x, y) where the y-values of these coordinates are the results of substituting x-values into the equation.

Graphing Equations by Plotting Points:
- we can make a table of values to organize these x-inputs and y-outputs of an equation.
- they then form the order pairs (x, y) for plotting on a Cartesian plane.

***x*-intercept**: - when the graph crosses the x-axis. (**Let $y = 0$ for x-intercept** – since all y-values are 0 on x-axis)

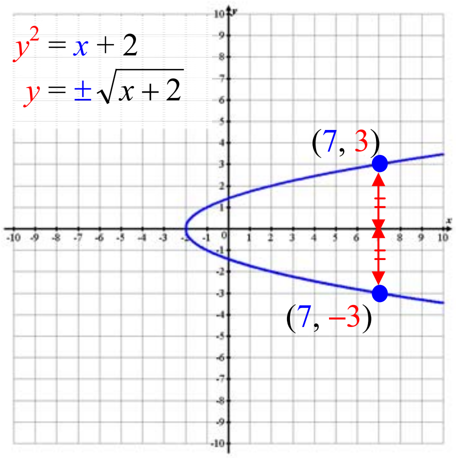
***y*-intercept**: - when the graph crosses the y-axis. (**Let $x = 0$ for y-intercept** – since all x-values are 0 on y-axis)

Symmetry: - there are three types of symmetries:

a. **Symmetry with respect to the *x*-axis**: - when the graph is **equidistant away from the *x*-axis**.
- when the **equation remains the same if *y* is replaced by (–*y*)**.

b. **Symmetry with respect to the *y*-axis**: - when the graph is **equidistant away from the *y*-axis**.
- when the **equation remains the same if *x* is replaced by (–*x*)**.

c. **Symmetry with respect to the *origin***: - when the graph is **equidistant away from the *origin***.
- when the **equation remains the same if *x* is replaced by (–*x*) and *y* is replaced by (–*y*)**.



Symmetry to *x*-axis

Original equation **remains the same** when ***y* is replaced by –*y*.**

(Same *x*-value gives ±*y* values)

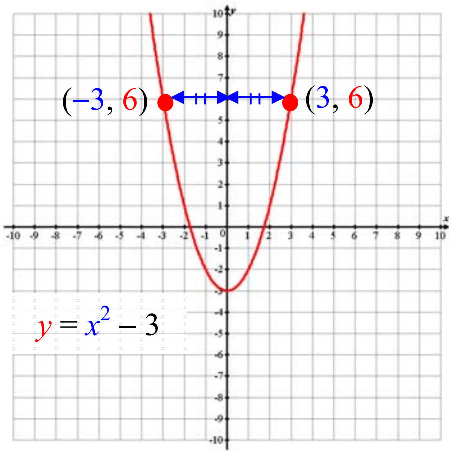
$y^2 = x + 2$

$(-y)^2 = x + 2$

$y^2 = x + 2$

(Replace y with $-y$ to test for symmetry to x-axis)

(Gives the exact equation as the original – it has symmetry to x-axis)



Symmetry to *y*-axis

Original equation **remains the same** when ***x* is replaced by –*x*.**

(Same *y*-value has ±*x* values)

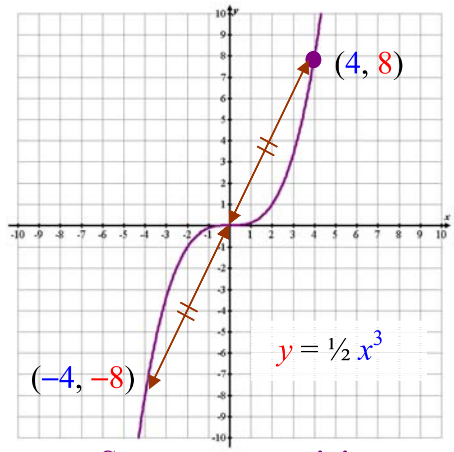
$y = x^2 - 3$

$y = (-x)^2 - 3$

$y = x^2 - 3$

(Replace x with $-x$ to test for symmetry to y-axis)

(Gives the exact equation as the original – it has symmetry to y-axis)



Symmetry to *origin*

Original equation **remains the same** when ***x* is replaced by –*x* and *y* is replaced by –*y*.**

(Same ±*x*-values with ±*y* values)

$y = ½\,x^3$

$(-y) = ½\,(-x)^3$

$-y = -½\,x^3$

$y = ½\,x^3$

(Replace x with $-x$ and y with $-y$ to test for symmetry to *origin*)

(Gives the exact equation as the original – it has symmetry to the *origin*)

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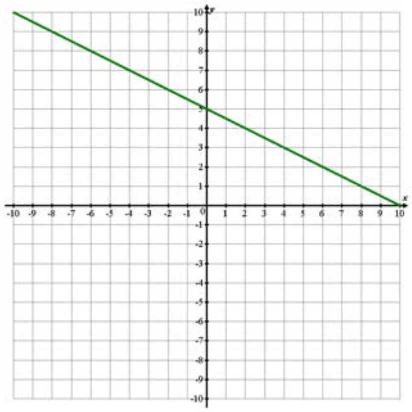
Example 1: Provide the table of values and a graph for the following equations. For each graph, state the x-intercept and y-intercept. Determine if there is any symmetry.

a. $y = -\frac{1}{2}x + 5$

x	y
−3	$-\frac{1}{2}(-3) + 5 = 6.5$
−2	$-\frac{1}{2}(-2) + 5 = 6$
−1	$-\frac{1}{2}(-1) + 5 = 5.5$
0	$-\frac{1}{2}(0) + 5 = 5$
1	$-\frac{1}{2}(1) + 5 = 4.5$
2	$-\frac{1}{2}(2) + 5 = 4$
3	$-\frac{1}{2}(3) + 5 = 3.5$



Let $y = 0$ for x-intercept

$(0) = -\frac{1}{2}x + 5$
$-5 = -\frac{1}{2}x$
$10 = x$ **$x = 10$ or at $(10, 0)$**

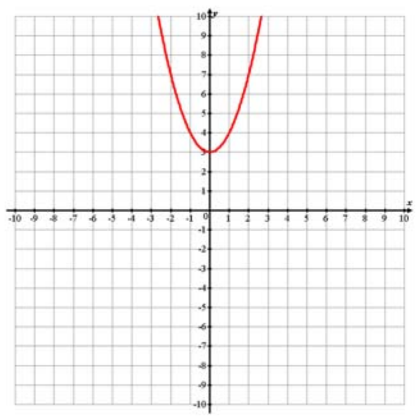
Let $x = 0$ for y-intercept

$y = -\frac{1}{2}(0) + 5$
$y = 5$ **$y = 5$ or at $(0, 5)$**

No Symmetry

b. $y = x^2 + 3$

x	y
−3	$(-3)^2 + 3 = 12$
−2	$(-2)^2 + 3 = 7$
−1	$(-1)^2 + 3 = 4$
0	$(0)^2 + 3 = 3$
1	$(1)^2 + 3 = 4$
2	$(2)^2 + 3 = 7$
3	$(3)^2 + 3 = 12$



Let $y = 0$ for x-intercept

$(0) = x^2 + 3$
$-3 = x^2$
$x = \pm\sqrt{-3} = \pm 3i$ **(Non-real)**

There is no real x-intercept

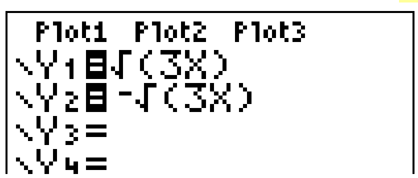
Let $x = 0$ for y-intercept

$y = (0)^2 + 3$
$y = 3$ **$y = 3$ or at $(0, 3)$**

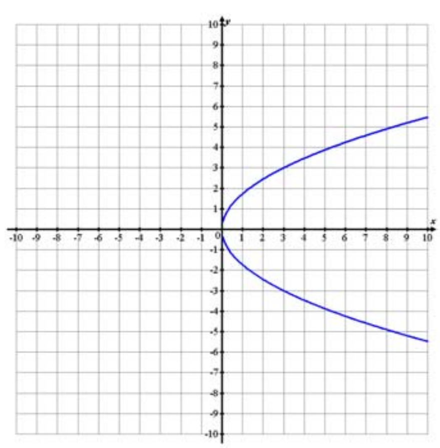
Symmetry to y-axis
$y = x^2 + 3$ is equivalent to $y = (-x)^2 + 3$

c. $y^2 = 3x$

$y = \pm\sqrt{3x}$



x	y
−3	$\pm\sqrt{3(-3)}$ = no soln
−2	$\pm\sqrt{3(-2)}$ = no soln
−1	$\pm\sqrt{3(-1)}$ = no soln
0	$\pm\sqrt{3(0)} = 0$
1	$\pm\sqrt{3(1)} = \pm 1.732$
2	$\pm\sqrt{3(2)} = \pm 2.449$
3	$\pm\sqrt{3(3)} = \pm 3$



Let $y = 0$ for x-intercept

$(0)^2 = 3x$
$0 = x$ **$x = 0$ or at $(0, 0)$**

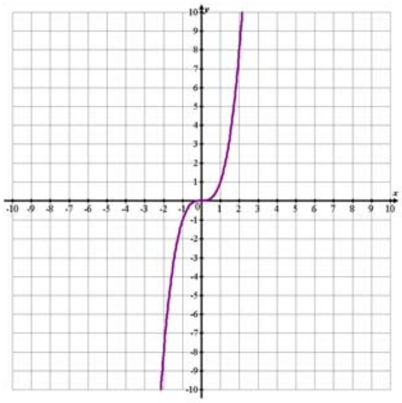
Let $x = 0$ for y-intercept

$y^2 = 3(0)$
$y = 0$ **$y = 0$ or at $(0, 0)$**

Symmetry to x-axis
$y^2 = 3x$ is equivalent to $(-y)^2 = 3x$

d. $y = x^3$

x	y
−3	$(-3)^3 = -27$
−2	$(-2)^3 = -8$
−1	$(-1)^3 = -1$
0	$(0)^3 = 0$
1	$(1)^3 = 1$
2	$(2)^3 = 8$
3	$(3)^3 = 27$



