Unit 3: Probability: Foundations for Inference

Chapter 6: Probability: The Study of Randomness

6.1: Randomness

Probability: - the study of the mathematics behind the pattern of chance outcome based on empirical (observational) data.

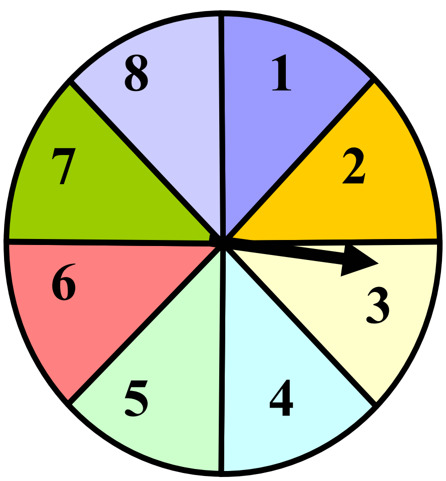
Randomness: - a state where the outcome may seems unpredictable (without pattern) at first, but with a large number of trials, the proportion of outcome will eventually emerge (random phenomenon).

Experimental Probability: - probability that came from a simulation such as tossing dice, coins … etc.
- in order to use experimental probability as an inference, many **independent trials (one trial cannot influence another)** will have to be performed in the simulation.

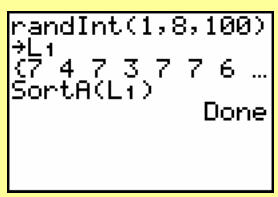
$$\textbf{Experimental Probability } P(A) = \frac{\textbf{Number of Favourable Outcomes}}{\textbf{Total Number of Trials}}$$

$P(A)$ = Probability (Proportion of Outcome) of Event A

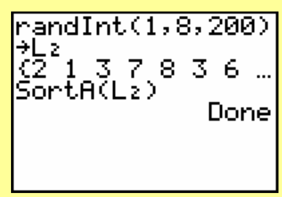
Example 1: Using the TI-83 Plus calculator, simulate spinning the spinner below 100 times and 200 times. Determine the experimental probability of spinning a 4 for each simulation.



Simulating using 100 trials:



Simulating using 200 trials:



To access →, press STO→

To access SortA, press 2nd LIST STAT ▶

Count the number of 4's in L_1

From $L_1(34)$ to $L_1(51)$, there are 18 fours.

$P(4) = \frac{18}{100}$ $\quad P(4) = \frac{9}{50} = 0.18$

Count the number of 4's in L_2

From $L_2(73)$ to $L_2(96)$, there are 24 fours.

$P(4) = \frac{24}{200}$ $\quad P(4) = \frac{3}{25} = 0.12$

6.1 Assignment
pg. 316–317 #6.2, 6.5 to 6.8

6.2A: Probability Models

Sample Space: - a list of all possible outcomes of an experiment.

Event: - an outcome or a set of occurrences from a subset of a sample space of a random phenomenon.

Complement: - probability of **ALL Non-Favourable** outcomes.

$P(A)$ = Probability of event A happening. $P(\overline{A})$ = Probability that event A will **NOT** happen.

In general, $$P(A) + P(\overline{A}) = 1$$ **or** $$P(\overline{A}) = 1 - P(A)$$

Example 1: Find the theoretical probability of rolling a 4 using a six-sided dice, and its compliment.

Sample Space of a dice = {1, 2, 3, 4, 5, 6}

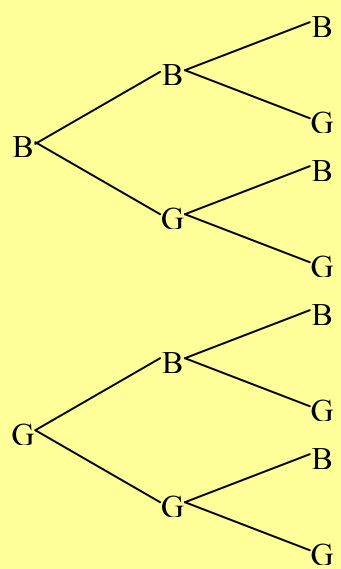
$\boldsymbol{P(4) = \frac{1}{6}}$ **(1 favorable outcome out of a total of 6 outcomes)**

$P(\overline{4}) = 1 - P(4) = 1 - \frac{1}{6}$ $\boldsymbol{P(\overline{4}) = \frac{5}{6}}$

Tree Diagram: - using branches to list all outcomes.
- following branches one path at a time to list all outcomes.
- its limitation lies in the fact that it can only handle individual events having **SMALL** number of outcomes.

Example 2: List the sample space of a family of 3 children. Let event A be at most having 2 girls, find $P(A)$ and $P(\overline{A})$.

1st child 2nd child 3rd child Outcomes



(B, B, B)
(B, B, G)
(B, G, B)
(B, G, G)
(G, B, B)
(G, B, G)
(G, G, B)

$P(A)$ = Probability of **at most** 2 girls **(no girls, 1 girl, and 2 girls)**

$\boldsymbol{P(A) = \frac{7}{8}}$

(7 favorable outcomes out of a total of 8 outcomes)

OR we can say $P(A) = 1 - P$ (all girls)

$\boldsymbol{P(A) = 1 - \frac{1}{8}}$ $\boldsymbol{P(A) = \frac{7}{8}}$

(G, G, G) $\longrightarrow$ $P(\overline{A}) = 1 - P(A) = 1 - \frac{7}{8}$ $\boldsymbol{P(\overline{A}) = \frac{1}{8}}$

Sample Space Table: useful when there are **TWO** items of **MANY** outcomes for each trial.

Example 3: List the sample space for the sums when 2 fair dice are thrown
a. Find the probability of rolling a sum of 8.
b. What is the probability of rolling at least an 8?

First Dice \ Second Dice						
Sum	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a. $\boldsymbol{P\,(8) = \frac{5}{36}}$

(5 favorable outcomes out of a total of 36 outcomes)

b. P (at least a sum of 8) = P (sum of 8 and higher)

P (at least a sum of 8) = $\frac{15}{36}$

$\boldsymbol{P}$ **(at least a sum of 8) =** $\boldsymbol{\frac{5}{12}}$

Example 4: Without drawing a tree diagram, determine the number outcomes that will be in the sample space when 4 coins are tossed. What is the probability of tossing **at least** one head?

From Example 2, we noticed that when there were 3 events (3 children) of 2 outcomes (boy or girl) each, there were a total of $2 \times 2 \times 2 = 8$ outcomes.

Therefore, it can be assumed that when there are 4 events (tossing four coins) of 2 outcomes (head or tail) each, there are a total of 2 × 2 × 2 × 2 = <u>16 outcomes</u>.

P (at least one head) = P (1 or 2 or 3 or 4 heads)
***P* (at least one head) = 1 – *P* (no head)**
***P* (at least one head) = 1 – *P* (all tails)**
P (at least one head) = $1 - \frac{1}{16}$
(There is only 1 outcome out of a total of 16 outcomes to get all tails.)

$\boldsymbol{P}$ **(at least one head) =** $\boldsymbol{\frac{15}{16}}$

Sampling With Replacement: - sometimes refer to as **independent events**.
- when the outcome of one event does NOT affect the outcomes of the events follow.

Example: Drawing a card out of a deck of 52 playing cards. Putting it back in the deck before drawing another one.

Sampling Without Replacement: - sometimes refer to as **dependent events**.
- when the outcome of the first event AFFECTS the outcome(s) of subsequent event(s).

Example: Drawing two cards out of a deck of 52 playing cards without putting the first card back in the deck.

Multiplication (Counting) Principle: - by multiplying the number of ways in each category of any particular outcome, we can find the total number if arrangements.

Example 5: How many outfits can you have if you have 3 different shirts, 2 pairs of pants, 4 pairs of socks, and 1 pair of shoes?

There are 4 categories: shirt, pants, socks and shoes.

$\underset{\text{shirts}}{\underline{3}} \times \underset{\text{pants}}{\underline{2}} \times \underset{\text{socks}}{\underline{4}} \times \underset{\text{shoes}}{\underline{1}} =$ **24 outfits**

When the outcome involves many restrictions, make sure to deal with the category that is the most restrictive first.

Example 6: How many 5-digits numbers are there if

a. there is no restriction?

1st digit	2nd digit	3rd digit	4th digit	5th digit
9 ×	10 ×	10 ×	10 ×	10
(1 to 9)	(0 to 9)	(0 to 9)	(0 to 9)	(0 to 9)

90000 numbers

b. they have to be divisible by 5?

1st digit	2nd digit	3rd digit	4th digit	5th digit
9 ×	10 ×	10 ×	10 ×	2
(1 to 9)	(0 to 9)	(0 to 9)	(0 to 9)	(0 or 5)
				restriction

18000 numbers

c. no digits are repeated?

1st digit	2nd digit	3rd digit	4th digit	5th digit
9 ×	9 ×	8 ×	7 ×	6
(1 to 9)	(0 to 9)	(0 to 9)	(0 to 9)	(0 to 9)
	− 1 digit	− 2 digits	− 3 digits	− 4 digits

27216 numbers

d. they have to be less than 40000 with non-repeated digits?

1st digit	2nd digit	3rd digit	4th digit	5th digit
3 ×	9 ×	8 ×	7 ×	6
(1 to 3)	(0 to 9)	(0 to 9)	(0 to 9)	(0 to 9)
	− 1 digit	− 2 digits	− 3 digits	− 4 digits

9072 numbers

e. at least two digits that are the same?

At least two digits the same = same 2 digits + same 3 digits + same 4 digits + same 5 digits

OR

At least two digits the same = Total number of ways with no restriction − No digits the same
= 90000 − 27216

62784 numbers

f. Find the probability of getting a 5-digits numbers if at least 2 digits have to be the same.

$$P\text{ (at least 2 same digits out of a 5-digits number)} = \frac{\text{Number of ways at least same 2 digits}}{\text{Total Number of ways}} = \frac{62784}{90000}$$

$$\mathbf{\frac{436}{625} \text{ or } 0.6976}$$

Example 7: In an Algebra Midterm Exam, there are 33 multiple-choice questions. Each question contains 4 choices. How many different combinations are there to complete the test, assuming all questions are answered? What is the probability that a student would guess all the answers correctly?

For each question, there are 4 ways (choices) to answer.

For 33 questions, there are $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times \ldots$ } There are 33 factors of fours.

Total number of ways to answer the test = 4^{33} or $7.378697629 \times 10^{19}$ ways

$$\mathbf{P\text{ (guessing all the right answers)} = \frac{1}{4^{33}} = 1.35525272 \times 10^{-20}}$$

(There is a way better chance to win the lottery compared to guessing all the answers correctly!)

Example 8: Find the number of ways to select 2 individuals from a group of 6 people.

Let's suppose the names of the people are A, B, C, D, E and F.

The sample space table would be

AB	BA	CA	DA	EA	FA
AC	**BC**	CB	DB	EB	FB
AD	**BD**	**CD**	DC	EC	FC
AE	**BE**	**CE**	**DE**	ED	FD
AF	**BF**	**CF**	**DF**	**EF**	FE

Order is NOT important! These repeated combinations do not need to be counted.

Total Number of ways = 15

Summary of the Multiplication (Counting) Principle

ORDER Important?

YES → **Restrictions?**

- YES → **Count the outcomes of the most restricted category** → **Multiply the number of ways in each category**
- NO → **Multiply the number of ways in each category**

NO → **1.Count Sample Spaces by using tables or tree diagram.**
2.Cross out any duplicated results.

6.2A Assignment

pg. 322–323 #6.9, 6.10, 6.12, 6.13, 6.15, 6.16

6.2B: Representing Probability

Basic Rules on Probability

1. **Probabilities in a Finite Sample Space**: - all values probability must be between 0 and 1.
   - when $P(A) = 1$, it is CERTAIN that Event A will occur.
   - when $P(A) = 0$, it is IMPOSSIBLE that Event A will happen.
   - probability of Event A can consists of many outcomes (**Example**: rolling a even number in a 6-sided dice – outcomes are 2, 4, 6 and the event is even numbers)

   $$\boxed{0 \leq P(A) \leq 1}$$

2. **All Probabilities in a Sample Space adds up to 1.** $\boxed{P(S) = 1}$

3. **If there are two events in a sample space, A and $\overline{A}$ (textbook refer to as A^c – Compliment of Event A – NOT A), then** $P(\overline{A}) = 1 - P(A)$

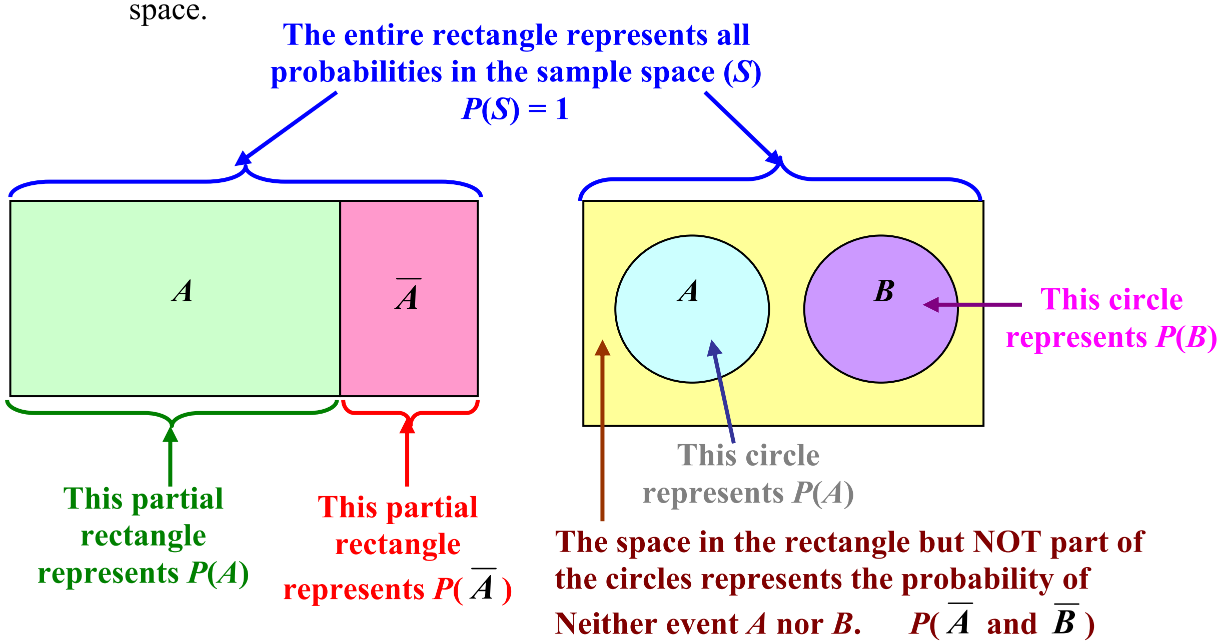
Equally Likely Outcomes: - when each individual outcome in a sample space has the same probability of happening.

Equally Likely Outcomes where k = number of equal outcomes
$P(\text{Each Outcome}) = \dfrac{1}{k}$ $\qquad$ $P(A) = \dfrac{\text{Number of Outcomes Satisfying Condition of Event } A}{k}$

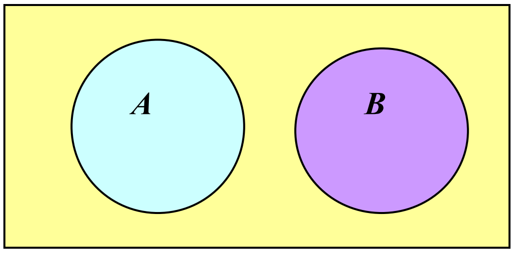
Example 1: rolling a even number in a 6-sided dice – outcomes are 2, 4, 6 and the event is even numbers

$$P\text{ (each outcome – numbers 1 to 6)} = \frac{1}{6} \qquad P(\text{Even Numbers}) = \frac{3}{6} = \frac{1}{2}$$

Venn Diagram: - a diagram that illustrate all the events and their relationships within the entire sample space.



Disjoint (Mutually Exclusive) Events: - when events A & B **CANNOT** occur at the **SAME TIME**.
- either event A occurs **OR** event B occurs.



Disjoint (Mutually Exclusive) Events

$$P(A \cup B) = P(A) + P(B)$$

$\cup$ means OR

NO Overlap

Example 2: A card is drawn from a standard deck of 52 cards. What is the probability that it is a red card or a spade?

Since there is no card that is BOTH a spade and red, the events are mutually exclusive.

$$P(\text{red}) = \frac{26}{52} = \frac{1}{2} \qquad P(\text{spade}) = \frac{13}{52} = \frac{1}{4}$$

$$P(\text{red} \cup \text{spade}) = P(\text{red}) + P(\text{spade})$$

$$P(\text{red} \cup \text{spade}) = \frac{1}{2} + \frac{1}{4}$$

$$\mathbf{P(\text{red} \cup \text{spade}) = \frac{3}{4}}$$

Example 3: Two dice are rolled. What is the probability that either the sum is 3 or the sum is 8?

Second Dice

First Dice: Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Since you can either roll a sum of 3 or a sum of 8, but NOT BOTH, the events are mutually exclusive.

$$P(3) = \frac{2}{36} \qquad P(8) = \frac{5}{36}$$

$$P(3 \cup 8) = P(3) + P(8)$$

$$P(3 \cup 8) = \frac{2}{36} + \frac{5}{36}$$

$$\mathbf{P(3 \cup 8) = \frac{7}{36}}$$

6.2B Assignment

pg. 330– 331 #6.18 to 6.23

6.2C: Independent and Dependent Events

Independent Events: - when the outcome of one event does NOT affect the outcomes of the events follow.

$$P(\underbrace{A \cap B}_{\text{Events } A \text{ and } B}) = P(A) \times P(B)$$

$\cap$ means AND

Example 1: What is the probability of rolling at most a "4" from a dice and selecting a diamond out of a standard deck of cards?

Rolling a Dice and Drawing a Card are Unrelated Events – Independent Events

P (at most "4" $\cap$ diamond) $= \frac{4}{6} \times \frac{13}{52}$

Reduce before multiply

P (at most "4" $\cap$ diamond) $= \frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$

$\boldsymbol{P}$ **(at most "4" $\cap$ diamond) = $\frac{1}{6}$**

Example 2: Find the probability of getting at least one number correct out a 3-numbers combination lock with markings from 0 to 59 inclusive.

Each number in the combination is unrelated to each other –Independent Events

All numbers are Incorrect (No number is correct) is the Compliment Event of At Least ONE number is correct.

P (All numbers are Incorrect) $= \frac{59}{60} \times \frac{59}{60} \times \frac{59}{60} = \left(\frac{59}{60}\right)^3$ (There are 60 numbers from 0 to 59 inclusive.)

P (At Least ONE number is correct) = 1 – P (All numbers are Incorrect)

P (At Least ONE number is correct) $= 1 - \left(\frac{59}{60}\right)^3$

$\boldsymbol{P}$ **(At Least ONE number is correct) ≈ 0.0492 ≈ 4.92%**

Example 3: What is the probability of drawing two aces if the first card is replaced (put back into the deck) before the second card is drawn?

P (drawing 2 aces) $= \frac{4}{52} \times \frac{4}{52}$

P (drawing 2 aces) $= \frac{1}{13} \times \frac{1}{13}$

$\boldsymbol{P}$ **(drawing 2 aces) = $\frac{1}{169}$ ≈ 0.005917 ≈ 0.592%**

Dependent Events: - when the outcome of the first event AFFECTS the outcome(s) of subsequent event(s).

Example 4: What is the probability of drawing two hearts from a standard deck of 52 cards?

If the question doe NOT say, assume the card is NOT Replaced! (WITHOUT REPLACEMENT is ALWAYS Dependent Events)

$$P\text{ (two hearts)} = \frac{13}{52} \times \frac{12}{51}$$

One less heart remains in the deck after the 1st draw.
One less card total remains in the deck after the 1st draw.

$$P\text{ (two hearts)} = \frac{1}{4} \times \frac{4}{17}$$

$$\boldsymbol{P\text{ (two hearts)} = \frac{1}{17} \approx 0.0588 \approx 5.88\%}$$

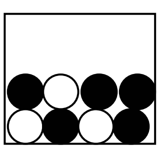
Example 5: About 40% of the population likes sci-fi movies, while 80% of the population likes comedies. If 35% of the population likes both sci-fi and comedies, determine whether preferences for sci-fi and comedies are independent events. Justify your answer mathematically.

P (sci-fi) = 40% = 0.40
P (comedies) = 80% = 0.80

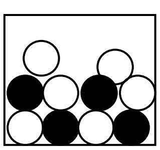
If sci-fi and comedies are independent events, then
P (sci-fi $\cap$ comedies) = P (sci-fi) $\times$ P (comedies)
P (sci-fi $\cap$ comedies) = 0.40 $\times$ 0.80
P (sci-fi $\cap$ comedies) = **0.32 (independent events)**

Since the question indicates P (sci-fi $\cap$ comedies) is 35% = 0.35, they should be considered as **DEPENDENT EVENTS**.

Example 6: There are two identical looking containers. Each contains different number of black and white balls of the same size. Container A has 8 balls in total and 3 of those are white. Container B has 10 balls in total and 4 of those are black. If a person has a choice to pick one ball out of these two containers, what is the probability of selecting a black ball?



Container A



Container B

P (black) = P (Container $A \cap$ Black) $\cup$ P (Container $B \cap$ Black)

OR means ADD

$$P\text{ (black)} = P(A) \times P\text{ (Black)} + P(B) \times P\text{ (Black)}$$

$$P\text{ (black)} = \left(\frac{1}{2}\right)\times\left(\frac{5}{8}\right) + \left(\frac{1}{2}\right)\times\left(\frac{4}{10}\right)$$

$$P\text{ (black)} = \frac{5}{16} + \frac{4}{20}$$

$$\boldsymbol{P\text{ (black)} = \frac{41}{80} = 0.5125 = 51.25\%}$$

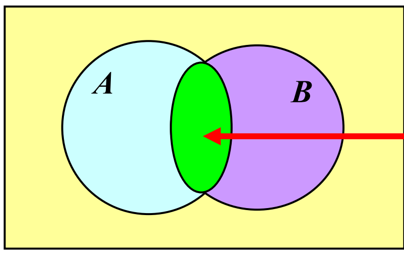
6.2C Assignment
pg. 335–336 #6.24, 6.25, 6.27
pg. 337–340 #6.29 to 6.36

6.3A: Union of Two Events

Joint Event: - when two (A and B) or more events can happen simultaneously.

Joint Probability: - the probability of a joint event $P(A \text{ and } B)$.
- **for Independent Events, $P(A \cap B) = P(A) \times P(B)$**
- **for Dependent Events, $P(A \cap B)$ are either given or found**

Union of Two (Non-Mutually Exclusive) Events: - the occurrence of **at least of one of the events A Or B** when **both events A & B can occur** at the **SAME TIME**.
- the probability of the overlapping area, ***P*** **(*A* and *B*), is subtracted**.



Union of Two (Non-Mutually Exclusive) Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Overlapping Area – Joint Probability

$P(A \text{ and } B) = P(A) \times P(B)$ (if A and B are independent events)

$P(A \text{ and } B)$ must be given or found if A and B are dependent

Example 1: Two dice are rolled. What is the probability that one of the dice is 3 or the sum is 8?

First Dice \ Second Dice	Sum	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Since you can have sums of 8 that have 3 on one dice, these are Union of Two (Non-Mutually Exclusive) Events.

$$P(\text{one of the dice is 3}) = \frac{10}{36}$$

(sum of 6 has two 3's – Not included)

$$P(8) = \frac{5}{36} \qquad P(\text{3 on one dice} \cap \text{sum is 8}) = \frac{2}{36}$$

$$P(\text{3 on one dice} \cup \text{sum of 8}) = P(3) + P(8) - P(\text{3 on one dice} \cap \text{sum of 8})$$

$$P(\text{3 on one dice} \cup \text{sum of 8}) = \frac{10}{36} + \frac{5}{36} - \frac{2}{36}$$

$$\mathbf{P(\text{3 on one dice} \cup \text{sum of 8}) = \frac{13}{36}}$$

Example 2: Out of 200 people, 80 of them like sci-fi and 160 like comedies. 70 of them like both sci-fi and comedies. Find the probability that someone will like either sci-fi or comedies.

Since you can have people prefer BOTH sci-fi and comedies, these events are non-mutually exclusive.

$$P(\text{sci-fi}) = \frac{80}{200} \qquad P(\text{comedies}) = \frac{160}{200} \qquad P(\text{sci-fi} \cap \text{comedies}) = \frac{70}{200}$$

$$P(\text{sci-fi} \cup \text{comedies}) = P(\text{sci-fi}) + P(\text{comedies}) - P(\text{sci-fi} \cap \text{comedies})$$

$$P(\text{sci-fi} \cup \text{comedies}) = \frac{80}{200} + \frac{160}{200} - \frac{70}{200}$$

$$P(\text{sci-fi} \cup \text{comedies}) = \frac{170}{200} = \frac{17}{20} = 0.85 = 85\%$$

Example 3: In our society, the probability of someone suffering from heart disease is 0.56 and the probability of developing cancer is 0.24. If these events are independent and non-mutually exclusive, what is the probability that you will suffer **neither** illness?

P (Heart Disease) = $P(H) = 0.56$ P (Cancer) = $P(C) = 0.24$

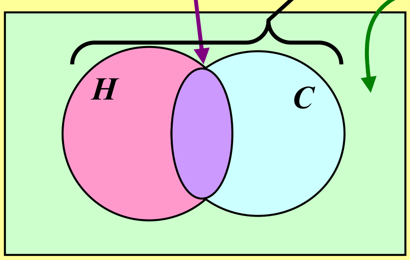
$P(H \cap C) = P(H) \times P(C) = 0.56 \times 0.24$ (independent events)

$\mathbf{P(H \cap C) = 0.1344}$

$P(H \cup C) = P(H) + P(C) - P(H \cap C)$ (non-mutually exclusive events)

$P(H \cup C) = 0.56 + 0.24 - 0.1344$

$\mathbf{P(H \cup C) = 0.6656}$



P (No Heart Disease NOR Cancer) = $P(\overline{H} \cap \overline{C})$

Area OUTSIDE the circles

$P(\overline{H} \cap \overline{C}) = 1 - P(H \cup C)$

$P(\overline{H} \cap \overline{C}) = 1 - 0.6656$

$\mathbf{P(\overline{H} \cap \overline{C}) = 0.3344 = 33.44\%}$

Example 4: Two cards are drawn from a standard deck of 52 cards. What is the probability that both cards are face cards or both are hearts?

The question did not mention replacement. Therefore, we can assume there is NO replacement (Dependent Events).

Since you can have BOTH face cards and hearts, these events are non-mutually exclusive.

P (face cards) = $\frac{12}{52} \times \frac{11}{51} = \frac{11}{221}$ P (hearts) = $\frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$

P (face cards $\cap$ hearts) = $\frac{3}{52} \times \frac{2}{51} = \frac{1}{442}$ (K, Q, and J of Hearts)

P (face cards $\cup$ hearts) = P (face cards) + P (hearts) − P (face cards $\cap$ hearts)

P (face cards $\cup$ hearts) = $\frac{11}{221} + \frac{1}{17} - \frac{1}{442}$

P (face cards $\cup$ hearts) = $\frac{47}{442}$ = 0.1063 = 10.63%

6.3A Assignment
pg. 345–346 #6.37 to 6.40

6.3B: Conditional Probabilities

Conditional Probability: - probability of an event B ***given that*** event A occurs first.

Conditional Probability for Dependent Events (*Bayes's Theorem*)

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \qquad P(A \cap B) \neq P(A) \times P(B) \text{ (for Dependent Events)}$$

$P(B \mid A)$ **= Conditional Probability for Event B "*GIVEN*" that Event A has occurred**

Example 1: An independent survey has indicated Mary has a probability of 0.652 of becoming the next student council president of a university. Joseph, however, has a 0.876 chance of becoming the vice-president of the same council. Suppose the probability that they are both elected is 0.758 because they campaign together under the guise of some divine approval (or they simply just look good together). On the day of the election, determine the probability that Mary will become the president given that Joseph has already been declared a winner.

First, label and define all probability notations involved.

$P(J)$ = Probability that Joseph wins = 0.876
$P(M)$ = Probability that Mary wins = 0.652

$P(J \cap M)$ = Probability that both Joseph and Mary win = 0.758
Note that they are dependent events. $P(J \cap M) \neq P(J) \times P(M)$ $[0.758 \neq 0.876 \times 0.652]$

$P(M \mid J)$ **= Probability of Mary winning given that Joseph has won = ?**

$$P(M \mid J) = \frac{P(J \cap M)}{P(J)} = \frac{0.758}{0.876} \qquad P(M \mid J) = 0.865$$

Therefore, in contrast to the survey, Mary has a way better chance of winning if Joseph has won.

Conditional Probability for Independent Events

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{\cancel{P(A)} \times P(B)}{\cancel{P(A)}} \qquad P(A \cap B) = P(A) \times P(B) \text{ (for Independent Events)}$$

$$P(B \mid A) = P(B)$$

$P(B \mid A)$ **= Conditional Probability for Event B "*GIVEN*" that Event A has occurred**

Intersection of Events:- the occurrence where ***all*** favourable events occur.
- basically it is $P(A \cap B)$, for **either independent or dependent** events.

Example 2: A dice is rolled once and then again. What is the probability that the second roll results a 6 given that the first roll was a 4?