Let $y = 0$ for x-intercept

$(0) = x^3$

$0 = x$ **$x = 0$ or at $(0, 0)$**

Let $x = 0$ for y-intercept

$y = (0)^3$

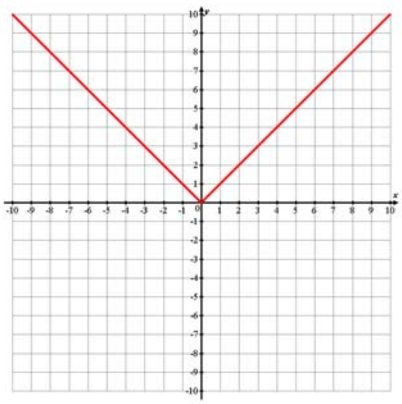
$y = 0$ **$y = 0$ or at $(0, 0)$**

Symmetry to the *origin*

$y = x^3$ is equivalent to $(-y) = (-x)^3$

e. $y = |x|$

x	y
−3	$\|(-3)\| = 3$
−2	$\|(-2)\| = 2$
−1	$\|(-1)\| = 1$
0	$\|(0)\| = 0$
1	$\|(1)\| = 1$
2	$\|(2)\| = 2$
3	$\|(3)\| = 3$



Let $y = 0$ for x-intercept

$(0) = |x|$

$0 = x$ **$x = 0$ or at $(0, 0)$**

Let $x = 0$ for y-intercept

$y = |(0)|$

$y = 0$ **$y = 0$ or at $(0, 0)$**

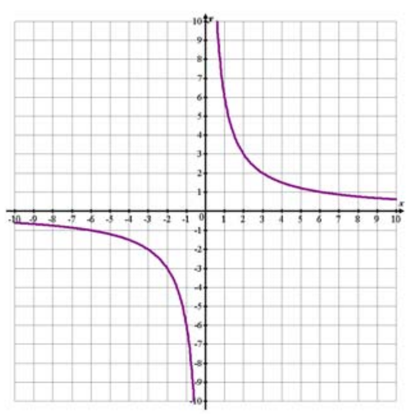
Symmetry to y-axis

$y = |x|$ is equivalent to $y = |(-x)|$

f. $xy = 6$

x	y
−3	$\frac{6}{(-3)} = -2$
−2	$\frac{6}{(-2)} = -3$
−1	$\frac{6}{(-1)} = -6$
0	$\frac{6}{(0)} =$ undefined
1	$\frac{6}{(1)} = 6$
2	$\frac{6}{(2)} = 3$
3	$\frac{6}{(3)} = 2$

$y = \frac{6}{x}$



Let $y = 0$ for x-intercept

$x(0) = 6$

$0 = 6$ *(FALSE)*

There is no x-intercept

Let $x = 0$ for y-intercept

$(0)y = 6$

$0 = 6$ *(FALSE)*

There is no y-intercept

Symmetry to the *origin*

$xy = 6$ is equivalent to $(-x)(-y) = 6$

2-2 Assignment (Part 1): pg. 162 # 3, 5, 7, 9, 13, 17, 19, 25, 27, 33, 39; Honours: #41

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2-2: Graphs of Equations in Two Variables (Part 2)

Reviewing Perfect Trinomial Squares

Perfect Trinomial Square

$$ax^2 + bx + c = (\sqrt{a}x + \sqrt{c})^2$$
$$ax^2 - bx + c = (\sqrt{a}x - \sqrt{c})^2$$

where a, c are square numbers, and $b = 2(\sqrt{a})(\sqrt{c})$

Example 1: Expand $(3x + 2)^2$.

$(3x + 2)^2 = (3x + 2)(3x + 2)$
$= 9x^2 + 6x + 6x + 4$
$= 9x^2 + 12x + 4$

$\sqrt{9} = 3$ | $2(\sqrt{9})(\sqrt{4}) = 12$ | $\sqrt{4} = 2$

Completing the Square: - the process to turn a quadratic equation into a perfect trinomial square.

Example 2: Find the value of c in order to make the following expressions perfect trinomial squares. Factor the resulting trinomials.

a. $x^2 - 8x + c$

$b = 2(\sqrt{a})(\sqrt{c})$
$-8 = 2(\sqrt{1})(\sqrt{c})$
$-4 = \sqrt{c}$ $\quad$ $c = 16$

$x^2 - 8x + 16$
$= (x - 4)(x - 4) = (x - 4)^2$

b. $x^2 + 20x + c$

$b = 2(\sqrt{a})(\sqrt{c})$
$20 = 2(\sqrt{1})(\sqrt{c})$
$10 = \sqrt{c}$ $\quad$ $c = 100$

$x^2 + 20x + 100$
$= (x + 10)(x + 10) = (x + 10)^2$

c. $x^2 - \frac{3x}{5} + c$

$b = 2(\sqrt{a})(\sqrt{c})$
$-\frac{3}{5} = 2(\sqrt{1})(\sqrt{c})$
$-\frac{3}{10} = \sqrt{c}$ $\quad$ $c = \frac{9}{100}$

$x^2 - \frac{3}{5}x + 100$
$= \left(x - \frac{3}{10}\right)\left(x - \frac{3}{10}\right) = \left(x - \frac{3}{10}\right)^2$

Note: To find c from b if $a = 1$, we can divide b by 2, then square.

To Complete the Squares of a Quadratic Equation

1. **Factor out the leading coefficient if a is NOT 1 out of the ax^2 and bx terms.**
2. **Group the first two terms in a bracket.**
3. ***Complete the Square in the bracket by adding another constant. Be sure to subtract this constant at the end of the equation. Also be aware that sometimes we have to subtract the PRODUCT of this constant and a (when a is NOT 1).***
4. **Factor the perfect trinomial and *Combine Like Terms* at the end of the equation.**

Example 3: Solve the following equations by completing the squares.

a. $0 = x^2 - 6x + 4$

$-4 = x^2 - 6x$	(Constant term to one side)
$-4 + 9 = x^2 - 6x + 9$	(Complete the Square)
$5 = (x + 3)^2$	(Factor Perfect Trinomial)
$\pm\sqrt{5} = (x + 3)$	(Take the Square Root)
$-3 \pm \sqrt{5} = x$	$x = -3 \pm \sqrt{5}$

b. $0 = -x^2 + 4x - 7$

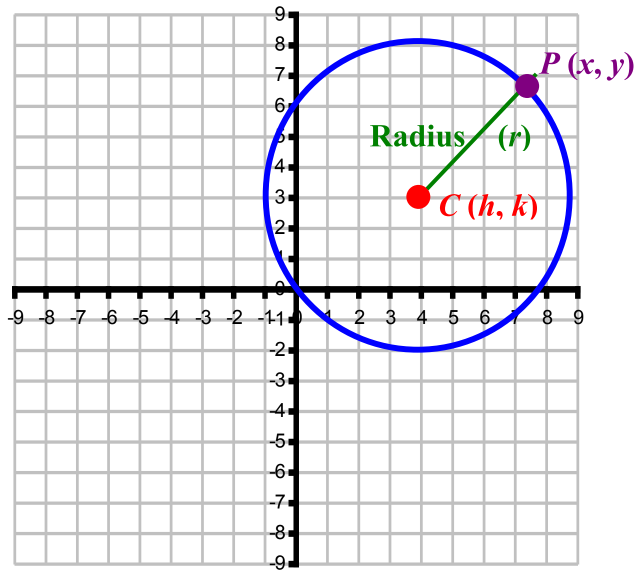
$x^2 - 4x = -7$	(x^2 term has positive coefficient)
$x^2 - 4x + 4 = -7 + 4$	(Complete the Square)
$(x - 2)^2 = -3$	(Factor Perfect Trinomial)
$(x - 2) = \pm\sqrt{-3}$	(Take the Square Root)
$x = 2 \pm i\sqrt{3}$	**(Complex Solutions)**

c. $0 = 3x^2 - 24x + 12$

$-12 = 3x^2 - 24x$	(Constant term to one side)
$-12 = 3(x^2 - 12x)$	(Factor out a)
$-4 = (x^2 - 12x)$	(Divide a on both sides)
$-4 + 36 = x^2 - 12x + 36$	**(Complete the square)**
$32 = (x - 6)^2$	(Factor Perfect Trinomial)
$\pm\sqrt{32} = (x - 6)$	$\boldsymbol{x = 6 \pm 4\sqrt{2}}$

Using the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ for the **general point P (x, y)** and the **centre C (h, k)**, the **radius**, r, would be

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$



Standard Equation for Circles

$$(x - h)^2 + (y - k)^2 = r^2$$

P (x, y) = any point on the path of the circle
C (h, k) = centre of the circle
r = length of the radius

Example 4: Write the equation of each circle below.

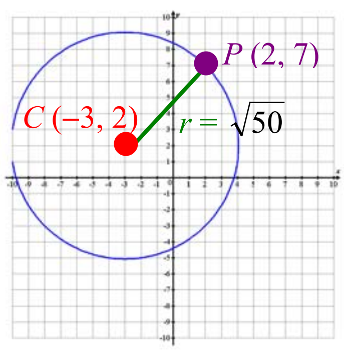
a. centre at origin, radius 7

C (0, 0) $h = 0$; $k = 0$ $r = 7$
$(x - 0)^2 + (y - 0)^2 = 7^2$
$\boldsymbol{x^2 + y^2 = 49}$

b. centre (–2, –5), radius = $3\sqrt{6}$

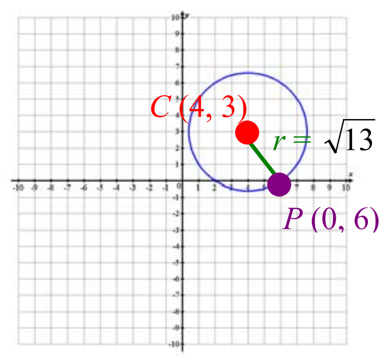
C (–2, –5) $h = -2$; $k = -5$ $r = 3\sqrt{6}$
$(x - -2)^2 + (y - -5)^2 = (3\sqrt{6})^2$
$\boldsymbol{(x + 2)^2 + (y + 5)^2 = 54}$

c. centre at (–3, 2), passing through (2, 7)



r = *distance* between (–3, 2) and (2, 7)
$r = \sqrt{(2 - -3)^2 + (7 - 2)^2}$ $r = \sqrt{50}$
C (–3, 2) $h = -3$; $k = 2$
$(x - -3)^2 + (y - 2)^2 = (\sqrt{50})^2$
$\boldsymbol{(x + 3)^2 + (y - 2)^2 = 50}$

d. centre (4, 3), x-intercept at 6



r = *distance* between (4, 3) and (6, 0)
$r = \sqrt{(6 - 4)^2 + (0 - 3)^2}$ $r = \sqrt{13}$
C (4, 3) $h = 4$; $k = 3$
$(x - 4)^2 + (y - 3)^2 = (\sqrt{13})^2$
$\boldsymbol{(x - 4)^2 + (y - 3)^2 = 13}$