$P(4)$ = First Roll is a 4 = $\frac{1}{6}$ $P(6)$ = Second Roll is a 6 = $\frac{1}{6}$ $P(4 \cap 6) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$P(6 \mid 4)$ **= Rolling a 6 given that a 4 is rolled first = ?**

$$P(6 \mid 4) = \frac{P(6 \cap 4)}{P(4)} = \frac{(1/36)}{(1/6)} \qquad P(6 \mid 4) = \frac{1}{6}$$

Note that First Roll was already CERTAIN (4 had happened), $1 \times \frac{1}{6} = \frac{1}{6}$

Example 3: The National Cancer Institute has stated that 2.278% of the population will develop colon cancer with their lives. A new test for colon cancer, without probing, has been developed. It has an accuracy rate of 95%.

a. Draw a tree diagram to illustrate a randomly selected patient the different outcomes and their probabilities while underwent this new, non-probing cancer test. Verify the results with one of the basic rule of probabilities.

b. Find the probability that you will have colon cancer given that your test was positive.

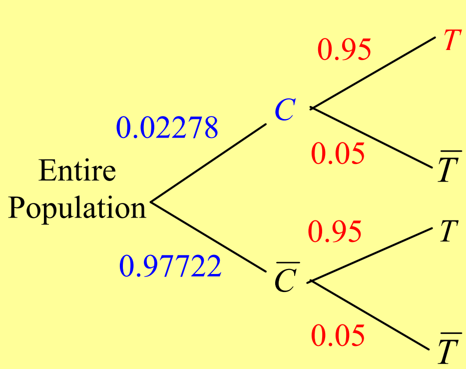
c. To test the accuracy rate, what is the probability that you will be tested negative given that you have no colon cancer?

a. First, label and define all probability notations involved.

$P(C)$ = Have Colon Cancer = 0.02278 $P(\bar{C})$ = NO Colon Cancer = 1 − 0.02278 = 0.97722

$P(T)$ = Test Accurate = 0.95 $P(\bar{T})$ = Test Inaccurate = 1 − 0.95 = 0.05

Colon Cancer Test Accuracy Outcomes



$P(C \cap T) = 0.02278 \times 0.95 = 0.021641$
Have Cancer and Test Accurate (Positive Result)

$P(C \cap \bar{T}) = 0.02278 \times 0.05 = 0.001139$
Have Cancer and Test Inaccurate (Negative Result)

$P(\bar{C} \cap T) = 0.97722 \times 0.95 = 0.928359$
No Cancer and Test Accurate (Negative Result)

$P(\bar{C} \cap \bar{T}) = 0.97722 \times 0.05 = 0.048861$
No Cancer and Test Inaccurate (Positive Result)

Note that whether the result is positive or negative depends on whether the patient actually has cancer or not. Test being accurate or inaccurate alone does NOT define positive or negative result.

Verification Using Probability Rule

$P(C \cap T) + P(C \cap \bar{T}) + P(\bar{C} \cap T) + P(\bar{C} \cap \bar{T}) = 1$

$0.021641 + 0.001139 + 0.928359 + 0.048861 = 1$

b. $$P(C \mid Positive) = \frac{P(C \cap Positive)}{P(Postive)} = \frac{P(C \cap T)}{P(C \cap T) + P(\bar{C} \cap \bar{T})} = \frac{0.021641}{0.021641 + 0.048861}$$

$P(C \mid Positive)$ = 0.306955831 = 30.69%
(There is a bigger chance you DON'T have cancer even if the test was positive!)
(Therefore, DON'T spend all your money because you think you will die soon!)

c. $$P(Negative \mid \bar{C}) = \frac{P(\bar{C} \cap Negative)}{P(\bar{C})} = \frac{P(\bar{C} \cap T)}{P(\bar{C} \cap T) + P(\bar{C} \cap \bar{T})} = \frac{0.928359}{0.928359 + 0.048861}$$

$P(Negative \mid \bar{C})$ = 0.95 = 95% (Same as question stated, but rather misleading as seen in part b.)

<u>6.3B Assignment</u>
pg. 350 #6.41 to 6.43
pg. 355–358 #6.44 to 6.49, 6.51, 6.53 to 6.55

<u>Chapter 6 Review</u>
pg. 361 to 364 #6.59 to 6.66

Chapter 7: Random Variables

7.1: Discrete and Continuous Random Variables

Random Variable (X): - a variable that has numerical values that were from some random incident.
- there are two types of random variables (**discrete** and **continuous**)

Discrete Random Variable: - a random variable where the set of values can be counted by their **specific** occurrences.

Examples: Rolling a six-sided dice is discrete. $X = \{1, 2, 3, 4, 5, 6\}$
The number of girls in a family of 5 children is discrete $X = \{0, 1, 2, 3, 4, 5\}$

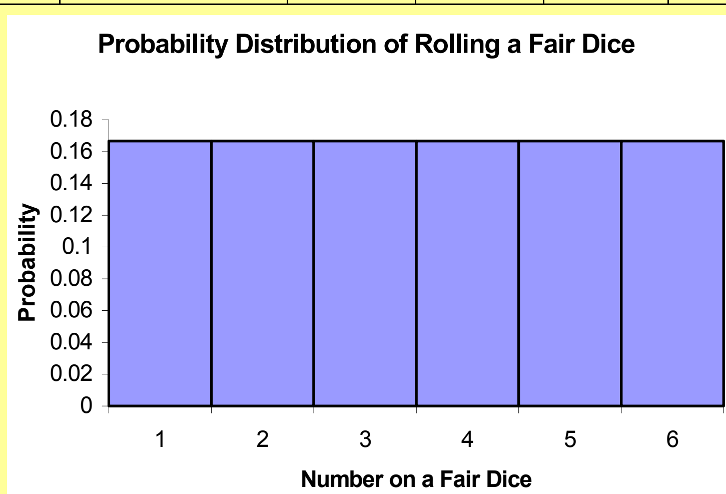
- the general list of specific occurrence in a set of discrete random variable (commonly known as sample space) are usually labelled as x_i, where i is the individual element of the sample space
- the corresponding probability for each occurrence is p_i.

Probability Histogram: - a histogram showing all the probabilities associated with all elements of a discrete random variable.
- P (any particular outcome) is between 0 and 1
- the sum of all the probabilities is always 1.

a. **Uniform Probability Histogram**: - a probability histogram where the probability of one occurrence is the same as all the others.

Example 1: List all the elements in the sample space (x_i) and their probabilities (p_i) for each outcome of rolling a fair 6-diced dice. Draw the corresponding probability histogram.

Occurrence (X)	$x_1 = 1$	$x_2 = 2$	$x_3 = 3$	$x_4 = 4$	$x_5 = 5$	$x_6 = 6$
Probability (p)	$p_1 = \frac{1}{6} = 0.167$	$p_2 = \frac{1}{6}$	$p_3 = \frac{1}{6}$	$p_4 = \frac{1}{6}$	$p_5 = \frac{1}{6}$	$p_6 = \frac{1}{6}$



b. Non-Uniform Probability Histogram: - a probability histogram where the probability of one occurrence is the different than others.

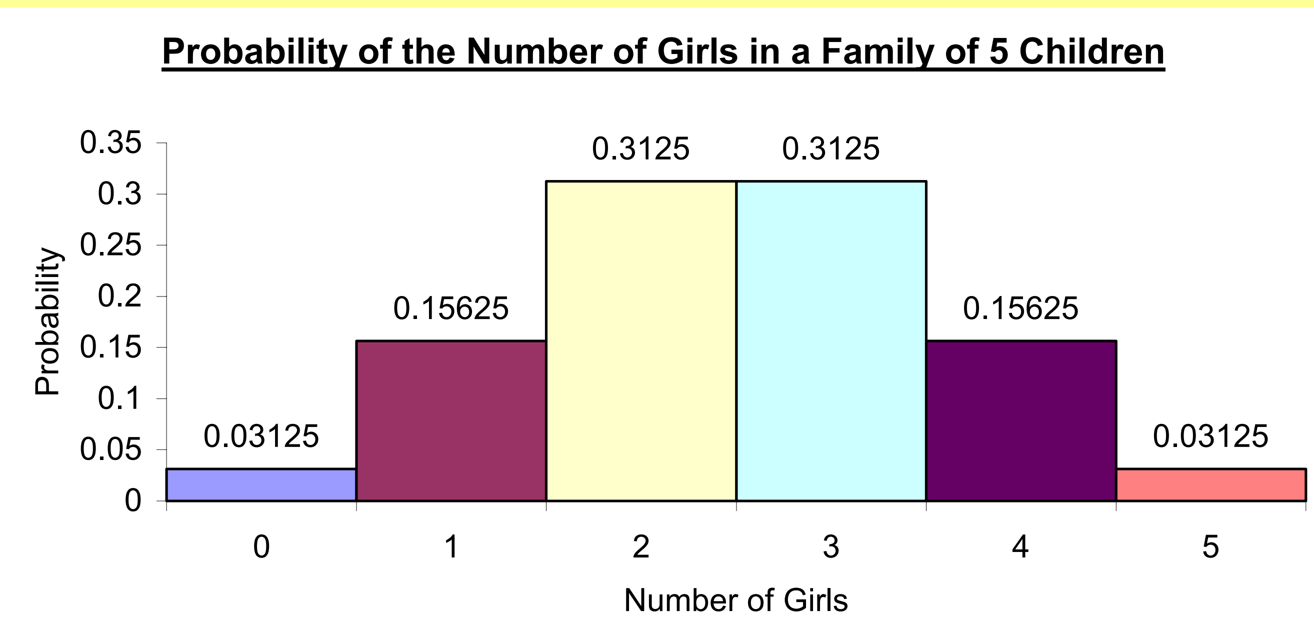
Example 2: List all the elements in the sample space (x_i) and their probabilities (p_i) for each outcome of number of girls in a family of 5 children. Draw the corresponding probability histogram.

First we need to know that number of possible outcomes. According to the multiplication rule, there are 2 × 2 × 2 × 2 × 2 = 32 different outcome (each child has 2 possibilities – boy or girl).

Therefore, the probability of each outcome is $\frac{1}{32}$. Listing them gives us the following table.

# of Girls (X)	x_1 = 0 Girl	x_2 = 1 Girl	x_3 = 2 Girls	x_4 = 3 Girls	x_5 = 4 Girls	x_6 = 5 Girls
Combinations	BBBBB	GBBBB BGBBB BBGBB BBBGB BBBBG	GGBBB GBGBB GBBGB GBBBG BGGBB BGBGB BGBBG BBGGB BBGBG BBBGG	BBGGG BGBGG BGGBG BGGGB GBBGG GBGBG GBGGB GGBBG GGBGB GGGBB	BGGGG GBGGG GGBGG GGGBG GGGGB	GGGGG
Probability (p)	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}=\frac{5}{16}$	$\frac{10}{32}=\frac{5}{16}$	$\frac{5}{32}$	$\frac{1}{32}$

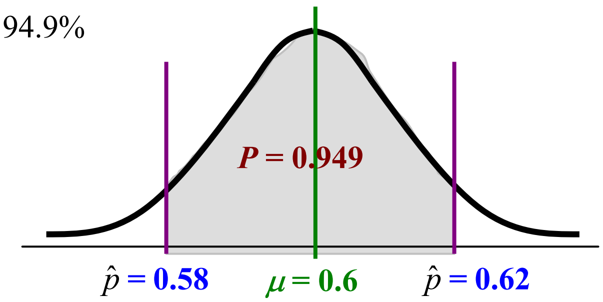
***Note*: There is an easier way to determine the theoretical p_i, using binomial probability function (Section 8.1).**



***Note*: Note the shape is normally distributed. If there are infinitely more bars, the histogram will be like a bell curve.**

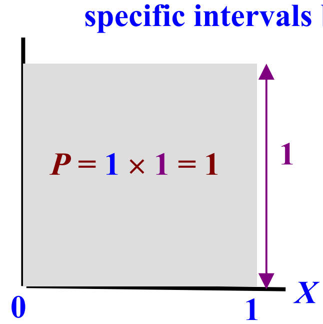
Continuous Random Variable (*X*): - a random variable where the set of values are specified by a **range** of numbers.
- there are no specific numbers and specific probabilities (no x_i and p_i), but rather a specific interval of numbers and the probabilities for that range, hence the term "***continuous***".

Example: The Proportion of the Population (p) that will agree to a national health care system is 0.6. The Range of Proportion of a Sample ($\hat{p}$) in a survey of 2000 people that will reflect this sentiment within 2% margin is ($0.58 \leq \hat{p} \leq 0.62$). This interval has a corresponding probability, P ($0.58 \leq \hat{p} \leq 0.62$), is 94.9%



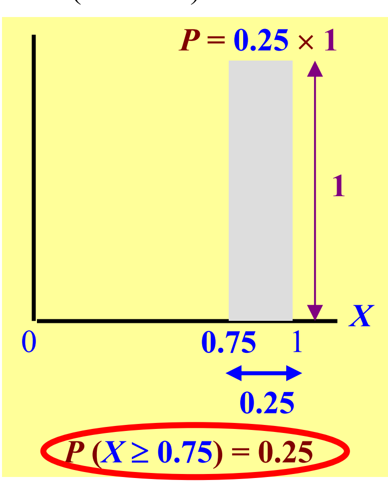
Probability Distribution: - a density curve that displays the **probability (area in the graph)** for a continuous random variable in an experiment.
- P (any range of numbers) is between 0 and 1.
- the sum of all ranges of probabilities is always 1.

a. Uniform Probability Distribution: - a probability distribution where its **height is 1** over any **specific intervals between 0 and 1**.

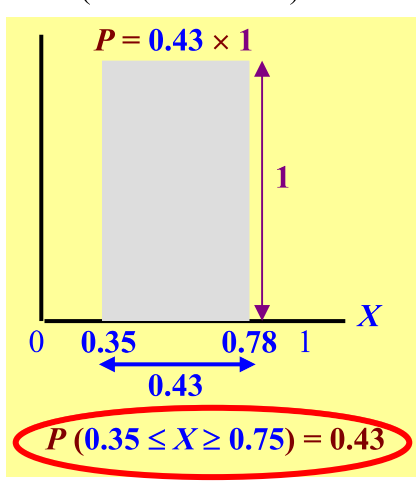


Example 3: Let X be a random variable between 0 and 1 with a uniform probability distribution. Find the following probabilities.