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Example 5: Locate the coordinates for the centre and determine the length of the radius given the equations.

a. $(x + 5)^2 + y^2 = 121$

$(x - -5)^2 + (y - 0)^2 = 121$

$h = -5 \quad k = 0 \quad r^2 = 121$

$C(-5, 0) \quad r = 11$

b. $3(x - 4)^2 + 3(y + 4)^2 = 30.72$

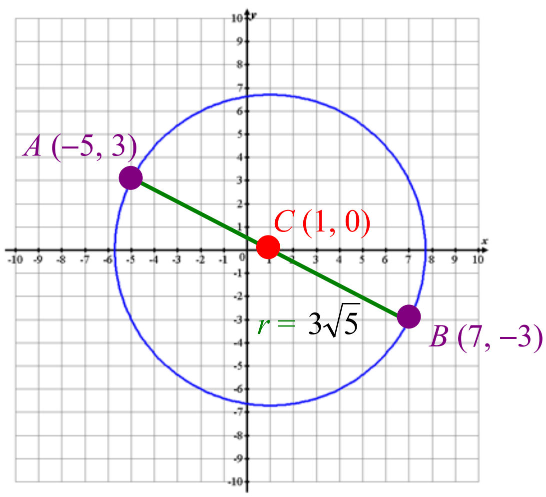
$(x - 4)^2 + (y + 4)^2 = 10.24$ (Divide both sides by 3)

$(x - 5)^2 + (y - -4)^2 = 10.24$

$h = 5 \quad k = -4 \quad r^2 = 121$

$C(5, -4) \quad r = 3.2$

Example 6: Write the equation of this circle that has a diameter with endpoints at $A(-5, 3)$ and $B(7, -3)$.



diameter = *distance* between (–5, 3) and (7, –3)

$d = \sqrt{(7 - -5)^2 + (-3 - 3)^2} \qquad d = \sqrt{180} = 6\sqrt{5}$

$r = \frac{d}{2} = \frac{6\sqrt{5}}{2} \qquad r = 3\sqrt{5}$

Centre = *Midpoint* = $\left(\frac{-5+7}{2}, \frac{3+-3}{2}\right) \qquad C(1, 0)$

$(x - 1)^2 + (y - 0)^2 = (3\sqrt{5})^2 \qquad h = 1\ ;\ k = 0$

$(x - 1)^2 + y^2 = 45$

Example 7: Show that $x^2 + y^2 + 4x - 10y - 52 = 0$ is a circle algebraically. Locate the centre, state the radius and 4 points on the path of this circle .

$x^2 + y^2 + 4x - 10y = \mathbf{52}$ (Constant to one side)

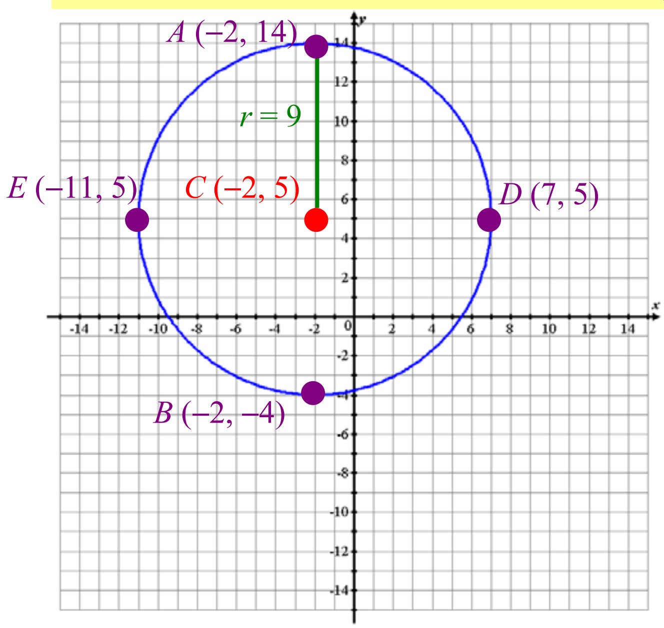
$(x^2 + 4x \quad) + (y^2 - 10y \quad) = 52$ (Group x and y terms in their own bracket)

$(x^2 + 4x + \mathbf{4}) + (y^2 - 10y + \mathbf{25}) = 52 + \mathbf{4} + \mathbf{25}$ **(Complete the squares for both x and y)**

$(x + \mathbf{2})^2 + (y - \mathbf{5})^2 = \mathbf{81}$ **(Factor each bracket)**

$(x - -2)^2 + (y - 5)^2 = 81$

$h = -2 \quad k = 5 \quad r^2 = 81$ **$C(-2, 5) \quad r = 9$**



2-2 Assignment (Part 2)
pg. 162– 164 #45, 49, 51, 52, 53, 57, 59, 61, 63, 67, 73, 83;
Honours: #77, 79, 87

2-3: Graphing Calculators; Solving Equations and Inequalities Graphically

Solutions of equation as well as intersecting points can be found graphically using a graphing calculator. One must remember to following when solving equations graphically.

1. **To Solve an Equation Graphically**, either:
   - ***Bring everything to one side of the equation and input it into Y_1*** of the **Y=** Screen. **Find the Zeros (x-intercepts) of the graph** using the **ZERO function** under **2nd** **TRACE** or **CALC** Screen.

     or

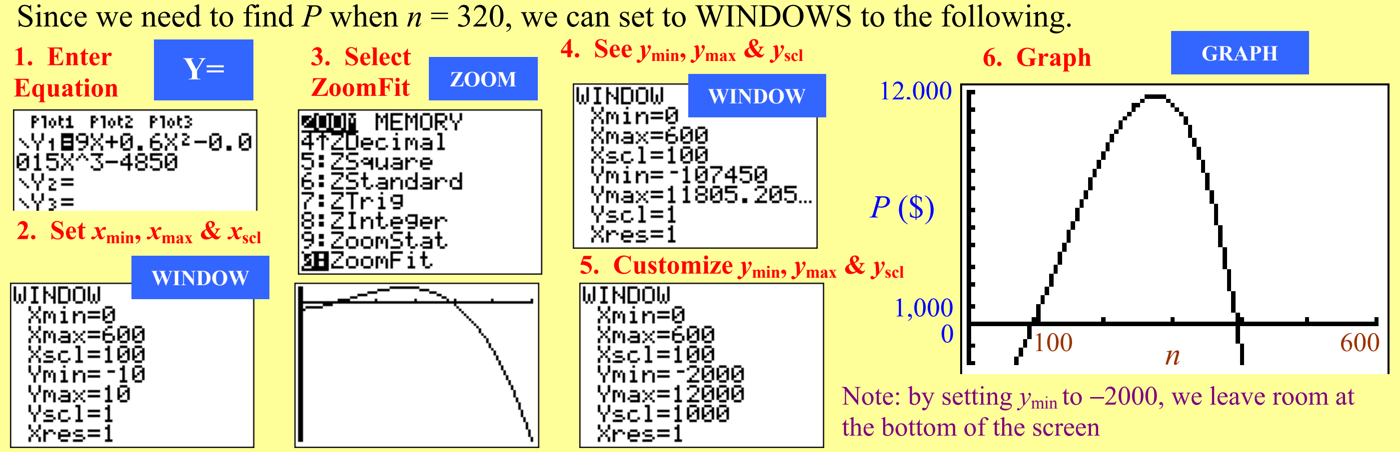
   - ***Graph one side of the equation as Y_1 and the other side of the equation as Y_2***. **Find the intersecting points of the two graphs** using the **INTERSECT function** under **2nd** **TRACE** or **CALC** Screen.

2. **Determine the scale of the axis (*WINDOW Size*).**
   - **Read the question and note the *x*-values (input).** This means that ***x_{max} value must be larger than the required input***.
   - Use the **ZOOM Fit** function to get an idea for the corresponding *y*-value. Go to the **WINDOW** Screen and better **Customize the y_{min}, y_{max}, and y_{scale}** to better fit the graph.
   - ***Sometimes, the context of the question can be interpreted that $x_{min} = 0$***. For examples, if *x* represents time elapsed, amount of product produced, or length of an object, we will have to assume $x \geq 0$. **Use Common Sense when determining WINDOW Size**.

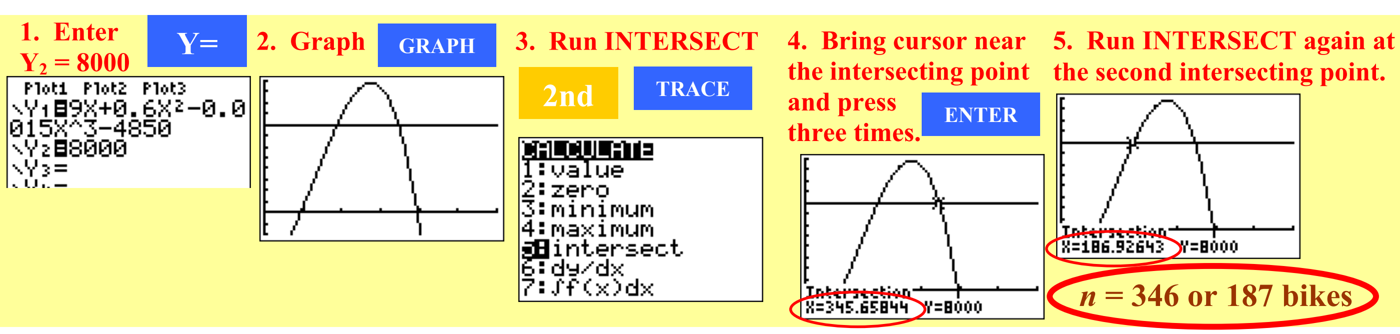
Example 1: A bicycle manufacturer estimates the profit (in dollars) generated by producing *n* units per month is given by the equation, $P = 9n + 0.6n^2 - 0.0015n^3 - 4850$.

a. Using the graph, find the estimated profit when there were 320 units produced. (State the WINDOW Size and explain the reason such choice.) Sketch the graph.