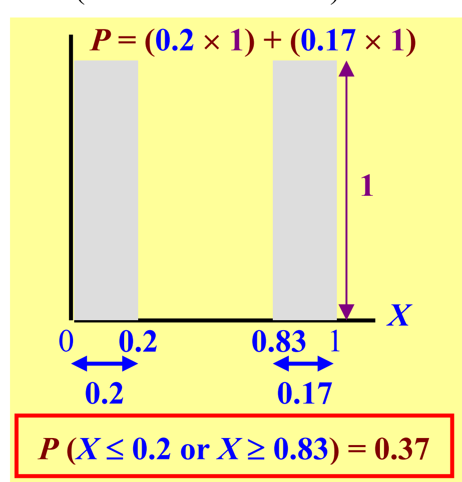
a. P ($X \geq 0.75$)



b. P ($0.35 \leq X \leq 0.78$)



c. P ($X \leq 0.2$ or $X \geq 0.83$)

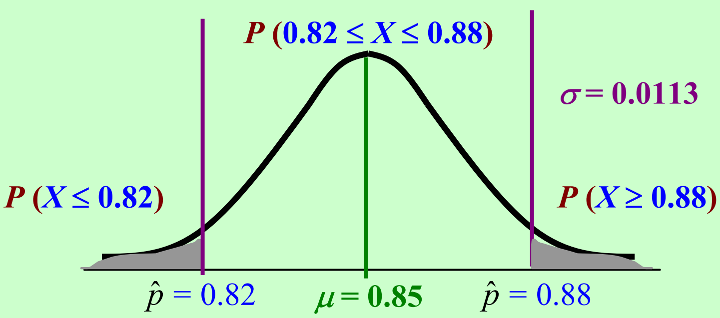


b. **Normal Distribution as Probability Distribution**: - when a continuous random variable distributed with the properties a normal distributed curve.

- we can say that X has a $N(\mu, \sigma)$ distribution and $Z = \dfrac{X - \mu}{\sigma}$

Example 4: The national poll on electricity regulation indicates that 0.85 surveyed believes governments should regulate electricity. In a random survey of 1000 Californians on their views of electricity regulation, what is the probability that the survey results are different than the national result by three percentage points if the standard deviation of the state survey is 0.0113?

First, we need to draw the probability distribution with N (0.85, 0.0113). Difference of three percentage points means $X \le 0.82$ or $X \ge 0.88$. We need to find the probability in this range.



Recall the TI-83 Plus function of normalcdf. We can use it to find the $P(0.82 \le X \le 0.88)$.

$\boldsymbol{P(0.82 \le X \le 0.88)}$ = normalcdf (0.82, 0.88, 0.85, 0.0113) = **0.9920659522**

$P(X \le 0.82 \text{ or } X \ge 0.88) = 1 - P(0.82 \le X \le 0.88)$
$P(X \le 0.82 \text{ or } X \ge 0.88) = 1 - 0.992065922$

$$\boldsymbol{P(X \le 0.82 \text{ or } X \ge 0.88) = 0.00793 = 0.793\%}$$

7.1 Assignment
pg. 373–374 #7.1 to 7.3; pg. 379 #7.4 and 7.5
pg. 380–384 #7.7 to 7.9, 7.13 and 7.15

7.2A: Mean of Random Variable and the Law of Large Numbers

Payoff: - the amount of won or lost for each possible outcome of an event.

Expected Value (Mathematical Expectation) *E*: - the **Average Winning** for each time an event occurs.

Mean of Discrete Random Variable (μ_X): - the **Expected Value** when the random variables are ran through infinite amount of trials.

When probability of each outcome is different:

$$E(X) = [P(A) \times (\text{Payoff for } A)] + [P(B) \times (\text{Payoff for } B)] + [P(C) \times (\text{Payoff for } C)] + \ldots$$

Mean of Discrete Random Variable (when each outcome is different):

$$E(X) = \mu_X = x_1p_1 + x_2p_2 + x_3p_3 + \ldots + x_kp_k = \Sigma x_ip_i$$

where k = total number of outcomes

When the probability of each outcome is the same:

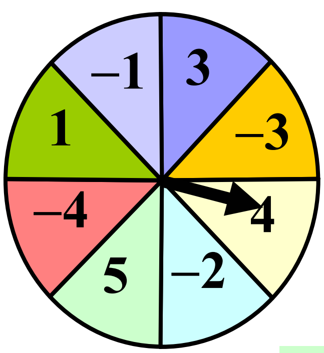
$$E(X) = \frac{\text{Sum of Payoffs of ALL Outcomes}}{\text{Total Number of Outcomes}}$$

Mean of Discrete Random Variable (when each outcome is the same):

$$E(X) = \mu_X = \frac{x_1 + x_2 + x_3 + \ldots + x_k}{k} = \frac{\sum x_i}{k}$$

Example 1: Find the expected value of the following spinners. The payoff of event is indicated on the sector of the circle.

a.



Since the probability of each sector is the same,

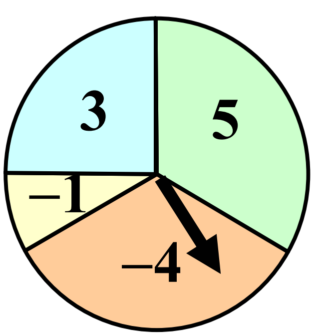
$$E(X) = \frac{\text{Sum of Payoffs of ALL Outcomes}}{\text{Total Number of Outcomes}}$$

$$E(X) = \frac{3+(-3)+4+(-2)+5+(-4)+1+(-1)}{8}$$

$$E(X) = \frac{3}{8}$$

We can expect that the **average payoff is 0.375 point per spin.**

b.



Since the probability of each sector is different,

$$E(X) = P(A)\times(\text{Payoff } A) + P(B)\times(\text{Payoff } B) + P(C)\times(\text{Payoff } C) + P(D)\times(\text{Payoff } D)$$

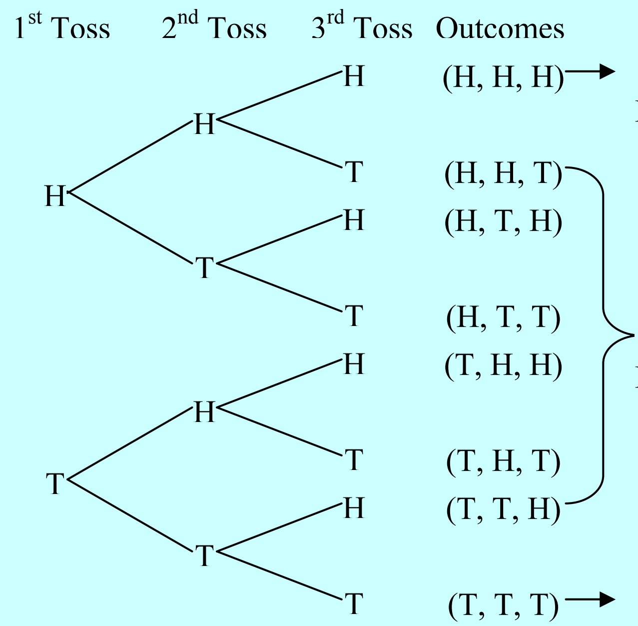
The sectors have the fractions $\frac{1}{4}, \frac{1}{3}, \frac{1}{3}$ and $\frac{1}{12}$ that add up to 1.

$$E(X) = \left(\frac{1}{4}\right)(3) + \left(\frac{1}{3}\right)(5) + \left(\frac{1}{3}\right)(-4) + \left(\frac{1}{12}\right)(-1) = \frac{3}{4} + \frac{5}{3} + \left(\frac{-4}{3}\right) + \left(\frac{-1}{12}\right)$$

$$E(X) = 1$$

We can expect that the **average payoff is 1 point per spin**.

Example 2: Three coins are tossed. He or she wins 5 points if all of the same kind appears; otherwise, the player losses 3 points. Using a tree diagram to determine the expected value.



$P(\text{all H}) = \frac{1}{8}$

Payoff of all H = 5

$P(\text{2H or 2T}) = \frac{6}{8} = \frac{3}{4}$

Payoff of 2H or 2T = –3

$P(\text{all T}) = \frac{1}{8}$

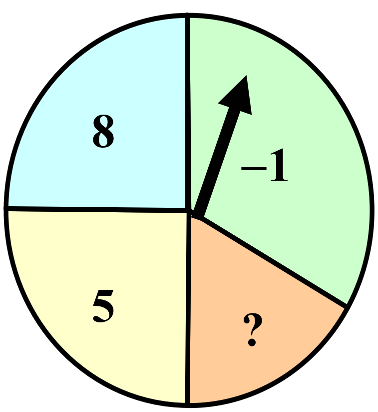
Payoff of all T = 5

$$E(X) = \left(\frac{1}{8}\right)(5) + \left(\frac{3}{4}\right)(-3) + \left(\frac{1}{8}\right)(5) = \frac{5}{8} + \left(\frac{-9}{4}\right) + \frac{5}{8}$$

$E(X) = -1$

We can expect, **on average, losing 1 point per spin**.

Example 3: If the expected value for a spin on the spinner below is 2, how many points should be awarded in the last sector?



Let x = payoff of the unknown sector

$P(\text{unknown sector}) = 1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{4} = \frac{1}{6}$

$E(X) = 2$

$$\left(\frac{1}{3}\right)(-1) + \left(\frac{1}{4}\right)(5) + \left(\frac{1}{4}\right)(8) + \left(\frac{1}{6}\right)(x) = 2$$

$$\left(\frac{-1}{3}\right) + \left(\frac{5}{4}\right) + 2 + \left(\frac{x}{6}\right) = 2$$

$$\frac{35}{12} + \frac{x}{6} = 2$$

$$\frac{x}{6} = 2 - \frac{35}{12}$$

$$\frac{x}{6} = \frac{-11}{12}$$

$$12x = -66$$

$$x = \frac{-66}{12}$$

$x = -5.5$

Example 4: In a game involving rolling two dice, a player pays \$1 per game. If the sum of a roll is 7 or if the two dice turn up with the same number, the player loses. If the sum is 3 or less, or 10 or more, the player doubles the money (\$2 back, which means \$1 gain). Any other sums mean a "push". The player gets the \$1 back, which means \$0 gain. Using a table, find the probability of the win, push and lose of this game. Evaluate if the game is fair.

		Second Dice					
	Sum	**1**	**2**	**3**	**4**	**5**	**6**
First Dice	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

$$P(\text{sum of 7 or same number dice}) = \frac{6}{36} + \frac{6}{36}$$

$$P(\text{sum} = 7 \text{ or same } \#) = \frac{1}{3}$$

Payoff of Sum of 7 and two dice with same number = –\$1

$$P(\text{sum} \le 3 \text{ or sum} \ge 10) = \frac{2}{36} + \frac{4}{36} \qquad P(\text{sum} \le 3 \text{ or sum} \ge 10) = \frac{1}{6}$$

(NOT including 1&1, 5&5, and 6&6)

Payoff for sum ≤ 3 or sum ≥ 10 = \$1

$$P(\text{other sums}) = 1 - \frac{1}{3} - \frac{1}{6} \qquad P(\text{other sums}) = \frac{1}{2}$$

Payoff for other sums = \$0

$$E(X) = \left(\frac{1}{3}\right)(-1) + \left(\frac{1}{6}\right)(1) + \left(\frac{1}{2}\right)(0) = \left(\frac{-1}{3}\right) + \left(\frac{1}{6}\right)$$

$$\boldsymbol{E(X) = -\frac{1}{6} = -\$0.166\ldots}$$

Since it costs \$1 to play and the expected value is about –0.17, the player will expect to lose on average \$1 – (–\$0.16) = \$1.16 to the dealer. <u>Therefore, it is NOT a fair game.</u>

<u>**Law of Large Numbers**</u>: - with a **large enough sample size or trials** drawn from a randomized fashion, the <u>**observed mean**</u> **($\bar{x}$)** will in time approach the <u>**mean of the population**</u> **(μ)**.

<u>**Law of Large Numbers**</u>

$$\mu \approx \bar{x} = \mu_X = x_1p_1 + x_2p_2 + x_3p_3 + \ldots + x_kp_k = \Sigma x_ip_i \quad \textbf{when } k = \infty \textbf{ (infinity)}$$

<u>**Law of Small Numbers**</u>: - when the sample size or number of trials is small, it is insignificant in determining the mean of the population because the observed mean is calculated by an extremely small fraction of an otherwise large sample.

Example: The belief that a tail has a high probability of showing up next when the previous 10 tosses of a fair coin were all heads. (Each toss is an independent event, and therefore the *P* (tail) = ½.)

Example: The belief that if a hockey team was having a good run of the last 5 games, that they will have a better chance of winning the next game. (The team's performance as a function of probability is determined by its long-term record with the same team composition. The 5 games winning streak is a small number compared the number of games the team will play in a season, not to mentioned winning records of playoffs.)

<u>**7.2A Assignment**</u>
pg. 389 #7.17 to 7.19
pg. 394–395 #7.21 and 7.23

7.2B: Rules of Means and Variances of Random Variables

Rules for Means: - when there are two independent random variables (**X and Y**), the total means (μ_{X+Y}) are simply the sum of the means of both random variables ($\mu_X + \mu_Y$).
- when the values of a particular random variable are increased by a constant factor (***a* or *b***), the mean of this random variable will also be increased by the same factor. ($\mu_{a+X} = a + \mu_X$ **and** $\mu_{bX} = b\mu_X$).

Rules for Means for Independent Random Variables

1. **Addition of Means:** $\mu_{X+Y} = \mu_X + \mu_Y$
2. **Constant Multiple:** $\mu_{bX} = b\mu_X$
3. **Constant Addition:** $\mu_{a+X} = a + \mu_X$

Example 1: A trip to the bank involves a car ride, waiting in line and processing the paperwork at the bank. Suppose the mean for the time of a car ride, waiting in line at the bank and processing the paperwork are 12.5, 6.2, and 3.7 minutes respectively. Assuming that all of the above variables are independent of each other.

a. Determine the combined mean for the entire trip to the bank.
b. If there was a delay due to road construction such that it took twice as long to get to the bank and the wait was increased by 5 minutes due to increase bank customers during the lunch hour, what is the new mean for the entire trip to the bank?

a. Let X = Time of Car Ride, Y = Time Waiting in Line, and Z = Time to Process Paperwork

$\mu_{X+Y+Z} = \mu_X + \mu_Y + \mu_Z = 12.5\text{ min} + 6.2\text{ min} + 3.7\text{ min}$ $\quad \mathbf{\mu_{X+Y+Z} = 22.4\text{ min}}$

b. Let $2X$ = Delayed Time of Car Ride, $Y + 5$ = Increased Time Waiting in Line, and Z = Time to Process Paperwork

$\mu_{2X+(Y+5)+Z} = 2\mu_X + (\mu_Y + 5) + \mu_Z = 2\ (12.5\text{ min}) + (6.2\text{ min} + 5\text{ min}) + 3.7\text{ min}$

$$\mathbf{\mu_{2X+(Y+5)+Z} = 39.9\text{ min}}$$

Variance of Discrete Random Variable (σ^2_X): - similar to variance of a sample (s^2), variance of discrete random variable is an average measure of how the variable is spread out about the mean (μ_X).

Standard Deviation of Discrete Random Variable (σ_X): - the square root of the variance of discrete random variable (σ^2_X).
- it measures the variability of the variable in a distribution and it's most commonly used in normal density curve with the mean (μ_X).

Variance of Discrete Random Variable

$$\text{Var}(X) = (\sigma_X^2) = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + (x_3 - \mu_X)^2 p_3 + \ldots + (x_k - \mu_X)^2 p_k$$

$$\text{Var}(X) = \sigma^2_X = \Sigma(x_i - \mu_X)^2 p_i$$

where k = total number of outcomes

Standard Deviation of Discrete Random Variable

$$\sigma_X = \sqrt{\sigma_X^2} = \sqrt{p_i \sum (x_i - \mu_X)^2}$$

Example 2: Using the game in Example 4 of Section 7.2 A, the following table is constructed. Adjusting with the $1 needed for the player to play this game, calculate the variance and the standard deviation of this particular game.

x_i	p_i	$x_i p_i$	$(x_i - \mu_X)^2 p_i$
−$1	$\frac{1}{3}$	$-\$\frac{1}{3}$	$\left(-1-\left(-\frac{1}{6}\right)\right)^2\left(\frac{1}{3}\right) = 0.2314814815$
$0	$\frac{1}{2}$	$0	$\left(0-\left(-\frac{1}{6}\right)\right)^2\left(\frac{1}{2}\right) = 0.0138888889$
$1	$\frac{1}{6}$	$\$\frac{1}{6}$	$\left(1-\left(-\frac{1}{6}\right)\right)^2\left(\frac{1}{6}\right) = 0.2268518519$
		$\mu_X = -\$\frac{1}{6}$	$\sigma^2_X = \Sigma(x_i - \mu_X)^2 p_i$ $\sigma^2_X = 0.4722222222$

$$\sigma_X = \sqrt{\sigma_X^2} = \sqrt{0.47222222222} \qquad \sigma_X = 0.687$$

Rules for Variances: - when there are two independent random variables (**X and Y**), the total variance (σ^2_{X+Y}) or the difference (σ^2_{X-Y}) in variance are simply the sum of the variances of both random variables ($\sigma^2_X + \sigma^2_Y$). This is because variance is calculated by the square of the difference between the individual outcome and the mean, $\Sigma(x_i - \mu_X)^2 p_i$.

- **in general, as one combined variances whether it is a total or difference, the resulting variance gets larger. (More Random Variables means Larger Variance).**
- when the values of a particular random variable are **increased by a constant multiple (b)**, the variance of this random variable will also be **increased by the square of the same multiple** ($\sigma^2_{bX} = b^2\sigma^2_X$).
- the increase on the random variable by **addition of a constant (a)** **does NOT affect** the **overall variance** ($\sigma^2_{a+X} = \sigma^2_X$).
- **the combined standard deviation (σ_{X+Y}) can be calculate AFTER the variances have been combined or modified. (Do NOT ADD OR MULTIPLE Standard Deviations!).**

Rules for Variances for Independent Random Variables

1. **Addition or Difference of Variances:** $\sigma^2_{X+Y} = \sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$
2. **Constant Multiple:** $\sigma^2_{bX} = b^2\sigma^2_X$
3. **Constant Addition:** $\sigma^2_{a+X} = \sigma^2_X$ **(NO Change on Variance)**

Do NOT Add or Multiply Standard Deviation (σ_X).
Go through Variance first than Square Root!

Example 3: The following table below shows the means and standard deviations from Example 1 in this section. Again, assuming that all of the above variables are independent of each other.

Driving to the Bank	$\mu_X = 12.5$ min	$\sigma_X = 2.8$ min
Waiting in Line	$\mu_Y = 6.2$ min	$\sigma_Y = 1.6$ min
Processing Paperwork	$\mu_Z = 3.7$ min	$\sigma_Z = 0.9$ min

a. Determine the combined variance and standard deviation for the entire trip to the bank.
b. Suppose again, the drive to the bank is doubled and the wait in line was 5 minutes longer, what are the new variance and standard deviation of the trip to the bank?

a. First, we have to <u>calculate the variance of each random variable</u> by <u>squaring the standard deviations given</u>.

$\sigma^2_X = (2.8)^2 = 7.84$ $\quad$ $\sigma^2_Y = (1.6)^2 = 2.56$ $\quad$ $\sigma^2_Z = (0.9)^2 = 0.81$

$\sigma^2_{X+Y+Z} = \sigma^2_X + \sigma^2_Y + \sigma^2_Z = 7.84 + 2.56 + 0.81$ $\quad$ $\sigma^2_{X+Y+Z} = 11.21$

$\sigma_{X+Y+Z} = \sqrt{11.21}$ $\quad$ $\sigma_{X+Y+Z} = 3.348$

b. We have to <u>examine the changes of the variances in question before combing them</u>.

$\sigma^2_{2X} = 2^2(2.8)^2 = 31.36$ $\quad$ $\sigma^2_{Y+5} = (1.6)^2 = 2.56$ (No Change) $\quad$ $\sigma^2_Z = (0.9)^2 = 0.81$

$\sigma^2_{2X+(Y+5)+Z} = 2^2\sigma^2_X + \sigma^2_{Y+5} + \sigma^2_Z = 31.36 + 2.56 + 0.81$ $\quad$ $\sigma^2_{2X+(Y+5)+Z} = 34.73$

$\sigma_{2X+(Y+5)+Z} = \sqrt{34.73}$ $\quad$ $\sigma_{2X+(Y+5)+Z} = 5.893$

<u>7.2B Assignment</u>
pg. 397 #7.24; pg. 402–403 #7.25 to 7.28; pg. 404–405 #7.29 to 7.32

<u>Chapter 7 Review</u>
pg. 406–410 #7.34 to 7.37, 7.39, 7.41, 7.42, 7.44

Chapter 8: The Binomial and Geometric Distributions

8.1A: The Binomial Distributions

Binomial Setting: - a sample space with a **finite number of trials (n)** where there is only **two outcomes for each trial (favorable and non-favorable or success and failure)**.
- the **probability of success (p)** of each trial is the **same and independent** of each other.

Binomial Random Variable (X): - is defined as the number of trials that was successful or favourable.
- the range of X is all whole numbers between 0 to n, where n represents the total number of trials.

Examples: tossing a coin, boy or girl, pass or fail, rolling a dice if specified a particle number versus not that number.

Binomial Distribution: - the probability distribution of a binomial random variable from **0 to n number of successes**.
- there is a **total of (n + 1) number of outcomes** for a binomial setting with n number of trials.
- commonly denotes as $B(n, p)$.

Example 1: Determine if the following are binomial distribution.

a. Having children until a boy is born where X = number of children.
b. The number of correct responses in a 30 multiple choice questions test with 5 choices for each question.
c. A survey of a proposition for an upcoming election where the only responses are yes or no.

a. Having children until a boy is born where X = number of children.

This is NOT a binomial distribution since in this case, X cannot be a range of numbers but rather the number of trials (n).

b. The number of correct responses in a 30 multiple choice questions test with 5 choices for each question.

This is a Binomial Distribution since there are two possible results from each question (right or wrong), X = whole numbers between 0 to 30, all questions have the same success probability, which is p = 0.2

c. A survey of a proposition for an upcoming election where the only responses are yes or no.

This is a Binomial Distribution since there are two results from each respondent (yes or no), X = whole numbers between 0 to n, all questions have the same success probability, which is p = 0.5

Binomial Probability (**$P(X)$**): - the individual probability of each possible outcome of a binomial distribution.

> **For Binomial Distribution where $\{X \mid 0 \le X \le n,\ n \in W\}$,**
> **P(at least 1) = $P(1 \le X \le n) = 1 - P(0)$**

Probability Distribution Function (**pdf**): - a mathematical function (formula) to calculate all probabilities for each value of X.

binompdf (Binomial Probability Distribution Function):

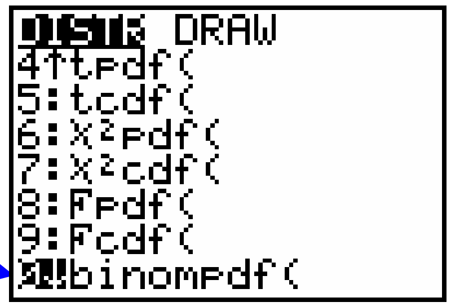
- displays a binomial probability distribution when the number of trials and the theoretical probability of the favourable outcome are specified.

$B(n, p)$ = binompdf (Number of Trials, Theoretical Probability of Favorable Outcome)

To access binompdf:

1. Press **2nd** **DISTR VARS**

2. Select Option 0



To Calculate the Binomial Probability of a Particular Outcome, X:

$P(X) = B(n, p, X)$ = binompdf (Number of Trials, Theoretical Probability of Favorable Outcome, Particular Outcome Desire)

Cumulative Distribution Function (**cdf**): - a mathematical function (formula) to sum up all probabilities for a specified range of X.

binomcdf (Binomial Cumulative Distribution Function):

- displays the sum of a specified range of binomial probabilities when the number of trials and the theoretical probability of the favourable outcome are defined.

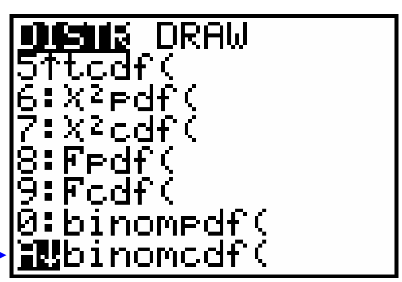
To Calculate the Binomial Cumulative Probabilities of a range of Outcome from 0 to X:

$P(0 \text{ to } X)$ = binomcdf(n, p, X) = binomcdf (Number of Trials, Theoretical Probability of Favorable Outcome, 0 to Particular Outcome Desire)

To access binomcdf:

1. Press **2nd** **DISTR VARS**

2. Select Option A



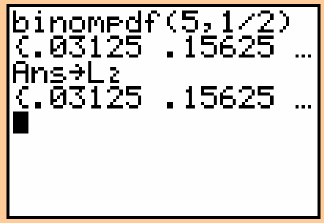
Example 2: Using your graphing calculator, determine the probabilities of having any number of girls in a family of 5 children.

1. binompdf (5, ½)

2. Store answer in L_2 of the STAT Editor.

STO→

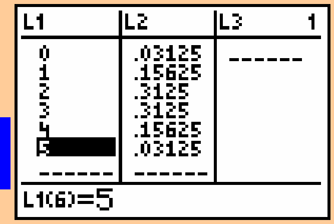
2nd | L2 / 2



3. Enter 0 to 5 in L_1 of the STAT Editor.

STAT

ENTER



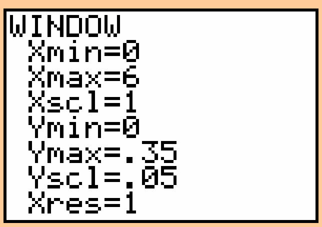
4. WINDOW Settings

x: [x_{min}, x_{max}, x_{scl}] = x: [0, 6, 1]

y: [y_{min}, y_{max}, y_{scl}] = y: [0, 0.35, 0.05]

WINDOW

Select a number slightly higher than the maximum in L_2.



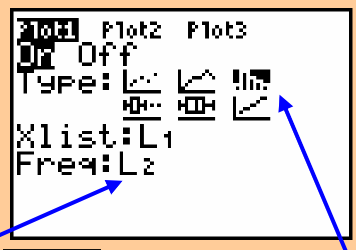
Must be 1 more than the number of trials.

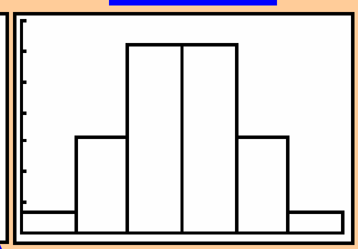
5. Select Histogram in STAT PLOT and graph.

2nd | STAT PLOT / Y= | ENTER

GRAPH

Plot1 is ON.





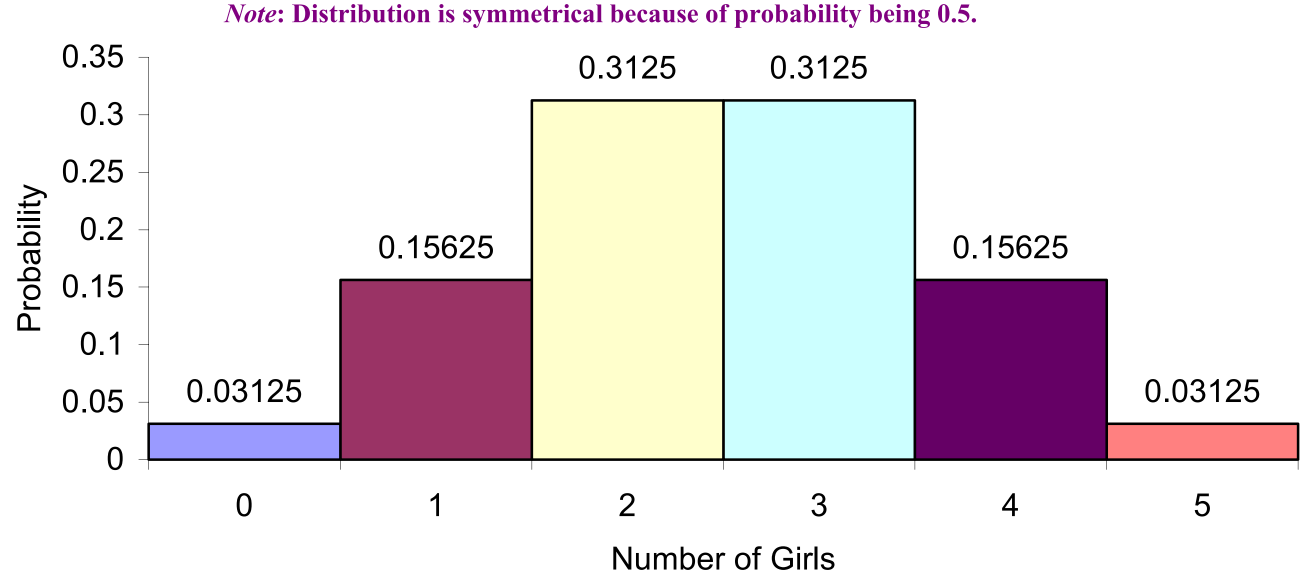
Type in L_2 as Frequency by pressing: 2nd | L2 / 2

Select Histogram

6. Transfer the graph on the calculator to paper.

Probability of the Number of Girls in a Family of 5 Children

***Note*: Distribution is symmetrical because of probability being 0.5.**

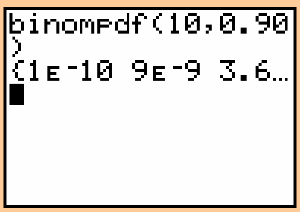


Example 3: The first week of February marks the tradition of Groundhog Day. If the groundhog sees its own shadow, it means 6 more weeks of winter. Otherwise, spring is just around the corner. Recent statistics has shown that the groundhog sees its shadow 90% of the time on Groundhog Day.

a. Graph a binomial distribution to illustrate the probability that the groundhog will see its shadow for the next ten years.
b. Find the probability that the groundhog will see its shadow 9 time out of the ten years.
c. Calculate the probability that the groundhog will see its shadow at least 6 times out of the next 10 years.
d. Determine the probability that "spring is just around the corner" at least 8 years out of the next ten years.