Since we need to find *P* when *n* = 320, we can set to WINDOWS to the following.



b. How many bicycles should be produced in a month so the profit will exceeds $8000?



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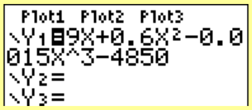
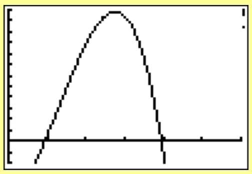
c. How many bicycles does the company need to make per month to break even?

To break even, profit = $0. Hence, we are looking for *x*-intercept.

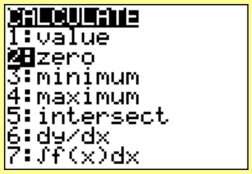
1. Erase Y_2 from before Y=



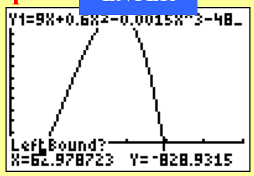
2. Graph GRAPH



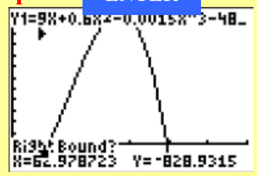
3. Run ZERO 2nd TRACE



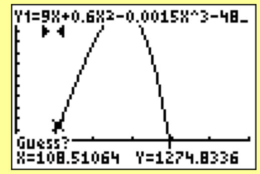
4. Bring cursor left of the first *x*-int and press ENTER



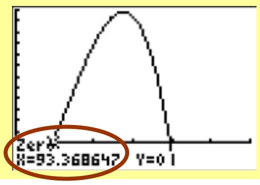
5. Bring cursor right of the first *x*-int and press ENTER



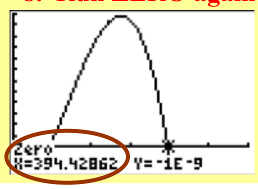
6. Press ENTER **again**



7. ZERO is shown



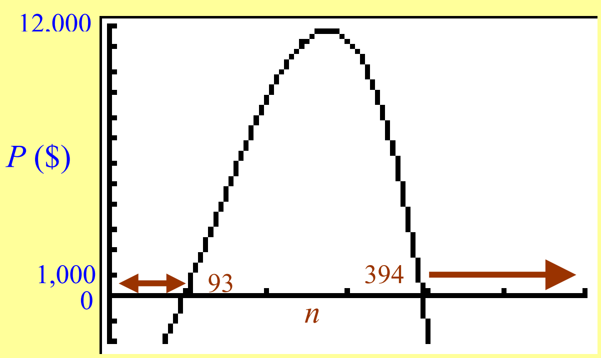
8. Run ZERO again for the second *x*-int



It will break even when

***n* = 94 or 395 bikes**

d. What is/are the range(s) of the monthly production will the manufacturer lose money? Why?



To lose money, *P* < 0. This means the ranges of *n* that is below the *x*-axis.

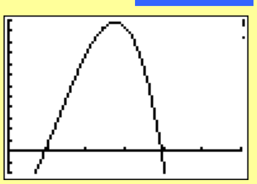
$0 \le n \le 93$ or $n \ge 394$

At $0 \le n \le 93$, there is not enough bikes made to cover the fixed cost of the operation.

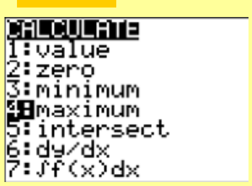
At $n \ge 394$, there are too many bikes made. Since supply increases, demand decrease ad the price would fall such that profit is not possible.

e. How many bicycle should the manufacturer produce to achieve maximum profit?

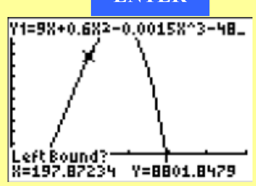
1. Graph GRAPH



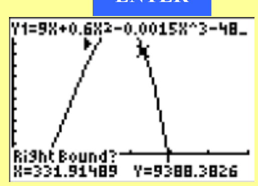
2. Run MAXIMUM 2nd TRACE



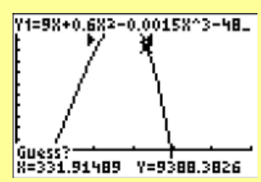
3. Bring cursor left of the maximum and press ENTER



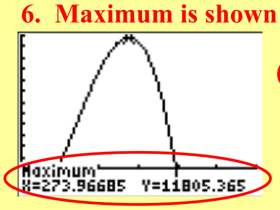
4. Bring cursor right of the maximum and press ENTER



5. Press ENTER **again**



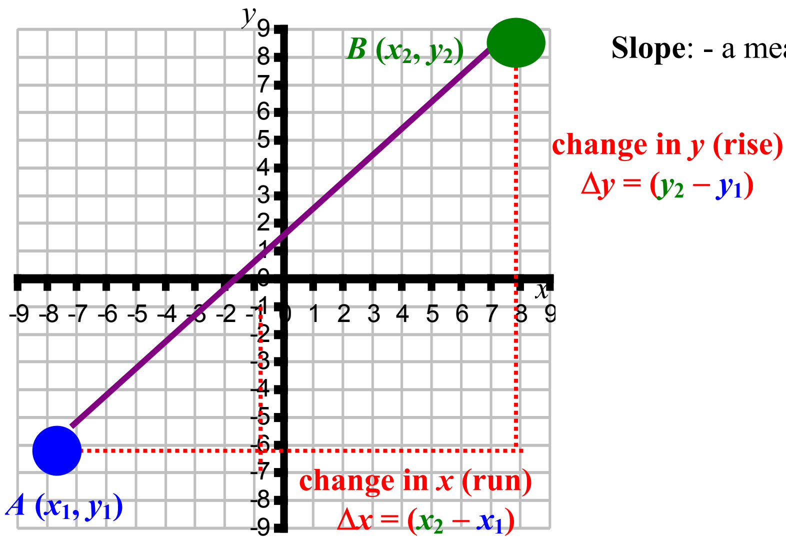
6. Maximum is shown



The maximum would be $11,805.37 when 274 bikes are built.

2-3 Assignment: 2-3 Calculator Worksheet

2-4: Lines



Slope: - a measure on the steepness of the line segment.

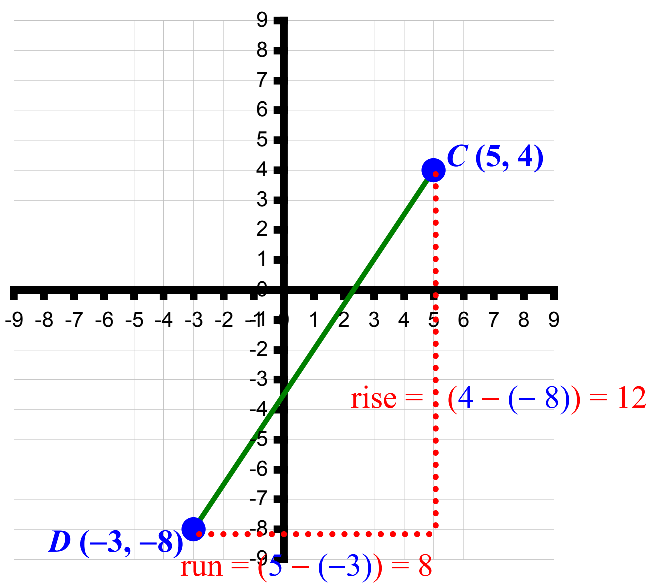
Slope of a Line Segment

$$m = \frac{rise}{run} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

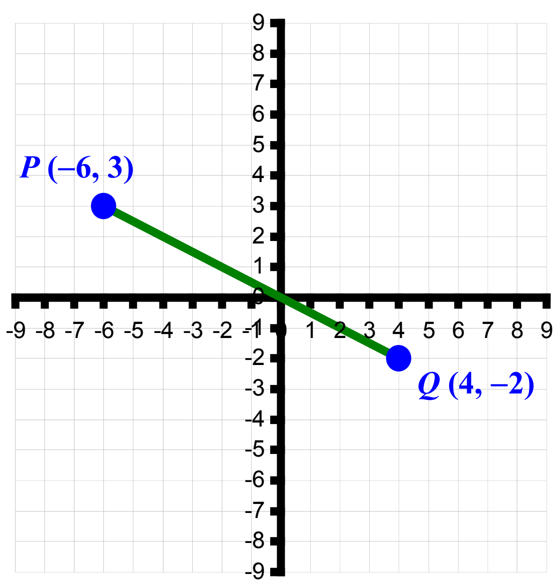
Example 1: Find the slope of the following line segments.

a. $\overline{CD}$ where C (5, 4) and D (−3, −8)   b. $\overline{PQ}$ where P (−6, 3) and Q (4, −2)



$$m_{\overline{CD}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-8)}{5-(-3)} = \frac{12}{8}$$

$$\boldsymbol{m_{\overline{CD}} = \frac{4}{3}}$$



$$m_{\overline{PQ}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{4-(-6)} = \frac{-5}{10}$$

$$\boldsymbol{m_{\overline{PQ}} = \frac{-1}{2}}$$

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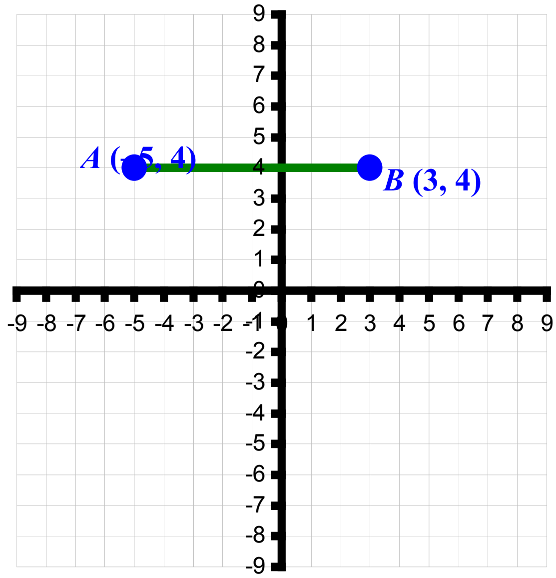
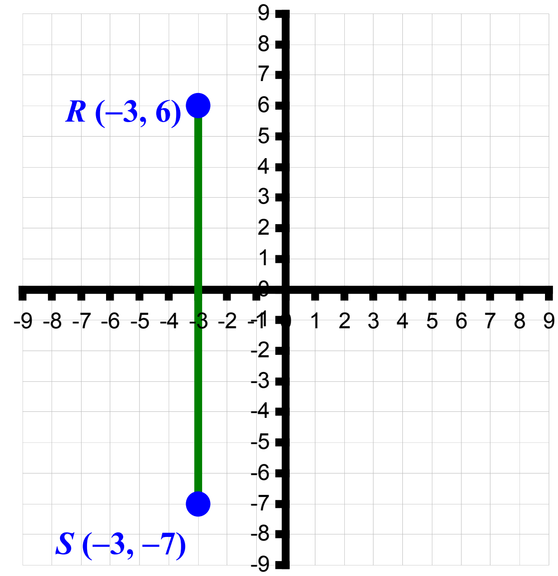
c. $\overline{AB}$ where $A\,(-5, 4)$ and $B\,(3, 4)$



$$m_{\overline{AB}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-4}{3-(-5)} = \frac{0}{8}$$

$$\boldsymbol{m_{\overline{AB}} = 0}$$

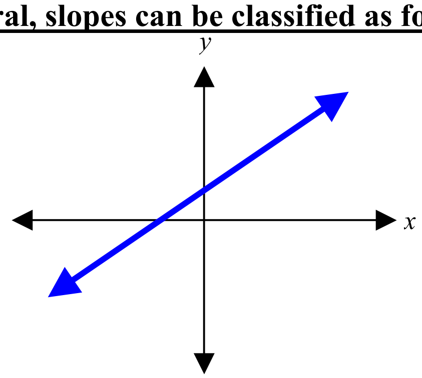
d. $\overline{RS}$ where $R\,(-3, 6)$ and $S\,(-3, -7)$



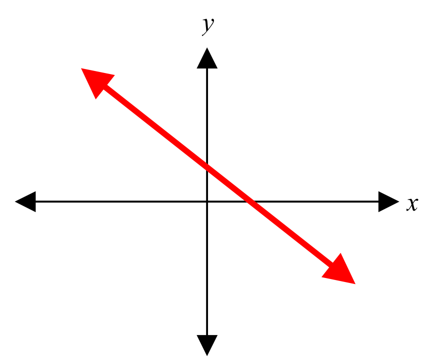
$$m_{\overline{RS}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-6}{-3-(-3)} = \frac{-13}{0}$$

$$\boldsymbol{m_{\overline{RS}} = \text{undefined}}$$

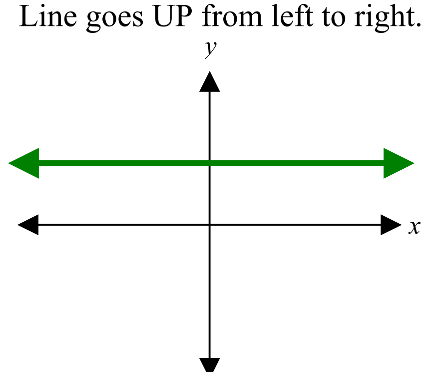
In general, slopes can be classified as follows:



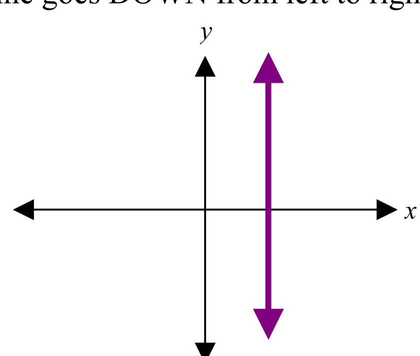
Positive Slope ($m > 0$)
Line goes UP from left to right.



Negative Slope ($m < 0$)
Line goes DOWN from left to right.



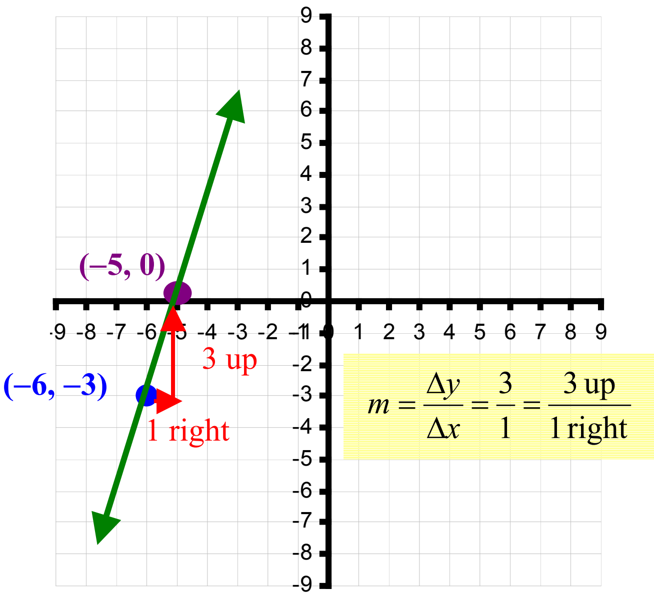
Zero Slope ($m = 0$)
Horizontal (Flat) Line [Rise = 0]



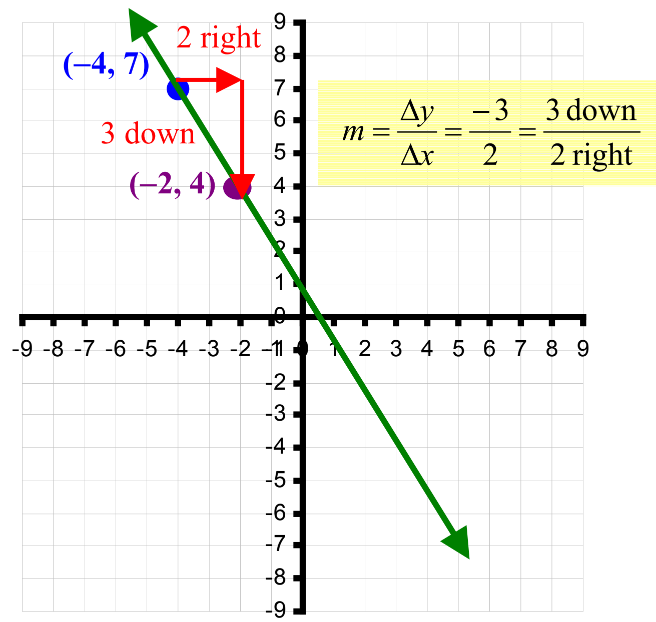
Undefined Slope
Vertical Line [Run = 0]

Example 2: Sketch the graph of a line given a point and a slope below.

a. $E(-6, -3)$ and $m = 3$



b. $F(-4, 7)$ and $m = -\frac{3}{2}$



Point-Slope form: - a form of a linear equation when given a slope (m) and a point (x_1, y_1) on the line

$$\frac{y - y_1}{x - x_1} = m \quad \text{(slope formula)} \qquad \boldsymbol{y - y_1 = m(x - x_1)} \quad \textbf{(Point-Slope form)}$$

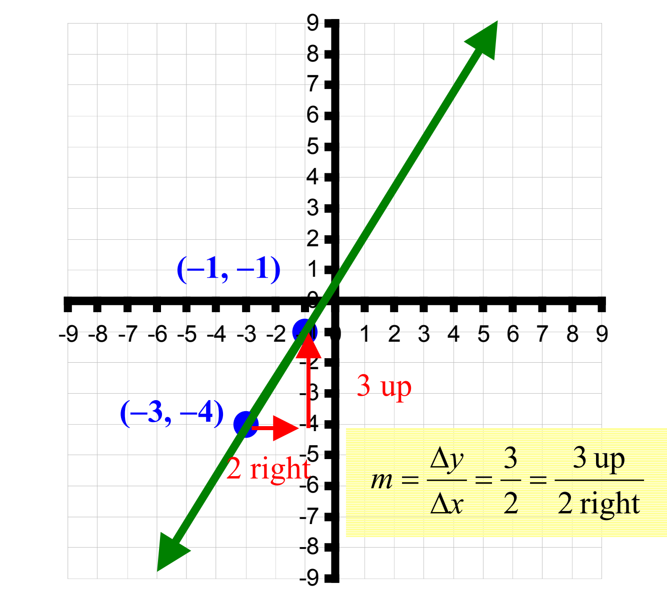
If we rearrange the equations so that all terms are on one side, it will be in **standard (general) form**:

$$\boldsymbol{Ax + By + C = 0} \qquad \textbf{(Standard or General form)}$$

($A \geq 0$, the leading coefficient for the x term must be positive)

Example 3: Find the equation in point-slope form and standard form given the following.

a. $(-3, -4)$ and $m = \frac{3}{2}$



For Slope-Point form:

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = \frac{3}{2}(x - (-3))$$

$$\boldsymbol{y + 4 = \frac{3}{2}(x + 3)}$$

For Standard form:

$$y + 4 = \frac{3}{2}(x + 3)$$

$$2(y + 4) = 3(x + 3)$$

$$2y + 8 = 3x + 9$$

$$0 = 3x - 2y + 9 - 8$$

$$\boldsymbol{0 = 3x - 2y + 1}$$

Bringing all the terms to the right-hand side of equation will ensure a positive coefficient for the x term.