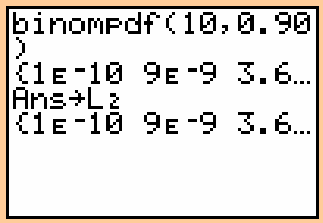
a. Graph Binomial Distribution

1. binompdf (10, 0.90) 2. Store answer in L_2 of the STAT Editor. 3. Enter 0 to 10 in L_1



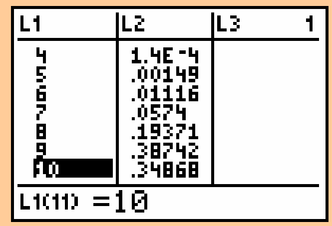
STO→

2nd **L2** **2**



STAT

ENTER

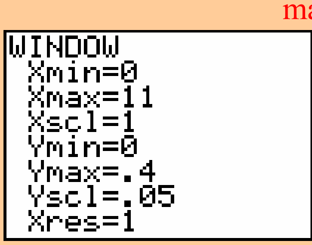


4. WINDOW Settings

x: [x_{min}, x_{max}, x_{scl}] = x: [0, 11, 1]
y: [y_{min}, y_{max}, y_{scl}] = y: [0, 0.4, 0.05]

Select a number slightly higher than the maximum in L_2.

WINDOW

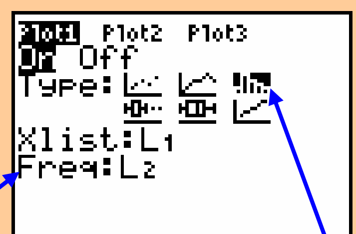


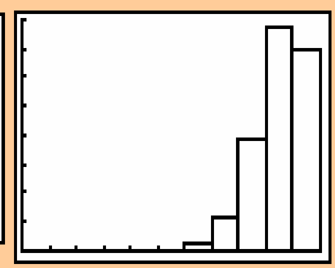
Must be 1 more than the number of trials.

5. Select Histogram in STAT PLOT and graph.

2nd **STAT PLOT / Y=** **ENTER** **GRAPH**

Plot1 is ON.



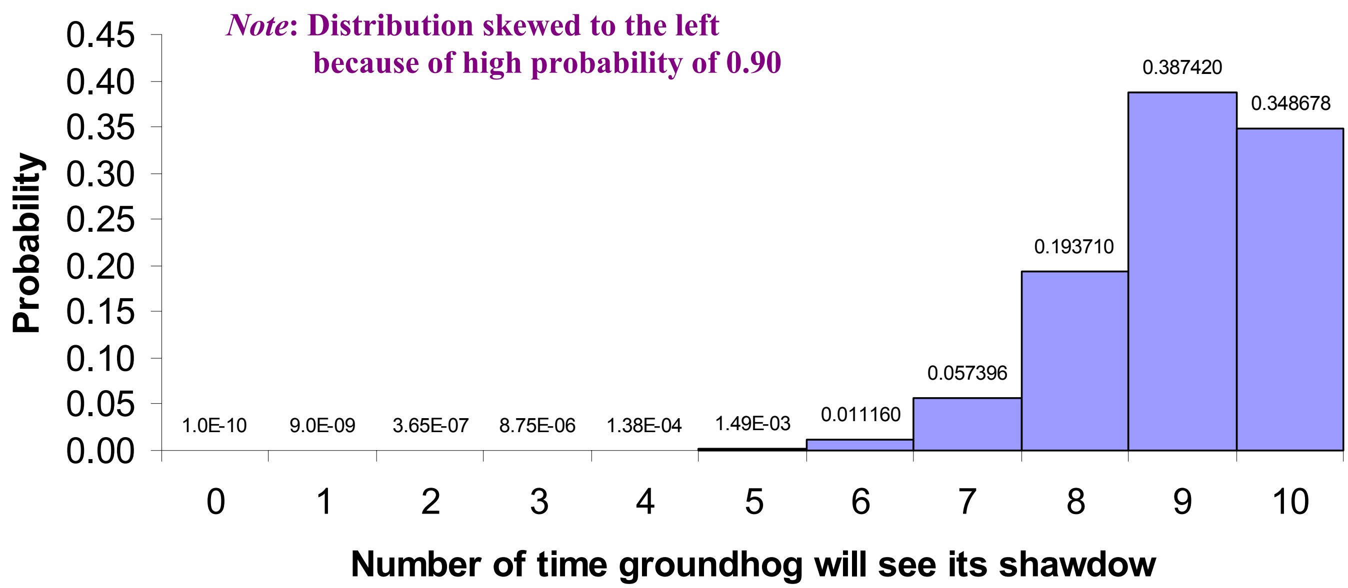


Type in L_2 as Frequency by pressing: **2nd** **L2** **2**

Select Histogram

6. Transfer the graph on the calculator to paper.

Groundhog Day Predictions for the next Ten Years



b. ***P*** **(Sees Shadow 9 times out of 10)**

$P(X = 9)$ = binompdf (10, 0.9, 9)

```
binompdf(10,0.9,
9)
          .387420489
```

$P(X = 9) = 0.387420 \approx 38.7\%$

(or read from Table or TRACE on Graph)

c. ***P*** **(Shadow at least 6 times)**

$P(6 \le X \le 10) = 1 - P(0 \le X \le 5)$
$P(6 \le X \le 10) = 1 -$ binomcdf (10, 0.9, 5)

```
1-binomcdf(10,0.
9,5)
        .9983650626
```

$P(6 \le X \le 10) = 0.998364 \approx 99.8\%$

d. ***P*** **(No Shadow at least 8 times) =** ***P*** **(Shadow at most 2 times)**

P (Shadow at most 2 times) = $P(0 \le X \le 2)$ = binomcdf (10, 0.9, 2)

```
binomcdf(10,0.9,
2)
           3.736E-7
```

P **(No Shadow at least 8 times) =** 3.736×10^{-7} **= 0.000 03736%**

8.1A Assignment

pg. 418 #8.1 to 8.4; pg. 423–424 #8.5 to 8.7

8.1B: The Binomial Formulas and Simulations

Factorial (***n*!**): - the number of ways to arrange *n* elements when they are in order.

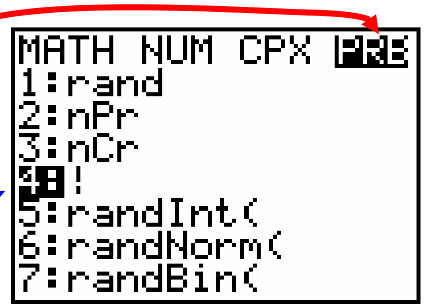
Factorial

$$n! = n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 3 \times 2 \times 1$$

where $n \in W$ $0! = 1$

To access $n!$:

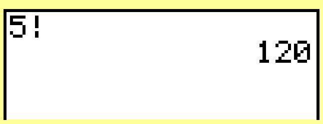
1. Press **MATH**
2. Press ▶ three times
3. Select Option 4



Example 1: Simplify or evaluate the following factorial expressions.

a. $5!$

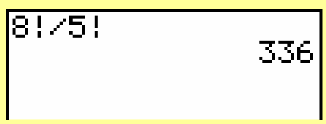
$5! = 5 \times 4 \times 3 \times 2 \times 1$



$5! = 120$

b. $\frac{8!}{5!}$

$$\frac{8!}{5!} = \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}} = 8 \times 7 \times 6$$



$\frac{8!}{5!} = 336$

c. $\frac{n!}{(n-2)!}$

$$\frac{n!}{(n-2)!} = \frac{n \times (n-1) \times \cancel{(n-2)!}}{\cancel{(n-2)!}}$$

$$= n\,(n-1)$$

$$\frac{n!}{(n-2)!} = n^2 - n$$

Combinations ($_nC_r$): - the number of ways *r* restrictions can be chosen from *n* elements where the order is unimportant.
- sometimes phrased as **"*n* choose *r*"**.

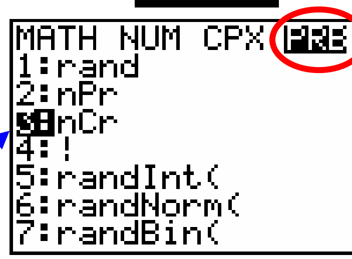
Binomial Coefficient ($_nC_k$): - the number of ways to *k* successes can be chosen form *n* trials disregarding order in a binomial experiment.
- it is an application of combinations in binomial probabilities.

Combinations

$$_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

n = total number of elements
r = number of restrictions
$r \le n$ where n and $r \in W$

To access $_nC_r$: **MATH**



Select Option 3

Binomial Coefficient

$$_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

n = total number of trials
k = number of successes
$k \le n$ where n and $k \in W$

Example 2: Evaluate the following binomial coefficients.

a. ${}_{8}C_{3}$

$${}_{8}C_{3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!\ 5!}$$

```
8 nCr 3
                56
```

$${}_{8}C_{3} = 56$$

b. ${}_{7}C_{7}$

$${}_{7}C_{7} = \frac{7!}{7!(7-7)!} = \frac{\cancel{7!}}{\cancel{7!}\ 0!} = \frac{1}{1}$$

```
7 nCr 7
                 1
```

$${}_{7}C_{7} = 1$$

c. ${}_{8}C_{5}$

$${}_{8}C_{5} = \frac{8!}{5!(8-5)!} = \frac{8!}{5!\ 3!}$$

```
8 nCr 5
                56
```

$${}_{8}C_{5} = 56$$

Notice ${}_{8}C_{3} = {}_{8}C_{5}$

Binomial Probability Formula

$$P(X = k) = \binom{n}{k} p^{k} (1-p)^{n-k}$$

***n* = total number of trials** ***X* = *k* = number of successes** **$k \le n$ where *n* and $k \in W$**

***p* = probability of success for each trial** **$(1 - p) = q$ = probability of failure for each trial**

$P(X = k)$ = binomial probability of *X* success for *n* trails

Example 3: It is estimated that 15% of the population suffers some form of mental illness at one point of their lives. In a small company that employs 20 workers,

a. determine the probability that exactly 2 employees suffer or had suffered some form of mental illness using the binomial probability formula and verify your answer using binompdf.

b. calculate the probability that at least 1 employee suffers or had suffered some form of mental illness using the binomial probability formula and verify your answer using binompdf.

a. $P(X = 2) = ?$

$n = 20$ $p = 0.15$ $(1-p) = 0.85$ $k = 2$

$$P(X = k) = \binom{n}{k} p^{k} (1-p)^{n-k}$$

$$P(X = 2) = \binom{20}{2} (0.15)^{2} (0.85)^{20-2}$$

$$P(X = 2) = {}_{20}C_{2}\ (0.15)^{2}\ (0.85)^{18}$$

Calculate and Verify using binompdf

```
20 nCr 2(0.15)^2
(0.85)^18
        .2293384019
binompdf(20,0.15
,2)
        .2293384019
```

$$P(X = 2) = 0.2293$$

b. $P(X \ge 1) = 1 - P(X = 0)$

$n = 20$ $p = 0.15$ $(1-p) = 0.85$ $k = 0$

$$1 - P(X = k) = 1 - \binom{n}{k} p^{k} (1-p)^{n-k}$$

$$1 - P(X = 0) = 1 - \binom{20}{0} (0.15)^{0} (0.85)^{20-0}$$

$$1 - P(X = 0) = 1 - [{}_{20}C_{0}\ (0.15)^{0}\ (0.85)^{20}]$$

$$1 - P(X = 0) = 1 - [{}_{20}C_{0}\ (0.85)^{20}]$$

Calculate and Verify using binompdf

```
1-20 nCr 0(0.85)
^20
        .9612404689
1-binompdf(20,0.
15,0)
        .9612404689
```

$$P(X \ge 1) = 0.9612$$

Simulating Binomial Experiments:

Experimental Probability: - probability that came from a simulation such as tossing dice, coins … etc.

randBin (Random Binomial): generates and displays a random integer from a specified binomial distribution.

randBin (1, *p*, *n*) = randBin (number assigned to represent success, Theoretical Probability of favorable outcome, number of trials)

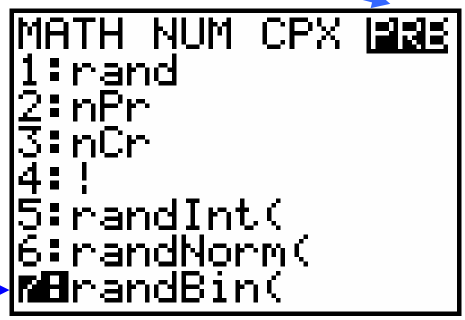
Always set to 1

To access randBin:

1. Press **MATH**

2. Use ▶ **to access PRB**

3. Select Option 7



Theoretical Probability: - probability from calculations by using sample space or other formulas.

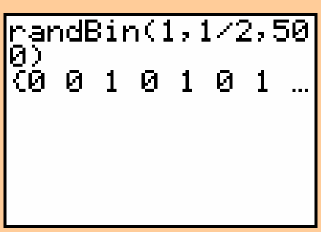
Example 4: Find the probability of obtaining tails when a coin is tossed 500 times theoretically and experimentally.

Theoretical Probability

Sample Space of Tossing a Coin = (Head), (Tail) $P\text{ (Tail)} = \dfrac{1}{2}$

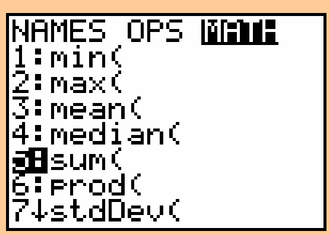
Experimental Probability using Graphing Calculator

1. randBin (1, ½, 500)



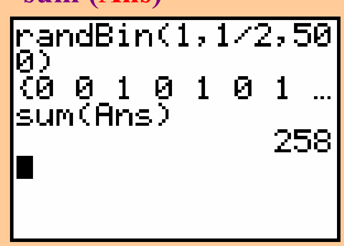
2. Summing all trails of favorable outcome.

2nd **LIST** **STAT**



2nd **ANS** **(−)**

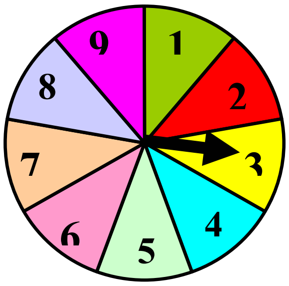
sum (Ans)



$$\text{Experimental } P\text{ (Tail)} = \frac{258}{500} = 0.516 = \frac{129}{250}$$

(Your answer might be different depending on when you last RESET your calculator.)

Example 5: Find the theoretical and experimental probability of **NOT** landing on a 5 using the spinner below if it was spun 100 times.



Theoretical Probability

$P(5) = \dfrac{1}{9}$ $\qquad P(\bar{5}) = 1 - \dfrac{1}{9}$ $\qquad P(\bar{5}) = \dfrac{8}{9}$

P (NOT 5)

Experimental Probability using Graphing Calculator

1. randBin (1, 8/9, 100) **2. Summing all simulations of favorable outcome.** **sum (Ans)**

```
randBin(1,8/9,10
0)
{1 1 1 1 1 1 1 …
```

2nd

LIST
STAT

```
NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:prod(
7↓stdDev(
```

2nd

ANS
(–)

```
randBin(1,8/9,10
0)
{1 1 1 1 1 1 1 …
sum(Ans)
              87
```

Experimental $P(\bar{5}) = \dfrac{87}{100} = 0.87$

Example 6: A world-class tennis player has an average of 60% win whenever she serves. In the most recent tennis match she served 80 times and has won 45 serves. Design an experiment that will simulate 10 matches of an upcoming tournament. What is the experimental probability that she will reproduce the same result or better compared to her last match? Contrast this result with the theoretical probability using the same parameters.

Experimental Probability ***n* = 80 serves per match; 10 matches = 10 simulations**

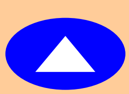
1. randBin (1, 0.60, 80), store results of each set simulation in L_1, sum L_1 and store the sum in the first cell of L_2.

randBin (1, 0.60, 80) → L_1: sum (L_1) → L_2 (1)

To access → press STO→

To access : press 2nd CATALOG 0

```
CATALOG
 )
 ⁝
 "
▸:
 π
 ?
```



a few times

Press ENTER **ten times (10 matches) and write down results as they appear.**

```
randBin(1,0.60,8
0)→L₁:sum(L₁)→L₂
(1)
                50
```

1st match

```
0)→L₁:sum(L₁)→L₂
(1)
                50
                46
                56
                44
                42
```

1st five matches

```
                44
                42
                52
                48
                46
                42
                51
```

Last five matches

There are 7 matches that she has will win 45 serves or more.

Experimental *P* ($X \geq 45$)

$\dfrac{7}{10} = 0.70$

Theoretical $P(X \geq 45) = 1 - P(X \leq 44)$

Theoretical $P(X \geq 45) = 1 -$ binomcdf (80, 0.60, 44)

Theoretical $P(X \geq 45) = 0.7885$

Mean and Standard Deviation of a Binomial Probability Distribution

When the **frequency distribution involves binomial probabilities**, we can use the following formulas to estimate the mean and standard deviation.

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}$$

where **n = number of trials** and **p = probability of favourable outcome**.

Example 7: Find the mean and standard deviation of the number of male students in a class of 35. Graph the binomial distribution.

$n = 35$ $p = 0.5$ (probability of a male student from any student)

$$\mu = np = 35(0.5)$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{(35)(0.5)(1-0.5)} = \sqrt{35 \times 0.5 \times 0.5}$$

$\mu = 17.5$ $\sigma = 2.96$

To Graph the Binomial Distribution:

1. binompdf (35, 0.5) **2. Store answer in L_2 of the STAT Editor.** **3. Enter 0 to 35 in L_1.**

```
binompdf(35,.5)
{2.910383046E-1…
```

STO→ 2nd L2 2

```
binompdf(35,.5)
{2.910383046E-1…
Ans→L2
{2.910383046E-1…
```

STAT ENTER

L1	L2	L3
29	4.7E-5	
30	9.4E-6	
31	1.5E-6	
32	1.9E-7	
33	1.7E-8	
34	1E-9	
35	3E-11	

L1(36) =35

4. WINDOW Settings

x: [x_{min}, x_{max}, x_{scl}] = x: [0, 36, 1]

y: [y_{min}, y_{max}, y_{scl}] = y: [0, 0.14, 0.01]

WINDOW

Select a number slightly higher than the maximum in L_2.

```
WINDOW
 Xmin=0
 Xmax=36
 Xscl=1
 Ymin=0
 Ymax=.14
 Yscl=.01
 Xres=1
```

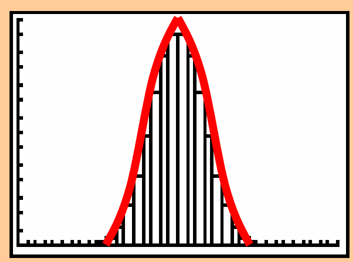
Must be 1 more than the number of trials.

5. Select Histogram in STAT PLOT and graph.

2nd STAT PLOT Y= ENTER GRAPH

Plot1 is ON.

```
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L1
Freq:L2
```



Type in L_2 as Frequency by pressing: 2nd L2 2

Select Histogram

8.1B Assignment

pg. 427 #8.8 to 8.10; pg. 429 #8.11, 8.13;
pg. 431–432 #8.15 to 8.18; pg. 433–434 #8.19 to 8.23

8.2: The Geometric Distributions

Geometric Setting: - where **the variable, X, is defined as the number of trials to realize the first success.**
- a sample space where there is only **two outcomes for each trial (favorable and non-favorable or success and failure)**.
- the **probability of success (p)** of each trial is the **same and independent** of each other.

Examples: tossing a coin until a head, keep having children until a boy is born, rolling a dice continuously until a specified number appears.

Geometric Distribution: - the probability distribution of a geometric random variable from **1 to X number of trials where $X \in N$.**
- there is an **infinite number of trials** and the **probability of each trial decreases as X increases exponentially**.

Example 1: Determine if the following are geometric distribution.

a. Guessing an answer to a single multiple-choice question with 5 choices until you get the right answer.
b. Testing each toaster coming of the same assembly line until one fails to meet specification.

a. Guessing an answer to a single multiple-choice question with 5 choices until you get the right answer.

This is NOT a geometric distribution since in this case, each time you guess on the same question, you have one less choice to choose from. Thus, the probability of each trial is not the same nor independent.

b. Testing each toaster coming of the same assembly line until one fails to meet specification.

This is a Geometric Distribution since there are two possible results from each toaster (pass or fail), X = natural numbers between 1 to infinity, all toasters have the same probability of failing the test.

Geometric Probability ($P(X = n)$): - the individual probability of each possible outcome of a geometric distribution.

Geometric Probability Formula

$$P(X = n) = (1 - p)^{n-1}p$$

$X = n$ = number of trials until success where $n \geq 1$ and $n \in N$
p = probability of success for each trial **$(1 - p) = q$ = probability of failure for each trial**
$P(X = n)$ = geometric probability of X or n trials until success is realized

geometpdf (Geometric Probability Distribution Function):

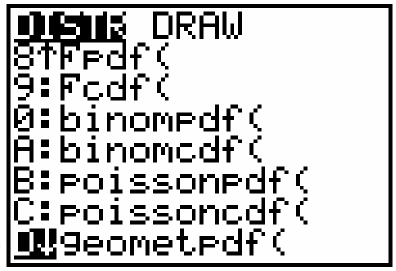
- displays a geometric probability distribution when the theoretical probability of the favourable outcome and the number of trials until success are specified

geometpdf (p, X) = geometpdf (Theoretical Probability of Success, Number of Trials Until Success)

To access geometpdf:

1. Press **2nd** **DISTR VARS**

2. Select Option D →



To Calculate the Geometric Probabilities of a Range of Trials, (1 to X): geometpdf (p, {List of Trials})

Geometric Cumulative Probabilities: - the sum of all geometric probabilities for a range of X from 1 to n.
- **the Sum of ALL Geometric Probabilities = 1**

$X = n$	Geometric Probability $P(X)$
1	p
2	$(1-p)\,p$
3	$(1-p)^2\,p$
4	$(1-p)^3\,p$
5	$(1-p)^4\,p$

Geometric Series (Sum of Geometric Sequence)

$$S_n = \frac{a(r^n - 1)}{r - 1} = P\,(X \le n)$$

a = value of 1st term = p **r = common ratio = $(1-p)$**
n = the number of terms in series

$$P\,(X \le n) = \frac{p\left((1-p)^n - 1\right)}{(1-p) - 1} = \frac{\cancel{p}\left((1-p)^n - 1\right)}{-\cancel{p}} = -[(1-p)^n - 1]$$

$$P\,(X \le n) = 1 - (1-p)^n$$

Geometric Cumulative Probabilities Formula

$$P(X \le n) = 1 - (1-p)^n \qquad \text{Sum of ALL Geometric Probability} = \sum_{i=1}^{\infty} P(x_i) = 1$$

$X \le n$ = a range of number of trials until success where $n \ge 1$ and $n \in N$
p = probability of success for each trial **$(1 - p) = q$ = probability of failure for each trial**
$P(X \le n)$ = geometric cumulative probability of a range of 1 to n trails until success is realized

geometcdf (Geometric Cumulative Distribution Function):
- displays the sum of a specified range of geometric probabilities when the theoretical probability of the favourable outcome and the list of number of trials within a range from 1 to n are defined.

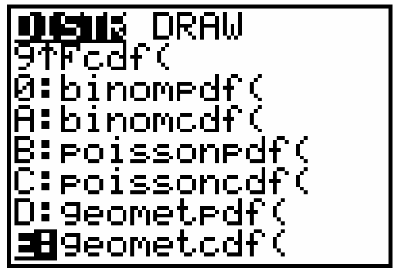
To Calculate the Geometric Cumulative Probabilities of a range of Outcome from 1 to n:

$P(X \leq n)$ = **geometcdf(p, X)**
$P(X \leq n)$ = **geometcdf (Theoretical Probability of Success, Number of Trials from 1 to X)**

To access geometcdf:

1. Press **2nd** **DISTR VARS**

2. Select Option E →



Example 2: The probability that anyone over the age of 50 had experienced heart attack at least once is 0.32. A room full of people who are over the age of 50 were asked if they had ever experienced a heart attack. You are to ask each person if they have experienced a heart attack until one admits it.

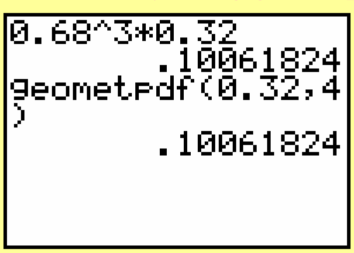
a. Using the formula and pdf of your calculator, determine the probability that the first person that experienced heart attack was the fourth person asked.
b. Construct a probability distribution graph of this experiment for the first 20 people asked.
c. Calculate the probability that **at most** four people are healthy before a respondent had a heart attack was asked. How is this question different than the one in part a?

a. $P(X = 4) = ?$

$n = X = 4 \quad p = 0.32 \quad (1 - p) = 0.68$

$P(X = n) = (1 - p)^{n-1}p$
$P(X = 4) = (1 - 0.32)^{4-1}(0.32)$
$P(X = 4) = (0.68)^3(0.32)$

Calculate and Verify using geometpdf



$P(X = 4) = 0.1006$

b. Graphing Geometric Probability Distribution

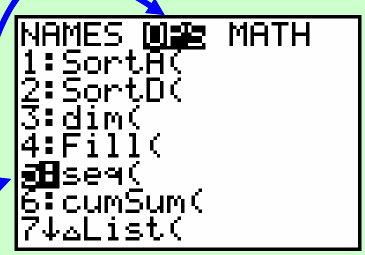
1. Type 1 to 20 in L_1 using seq (X, X, 1, 20, 1) → L_1

To access seq:

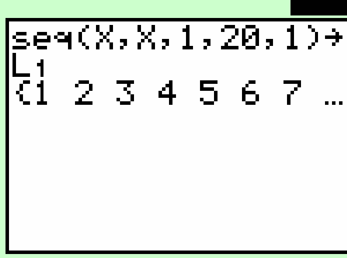
a. Press **2nd** **LIST STAT**

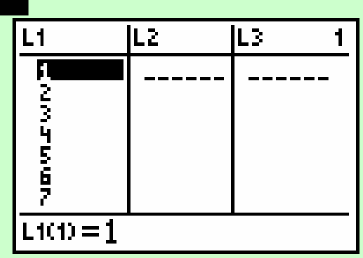
b. Use ► **to access OPS**

c. Select Option 5



To access X: **press** **X,T,θ,n**





2. Generate Geometric Probabilities and store in L_2

a. Use geometpdf (0.32, {1, 2, 3,…, 19, 20}) → L_2 OR

b. Use geometpdf (0.32, L_1) → L_2

```
seq(X,X,1,20,1)→
L1
{1 2 3 4 5 6 7 …
geometpdf(0.32,L
1)→L2
{.32 .2176 .147…
```

L1	L2	L3
1	.32	------
2	.2176	
3	.14797	
4	.10062	
5	.06842	
6	.04653	
7	.03164	

L1(1) = 1

3. WINDOW Settings

x: $[x_{min}, x_{max}, x_{scl}] = x$: [1, 21, 1]

y: $[y_{min}, y_{max}, y_{scl}] = y$: [0, 0.40, 0.05]

WINDOW

Select a number slightly higher than the maximum in L_2.

```
WINDOW
 Xmin=1
 Xmax=21
 Xscl=1
 Ymin=0
 Ymax=.4
 Yscl=.05
 Xres=1
```

Must be 1 more than the number of trials.

4. Select Histogram in STAT PLOT and graph.

2nd Plot1 is ON.