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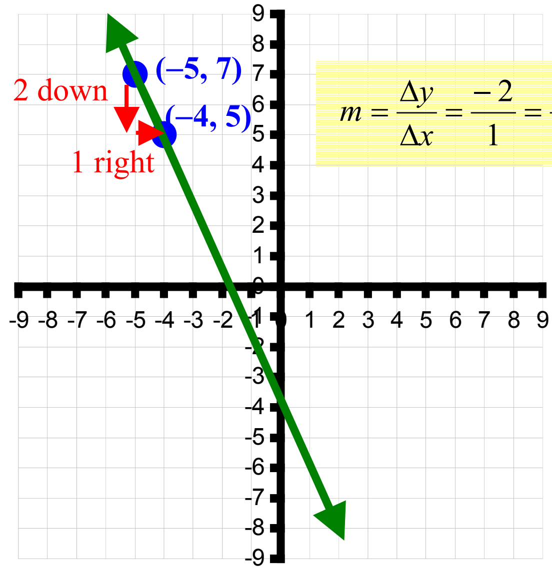
b. (–5, 7) and $m = -2$



$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = \frac{2\text{ down}}{1\text{ right}}$$

For Slope-Point form:

$$y - y_1 = m(x - x_1)$$
$$y - 7 = -2(x - (-5))$$
$$\boldsymbol{y - 7 = -2(x + 5)}$$

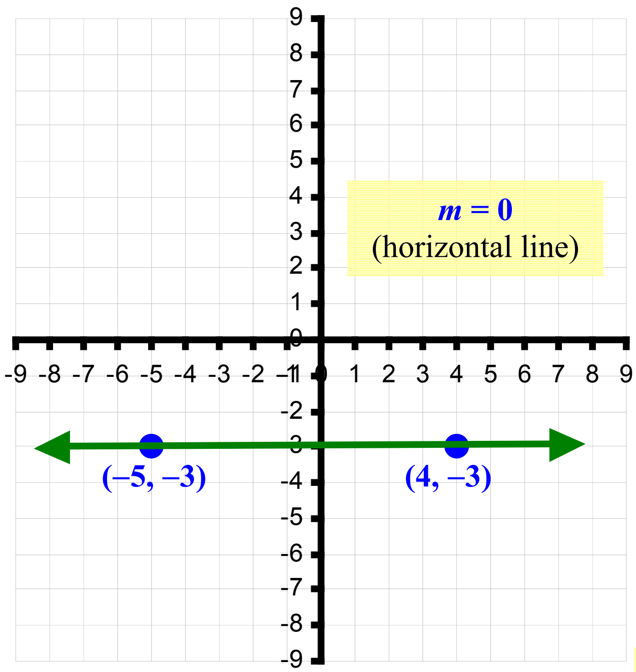
For Standard form:

$$y - 7 = -2(x + 5)$$
$$y - 7 = -2x - 10$$
$$2x + y - 7 + 10 = 0$$
$$\boldsymbol{2x + y + 3 = 0}$$

Bringing all the terms to the left-hand side of equation will ensure a positive coefficient for the x term.

Example 4: Find the equation in point-slope form and standard form given the following points.

a. (–5, –3) and (4, –3)

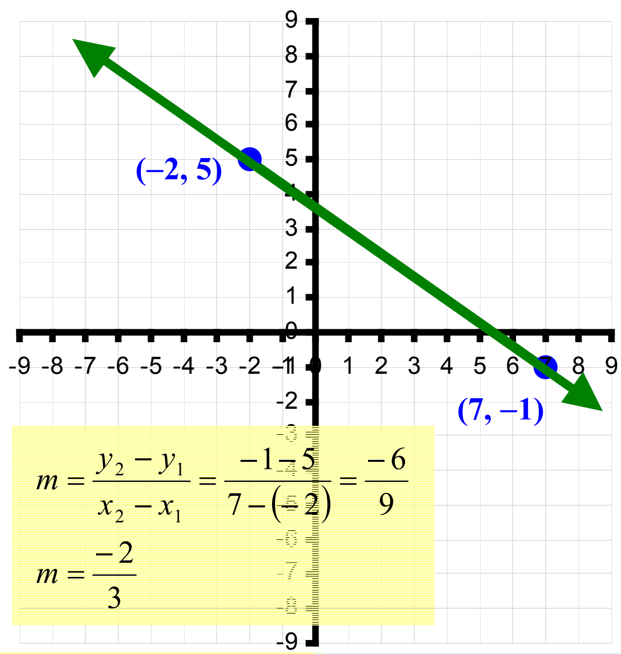


For Slope-Point form:

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = 0(x - (-5))$$
$$\boldsymbol{y + 3 = 0}$$

It is in standard form, also.
(Everything is already on one side.)

b. (–2, 5) and (7, –1)



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{7 - (-2)} = \frac{-6}{9}$$
$$m = \frac{-2}{3}$$

For Slope-Point form:

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{-2}{3}(x - (-2))$$
$$\boldsymbol{y - 5 = -\frac{2}{3}(x + 2)}$$

For Standard form:

$$y - 5 = \frac{-2}{3}(x + 2)$$
$$3(y - 5) = -2(x + 2)$$
$$3y - 15 = -2x - 4$$
$$2x + 3y - 15 + 4 = 0$$
$$\boldsymbol{2x + 3y - 11 = 0}$$

When given a slope (m) and the y-intercept $(0, b)$ of the line, we can find the equation of the line using the **slope and y-intercept form**:

$$y = mx + b \qquad \text{where } m = \text{slope and } b = y\text{-intercept}$$

Example 5: Given the y-intercept and slope, write the equation of the line in slope and y-intercept form, and standard form. Sketch a graph of the resulting equation.

a. $(0, -4)$ and $m = \frac{3}{2}$

y-intercept $= b = -4$

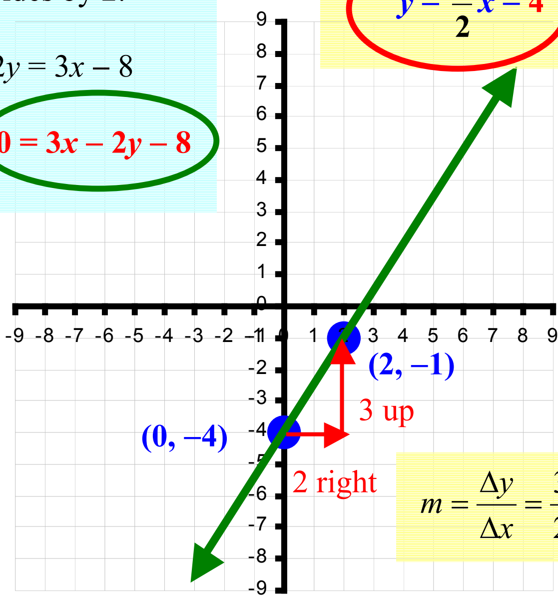
$y = \frac{3}{2}x - 4$

Multiply both sides by 2.

$2y = 3x - 8$

$0 = 3x - 2y - 8$

$m = \frac{\Delta y}{\Delta x} = \frac{3}{2} = \frac{3 \text{ up}}{2 \text{ right}}$



b. $b = 7$ and $m = -2$

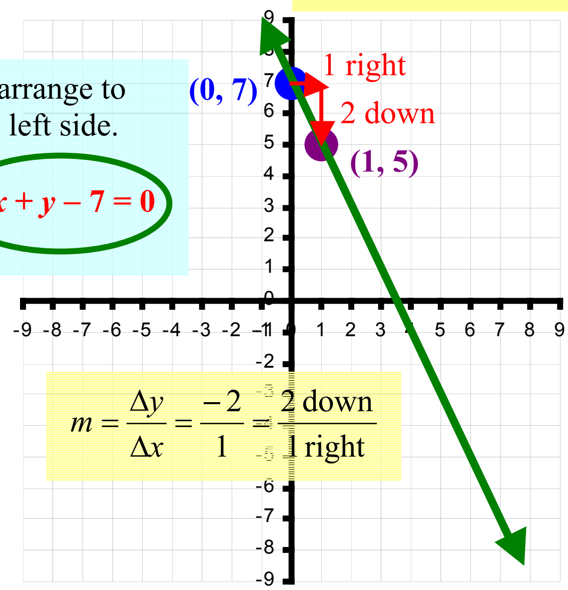
y-intercept $= b = 7$

$y = -2x + 7$

Rearrange to the left side.

$2x + y - 7 = 0$

$m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = \frac{2 \text{ down}}{1 \text{ right}}$



c. $b = 3$ and $m = 0$

y-intercept $= b = 3$

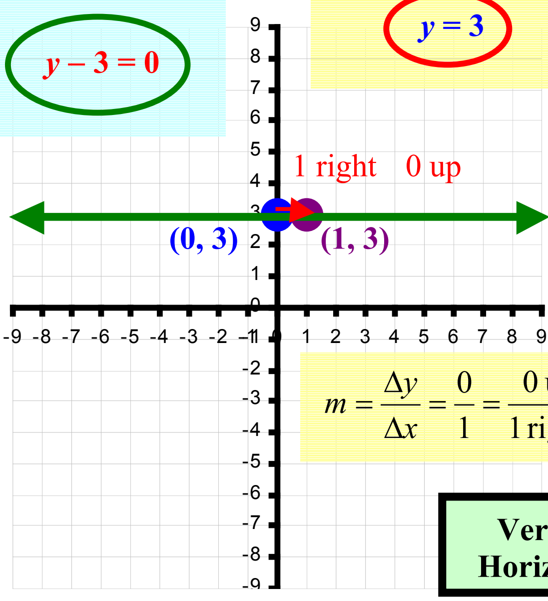
$y = 0x + 3$

$y = 3$

Rearrange to the left side.

$y - 3 = 0$

$m = \frac{\Delta y}{\Delta x} = \frac{0}{1} = \frac{0 \text{ up}}{1 \text{ right}}$



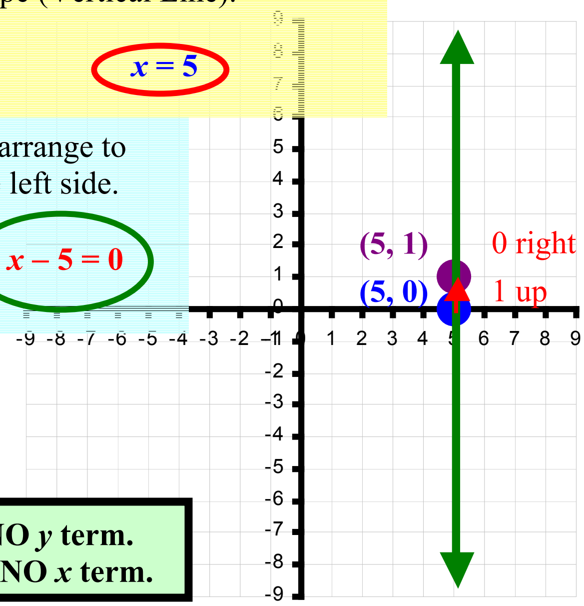
d. x-intercept $= 5$ and $m =$ undefined

x-intercept $= 5$ and undefined slope (Vertical Line).

$x = 5$

Rearrange to the left side.

$x - 5 = 0$



Vertical Line – NO y term.
Horizontal Line – NO x term.

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Family of Lines: - when lines are parallel (having the same slope) **or** have the same *y*-intercept.

Example 6: Given the following equations, graph them on the same grid and determine which lines belong to a family.

Line 1: $y = 2x - 5$ Line 2: $6x - 3y = -12$ Line 3: $0 = 2x - 4y + 8$ Line 4: $4x - 2y + 16 = 0$

Line 1: $y = 2x - 5$

$\boldsymbol{m = 2}$ $b = -5$

Line 2:

$6x - 3y = -12$

$-3y = -6x - 12$

$y = \dfrac{(-6x - 12)}{-3}$

$y = 2x + 4$

$\boldsymbol{m = 2}$ $b = 4$

Line 3:

$0 = 2x - 4y + 8$

$4y = 2x + 8$

$y = \dfrac{(2x + 8)}{4}$

$y = \dfrac{1}{2}x + 2$

$\boldsymbol{m = ½}$ $b = 2$

Line 4:

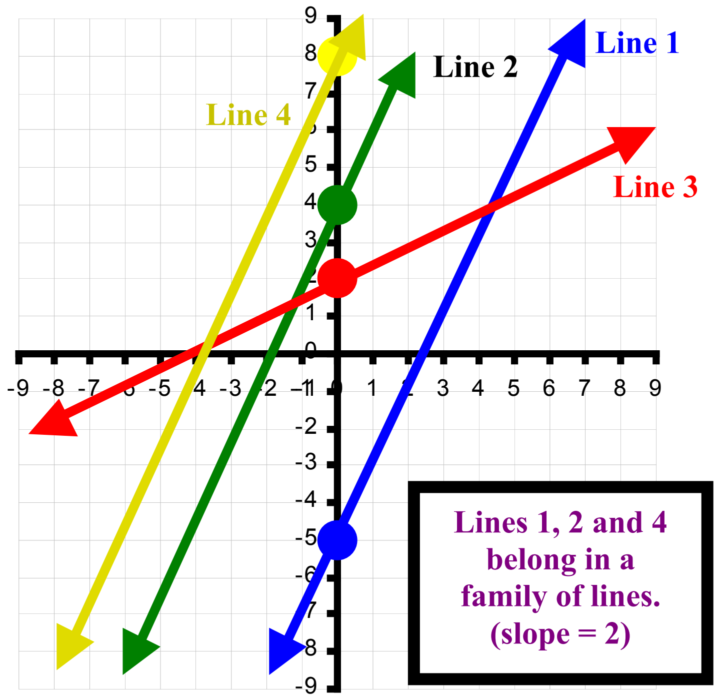
$4x - 2y + 16 = 0$

$-2y = -4x - 16$

$y = \dfrac{(-4x - 16)}{-2}$

$y = 2x + 8$

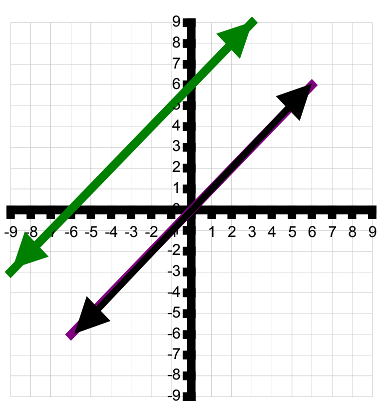
$\boldsymbol{m = 2}$ $b = 2$



Parallel Lines

slope of line 1 = slope of line 2

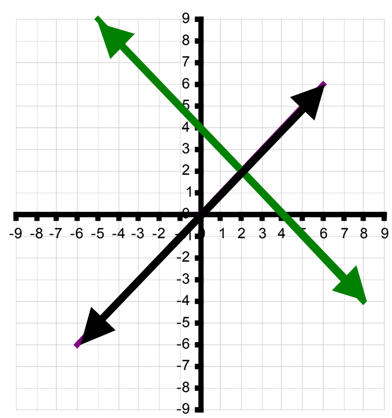
$$\boldsymbol{m_{l_1} = m_{l_2}}$$



Perpendicular Lines

slope of line 1 = negative reciprocal slope of line 2

$$\boldsymbol{m_{l_1} = \frac{-1}{m_{l_2}}}$$



Example 7: Given the slope of two lines below, determine whether the lines are parallel or perpendicular.

a. $m_1 = \dfrac{-3}{4}$ and $m_2 = \dfrac{8}{6}$

$m_1 = \dfrac{-3}{4}$ and $m_2 = \dfrac{4}{3}$

(negative reciprocal slopes)

Perpendicular Lines

b. $m_1 = \dfrac{4}{6}$ and $m_2 = \dfrac{6}{9}$

$m_1 = \dfrac{2}{3}$ and $m_2 = \dfrac{2}{3}$

(same slopes)

Parallel Lines

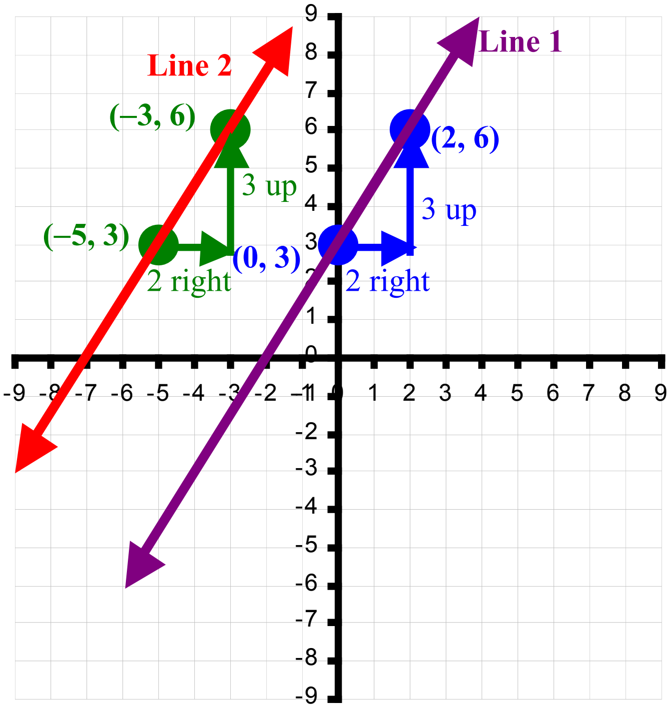
c. $m_1 = \dfrac{1}{2}$ and $m_2 = -\dfrac{1}{2}$

$m_1 = \dfrac{1}{2}$ and $m_2 = \dfrac{-1}{2}$

(neither the same nor negative reciprocal)

Neither Parallel nor Perpendicular Lines

Example 8: Find the equation of a line parallel to $3x - 2y + 6 = 0$ and passes through $(-5, 3)$.



Line 1:

$$-2y = -3x - 6 \qquad y = \frac{-3x-6}{-2} \qquad y = \frac{3}{2}x + 3 \qquad m_1 = \frac{3}{2}$$

Line 2:

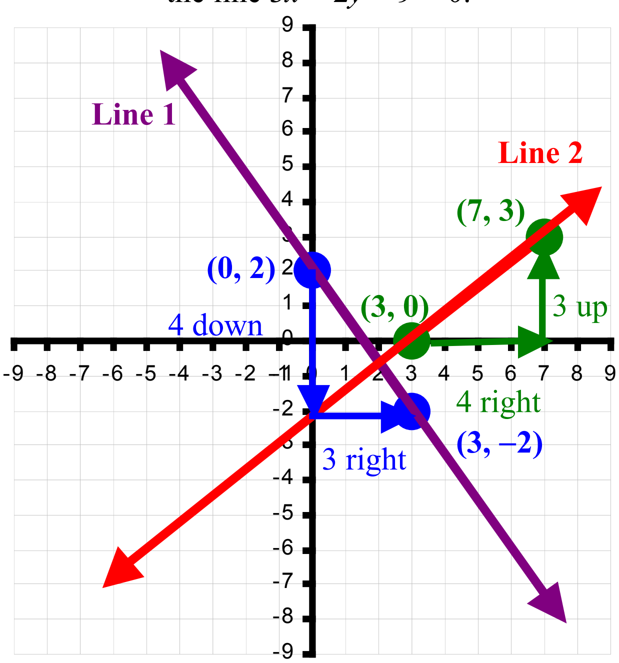
$m_2 = \frac{3}{2}$ (parallel lines – same slope as m_1)

Using $(-5, 3)$ as (x, y) and the form $y = mx + b$, we have:

$$(3) = \frac{3}{2}(-5) + b$$
$$3 = \frac{-15}{2} + b$$
$$3 + \frac{15}{2} = b$$
$$\frac{6}{2} + \frac{15}{2} = b$$
$$b = \frac{21}{2}$$

$$\boldsymbol{y = \frac{3}{2}x + \frac{21}{2}}$$

Example 9: Find the equation of a line perpendicular to $4x + 3y - 6 = 0$ and having the same x- intercept as the line $3x - 2y - 9 = 0$.



Line 1:

$$3y = -4x + 6 \qquad y = \frac{-4x+6}{3} \qquad y = \frac{-4}{3}x + 2 \qquad m_1 = \frac{-4}{3}$$

Line 2:

$m_2 = \frac{3}{4}$ (perpendicular lines – negative reciprocal of m_1)

To find x-intercept of $3x - 2y - 9 = 0$, we let $y = 0$.
$3x - 2(0) - 9 = 0$ $\quad 3x = 9$ $\quad x$-int = 3 means **(3, 0)**

Using $(3, 0)$ as (x, y) and the form $y = mx + b$, we have:

$$(0) = \frac{3}{4}(3) + b$$
$$0 = \frac{9}{4} + b$$
$$b = \frac{-9}{4}$$

$$\boldsymbol{y = \frac{3}{4}x - \frac{9}{4}}$$

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Example 10: In the world of economics, the price of an item sold is mainly depended on the supply and demand of the market place. Suppose the supply equation of a particular Star Trek model is $P = 0.001n + 40$ and the demand equation of the same model is $11n + 5000P - 1{,}000{,}000 = 0$, where P = price (in \$) and n is the quantity manufactured or sold.

a. Determine the WINDOW settings necessary to draw the graph of the supply and demand equations. Sketch the resulting graph of P versus. n.
b. What do the slope and y-intercept of the supply line represent?
c. What do the x- and y-intercepts of the demand line represent?
d. What does the intersecting point of the two linear equations represent?

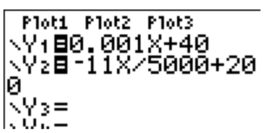
Supply Equation

$P = 0.001n + 40$ y-int or P-int = \$40

slope = 0.001

$$slope = \frac{1}{1000} \text{ or } \frac{\$10\text{ increase}}{10{,}000\text{ units increase}}$$

B (0 + 10000, \$40 + \$10)
= B (10000, \$50)



Demand Equation

$11n + 5000P - 1{,}000{,}000 = 0$ or $P = -\frac{11}{5000}n + 200$

For n-int (or x-int), let $P = 0$

$0 = 11n + 5000(0) - 1{,}000{,}000$

$-11n = -1{,}000{,}000$

$n = \frac{-1{,}000{,}000}{-11}$

***n*-int = 90909 units**

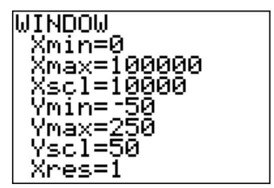
For P-int (or y-int), let $n = 0$

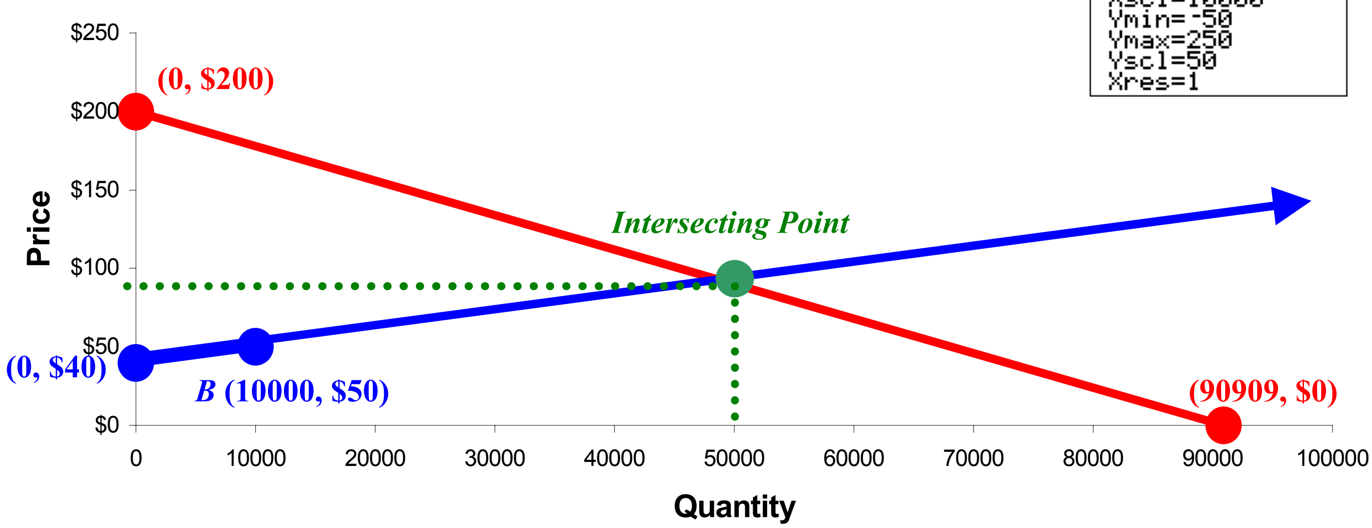
$P = -\frac{11}{5000}(0) + 200$

$P = 200$

***P*-int = \$200**

Supply and Demand of a Star Trek Model





b) Supply Line
slope = Manufacturing Variable Cost (labor, material)
***y*-int = Manufacturing Fixed Cost (rent, heat, license)**

c) Demand Line
***x*-int = quantity available when item becomes worthless.**
***y*-int = price of item when it becomes absolutely rare.**

d) Intersecting Point of Supply and Demand Lines
- Optimal Price at Optimal Amount Manufactured.

2-4 Assignment: pg. 184–187 #5, 9, 13, 17, 21, 25, 29, 31, 33, 43, 49, 57, 62, 65;
Honours: #69, 71

2-5: Modeling Variation

Direct Variation: - a variable that ***varies directly*** **(by a constant rate of change)** with another variable.

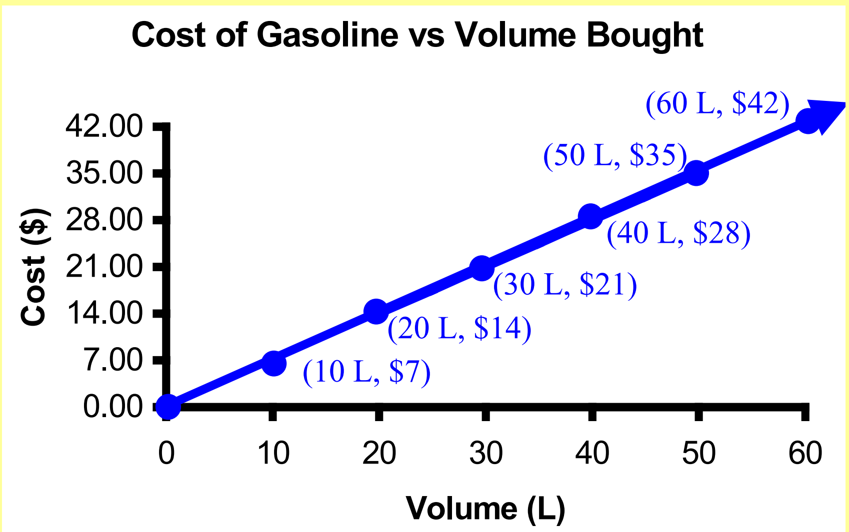
> $y \propto x$ (**y is directly proportional to x**)
>
> $$y = kx$$
>
> **where k = constant of variation (constant of proportionality – rate of change)**

Example 1: Gasoline at one time costs $0.70 per Litre.

a. What is the constant of variation and write the equation relating cost of gasoline and volume bought.
b. Graph the equation.
c. Find the cost of 63 L of gasoline.
d. How much gasoline can you buy with $26.43?

a. Constant of Variation = **$0.70/L (unit price of gasoline)** Equation: **C = ($0.70/L)$V$**

b.



c. V = 63 L
C = ?

C = ($0.70/L)$V$
C = ($0.70/L)(63 L)
C = $44.10

d. V = ?
C = $26.43

C = ($0.70/L)$V$
$26.43 = ($0.70/L)V

$$\frac{\$26.43}{\$0.70/\text{L}} = V$$

V = 37.757 L

Inverse Variation: - a variable that ***varies inversely*** **(where the input is in the denominator of a fraction)** with another variable.

> $$y \propto \frac{1}{x}$$ (**y is inversely proportional to x**)
>
> $$y = \frac{k}{x}$$
>
> **where k = constant of variation (constant of proportionality)**

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Example 2: Concentration of a solution varies inversely with its volume. A 300 mL salt solution has a concentration of 23%.

a. Find the constant of proportionality, and write the equation to express this inverse proportion.
b. Graph the equation.
c. What is the new concentration if 500 mL of water is added to the original solution?

a. C is inversely proportional to V

$$C = \frac{k}{V}$$

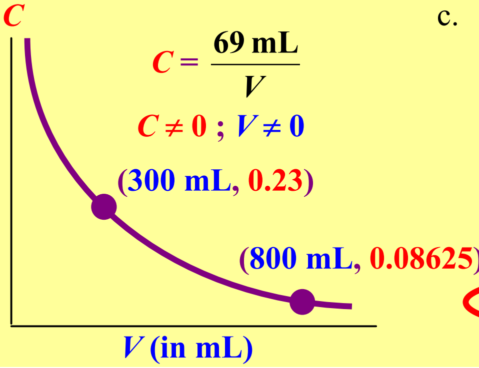
For $V = 300$ mL, $C = 23\% = 0.23$

$$0.23 = \frac{k}{(300\text{ mL})}$$

$$k = (0.23)(300\text{ mL})$$

$$\boldsymbol{k = 69\text{ mL}} \qquad \boldsymbol{C = \frac{69\text{ mL}}{V}}$$

b. C



c. $C = ?$

$V = 300\text{ mL} + 500\text{ mL}$

$V = 800$ mL

$$C = \frac{69\text{ mL}}{V} = \frac{69\text{ mL}}{800\text{ mL}}$$

$$\boldsymbol{C = 0.08625 = 8.625\%}$$

Joint Variation: - a variable that ***varies jointly*** (**have more than one inputs**) with another variable.

$$y \propto \frac{xz}{w}$$ **(y is jointly proportional to x, z and w)**

$$y = k\frac{xz}{w}$$

where k = constant of variation (constant of proportionality)

Example 3: The Ideal Gas Law states that the pressure of a gas in kPa (kilopascal) is inversely proportional to its volume, V (in L) and directly proportional to the amount of the gas, n (in mol) along with its temperature, T (in Kelvin).

a. Write the equation to express the Ideal Gas Law. Use k as the constant of variation.
b. A 5 L container of 0.35 mol neon gas has a pressure of 174.6 kPa at a temperature of 300 K. Determine the constant of variation.
c. A 25 L container of helium gas has a pressure of 200 kPa at a temperature of 275 K. Find the amount of helium gas in this container.

a. P = Pressure (in kPa); V = Volume (in L)
n = amount (in mol); T = Temp (in K) ; k = Constant of Proportionality

$$\boldsymbol{P = k\frac{nT}{V}}$$ **(Directly proportional to n and T)** **(Inversely proportional to V)**

b. $P = 174.6$ kPa
$V = 5$ L
$n = 0.35$ mol
$T = 300$ K
$k = ?$

$$P = k\frac{nT}{V}$$

$$PV = knT$$

$$\frac{PV}{nT} = k$$

$$k = \frac{(174.6\text{ kPa})(5\text{ L})}{(0.35\text{ mol})(300\text{ K})}$$

$$\boldsymbol{k = 8.314\ \frac{\text{kPa}\bullet\text{L}}{\text{mol}\bullet\text{K}}}$$

c. $k = 8.314\ \frac{\text{kPa}\bullet\text{L}}{\text{mol}\bullet\text{K}}$
$P = 200$ kPa ; $V = 25$ L
$T = 275$ K ; $n = ?$

$$P = k\frac{nT}{V}$$

$$\frac{PV}{kT} = n$$

$$n = \frac{(200\ \cancel{\text{kPa}})(25\ \cancel{\text{L}})}{\left(8.314\ \frac{\cancel{\text{kPa}}\bullet\cancel{\text{L}}}{\text{mol}\bullet\cancel{\text{K}}}\right)(275\ \cancel{\text{K}})}$$

$$\boldsymbol{n = 2.19\text{ mol}}$$

2-5 Assignment: pg. 191–193 #3, 7, 11, 13, 17, 19, 23, 27, 29, 33; Honours: #39, 42