STAT PLOT

Y=

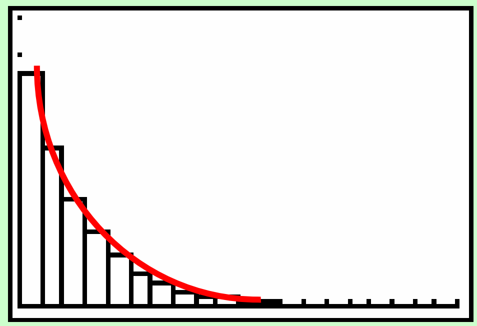
ENTER

```
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L1
Freq:L2
```

Type in L_2 as Frequency by pressing: **2nd** **L2** **2**

Select Histogram

GRAPH



Note: **The geometric distribution graph forms an exponential curve.**

c. $P(X \leq 5)$ **because at most 4 healthy people means maximum of 5 people asked (4 healthy and 1 had heart attack) = 5 trials.**

$$P(X \leq 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

$$P(X \leq 5) = 0.32 + (0.68)(0.32) + (0.68)^2(0.32) + (0.68)^3(0.32) + (0.68)^4(0.32)$$

Or Using the Formula: $p = 0.32$ and $n = 5$

$$P(X \leq n) = 1 - (1 - p)^n$$

$$P(X \leq 5) = 1 - (1 - 0.32)^5$$

$$\mathbf{P(X \leq 5) = 0.8546}$$

Calculate and Verify using geometcdf

```
1-(1-0.32)^5
         .8546066432
geometcdf(0.32,5
)
         .8546066432
```

Mean, Variance, Standard Deviation of a Geometric Probability Distribution

When the frequency distribution involves geometric probabilities,

$$\mu = \frac{1}{p} \qquad \sigma^2 = \frac{1-p}{p^2} \qquad \sigma = \frac{\sqrt{1-p}}{p}$$

μ = **mean or expected value of a geometric probability distribution**

σ^2 = **variance of a geometric probability distribution**

σ = **standard deviation of a geometric probability distribution**

p = **probability of favourable outcome**

Cumulative Geometric Probabilities of More than *n* Trials $P(X > n)$

$P(X > n) = 1 - P(X \le n) = 1 - [1 - (1 - p)^n]$
$P(X > n) = 1 - 1 + (1 - p)^n$

Cumulative Geometric Probabilities of More than *n* Trials

$$P(X > n) = (1 - p)^n$$

$X > n$ = a range of number of trials until success where $n \ge 1$ and $n \in N$
p = probability of success for each trial $(1 - p) = q$ = probability of failure for each trial
$P(X > n)$ = geometric cumulative probability of n to infinity trials until success is realized

Example 3: A company that manufactures high precision communication products finds that 98% of its newest product meets the pre-set standard. The manager decides to test each unit as it comes off the assembly line. As soon as a unit is found to be defective, the line will be shut down and engineers will analyze the process before production resumes.

a. How many units are expected to come off the assembly line before a defective one is found? Calculate its variance and standard deviation of the assembly line.
b. What is the probability that the first 50 units are produced before the line is shut down due to an appearance of a defective unit?
c. Calculate the probability that more than 50 units are produced before the line is shut down.
d. Determine the probability that at most 50 units are produced before a defective one is found.

a. Expected Number = μ = ?
$p = 0.02$ defective (98% passes)

$\mu = \dfrac{1}{p} = \dfrac{1}{0.02}$ **$\mu = 50$**

$\sigma^2 = \dfrac{1-p}{p^2} = \dfrac{1-0.02}{(0.02)^2}$ **$\sigma^2 = 2450$**

$\sigma = \sqrt{\sigma^2} = \sqrt{2450}$ **$\sigma = 49.5$**

b. $P(X = 50)$ = ?
$n = X = 50$ $p = 0.02$
$(1 - p) = 0.98$

$P(X = n) = (1 - p)^{n-1}p$
$P(X = 50) = (1 - 0.02)^{50-1}(0.02)$
$P(X = 4) = (0.98)^{49}(0.02)$

Calculate and Verify using geometpdf

```
0.98^49*0.02
        .0074320343
geometpdf(0.02,5
0)
        .0074320343
```

$P(X = 50) = 0.007432$

c. $P(X > 50)$ = ?
$p = 0.02$ defective $n = 50$

$P(X > n) = (1 - p)^n$
$P(X > 50) = (1 - 0.02)^{50}$

$P(X > 50) = 0.3642$

d. $P(X \le 51)$ because <u>at most 50 good units</u> means maximum of 51 units produced (50 good ones and 1 defective) = 51 trials.

$p = 0.02$ and $n = 51$ Calculate and Verify using geometcdf

$P(X \le n) = 1 - (1 - p)^n$
$P(X \le 51) = 1 - (1 - 0.02)^{51}$

$P(X \le 51) = 0.6431$

```
1-(1-0.02)^51
        .6431137135
geometcdf(0.02,5
1)
        .6431137135
```

***Note* that $P(X \le 51) \ne 1 - P(X > 50)$**

<u>8.2 Assignment</u>
pg. 439–440 #8.24 to 8.26;
pg. 443 #8.27 to 8.29;
pg. 444–446 #8.30 to 8.35

<u>Chapter 8 Review</u>
pg. 447–451 #8.37, 8.39, 8.41, 8.43 and 8.44

Chapter 9: Sampling Distributions

9.1: Sampling Distributions

Parameters: - any values or measures that describe the population.

Statistic: - a value or measure was obtained from data that were samples of the population.

Example 1: It was found that the mean age for all teachers in Suburbia High School was 42 years old. This is low compared to the national average taken from a recent survey to be 48 years old. Identify the statistic and parameter from the above statements.

Parameter: Mean Age of All Teachers in Suburbia High School being 42 years old.
(Population defined as all teachers in Suburbia High.)
Statistic: National Average of Teachers Age from Recent Survey to be 48 years old.
(The data was from a Sample of the Population of all Teachers in the Nation.)

Population Proportion (***p***): - a parameter that describes the proportion of the population that fits a certain category on any topic.

Example: From all income tax filed last year, the government has published that 14% of all two parents families has one parent working.

Sample Proportion ($\hat{p}$): - a statistic that describes the proportion of a sample that fits a certain category on any topic.

Example: A sample of 2500 college students was randomly chosen, and it is found that 64% have college loans to support their tuitions.

Sampling Variability: - the natural difference between the similar repeated random samples on the same survey topic.

Sampling Variability is NOT a Concern as long as:

1. The random sample is taken from a large population, and the sample size is big.
2. If repeated random samples are taken, they are of the same size and from the same population.

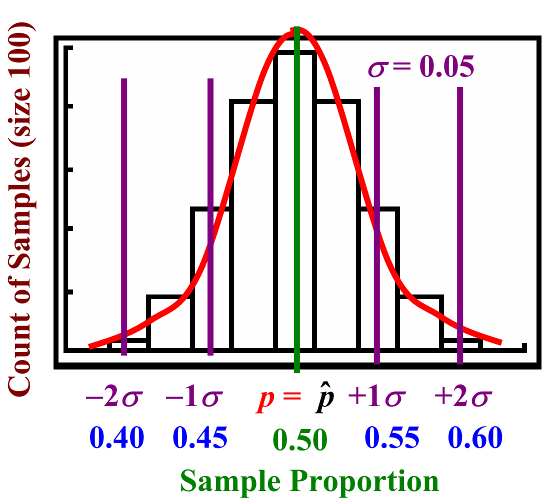
When Sampling Variability is Minimized, Sample Proportion ($\hat{p}$) = Population Proportion (*p*)

Sampling Distribution: - the distribution of the all sample proportions from repeated random sampling surveys from the same population.
- have the same features as other distributions (such as shape, outliers, centre and spread).

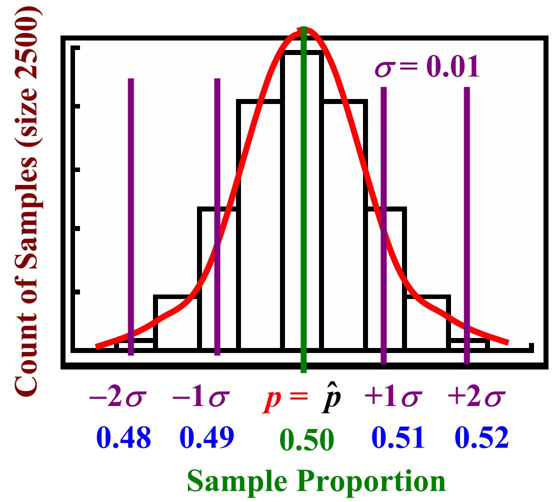
Describing Sampling Distributions:

If a fair sized sample is taken randomly from a large population, then

1. The **Overall SHAPE** of the distribution is **Symmetrical** and very similar to a **Normalized** curve.
2. There will be **No Outliers**.
3. The **CENTRE of the Sampling Distribution** is the **Sample Proportion ($\hat{p}$),** which is the same as the **Population Proportion (p).**
4. The **SPREAD of the Sampling Distribution** is the same as the **Standard Deviation** of the curve. It depends on the **Size of the Sample**. **The bigger the sample size, the smaller is the spread.**



The Standard Deviation of Sample Size of 100 for Sample Proportion of 0.50 is 0.05. This means 95% of the time (19 times out of 20), the accuracy is only within 10% of the Population Proportion.



The Standard Deviation of Sample Size of 2500 for Sample Proportion of 0.50 is 0.01. This means 95% of the time (19 times out of 20), the accuracy is only within 2% of the Population Proportion.

Example 2: Simulate an opinion poll of 40 people with the population proportion of 0.3 says "yes" using line 106 of the Random Digits Table.

a. Decide on a method to select the digits that will represent the "yes" response.
b. Repeat this simulation 20 times by using successive rows. Record the results in the table.
c. Input your data to your calculator and graph the resulting sampling distributions.
d. Determine the mean from the 20 sampling proportions. How does it compare with the actual population proportion of 0.3?

a. Since p = 0.3 or 30%, we can define digits 1, 2, and 3 as "yes", where as digits 4 to 9 and 0 as "no".

b. Line 106: 68417 35013 15529 72765 85089 57067 50211 47487 $\hat{p}$ = 10/40 = **0.25**

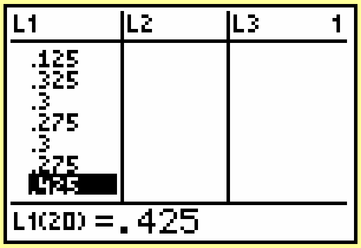
Line 107: 82739 57890 20807 47511 81676 55300 94383 14893 $\hat{p}$ = 11/40 = **0.275**

(Repeat to Line 125 and record in Table)

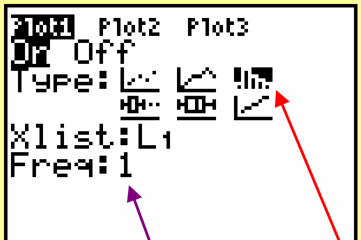
Line #	$\hat{p}$	Line #	$\hat{p}$	Line #	$\hat{p}$	Line #	$\hat{p}$
106	10/40 = **0.25**	111	10/40 = **0.25**	116	15/40 = **0.375**	121	12/40 = **0.3**
107	11/40 = **0.275**	112	8/40 = **0.2**	117	16/40 = **0.4**	122	11/40 = **0.275**
108	8/40 = **0.2**	113	13/40 = **0.325**	118	8/40 = **0.2**	123	12/40 = **0.3**
109	12/40 = **0.3**	114	13/40 = **0.325**	119	5/40 = **0.125**	124	11/40 = **0.275**
110	8/40 = **0.2**	115	15/40 = **0.375**	120	13/40 = **0.325**	125	17/40 = **0.425**

c. Input Data into Calculator and Graph Sampling Distribution.

1. Enter data in L_1

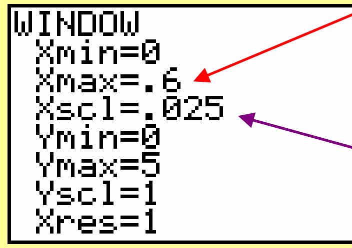


2. Turn STAT PLOT On and Set up Window



There is no L_2. (Leave it as 1)

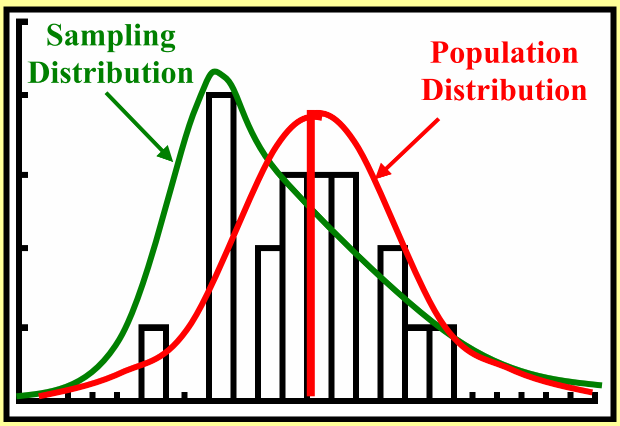
Select Histogram



Since p = 0.3, we double it for a symmetric view

The smallest interval is 1/40 = 0.025

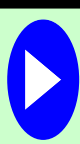
3. Graph



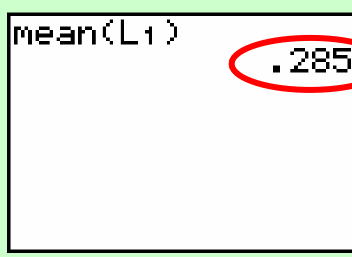
***p* = 0.30**

Notice that the <u>Sampling Distribution is Skewed to the right and NOT Normally Distributed</u>. This is due to the <u>Sample Size (40) and the Number of Repetitions (20) are relatively SMALL</u>. As they increase, the sampling distribution will become the normal distribution of the population.

d. Mean of 20 repetitions of Sample Size of 40.

1. Press  **twice**

2. Select Option 3



$\hat{p}$ = 0.285 where as ***p* = 0.30**. Again, the reason for the difference is because the <u>Sampling Distribution is Skewed to the right</u> due to <u>Small Sample Size and Small Number of Sampling Repetitions</u>.

Bias Statistic: - occurs when the **sample proportion is somewhat above or below the population proportion** due to overestimation, undercoverage or underestimation.

Unbiased Statistic: - when the **sample proportion matches very close or exactly with the population proportion**.

Variability of a Statistic: - sometimes refer to as the spread of the sampling distribution.
- **the higher the variability, the more spread out are the sampling proportions** if the samples are taken repeatedly.

Example 3: Classify the following statistics as bias versus unbiased along with their variability as high or low.

a. A state wide survey of a 100 college students has found that 30% still smokes regularly in Arizona. The survey is accurate within 9%, 19 times out of 20. A state university administration has found through their mandatory health plan that 16% of its students actually smokes.
b. Another statewide survey of 2000 college students has found the same result with 30% of the respondent smokes regularly in California. This survey is accurate within 1.8%, 19 times out of 20. The same university administration analyzed its student population still finds 16% of them smokes.
c. A poll to predict the upcoming proposition on regulating electricity in California reported 80% would vote yes. The poll uses 200 people across the state and is accurate with 5.7%, 19 times out of 20. The actual proposition result was 81% in favour of regulating electricity.
d. The same poll on the regulating electricity in California is now done with 1000 people. The same result was found to be 80% in favour and the reported accuracy is within 2.5%, 19 times out of 20. Again, the actual proposition was 81% in favour of regulating electricity.

a. High Bias ($\hat{p}$ = 30% versus p = 16%): Sampling Proportion is too far from Population Proportion.
High Variability: Standard Deviation is 4.5% (±9% at $\pm 2\sigma$ = 95% of the time; ±4.5% at $\pm 1\sigma$).

b. High Bias ($\hat{p}$ = 30% versus p = 16%): Sampling Proportion is too far from Population Proportion.
Low Variability: Standard Deviation is 0.9% (±1.8% at $\pm 2\sigma$ = 95% of the time; ±0.9% at $\pm 1\sigma$).

c. Low Bias ($\hat{p}$ = 80% versus p = 81%): Sampling Proportion is very close to Population Proportion.
High Variability: Standard Deviation is 2.85% (±5.7% at $\pm 2\sigma$ = 95% of the time; ±2.85% at $\pm 1\sigma$).

d. Low Bias ($\hat{p}$ = 80% versus p = 81%): Sampling Proportion is very close to Population Proportion.
Low Variability: Standard Deviation is 1.25% (±2.5% at $\pm 2\sigma$ = 95% of the time; ±1.25% at $\pm 1\sigma$).

***Note*: For a random sample, the Sample Size does NOT necessary affect the Sample Proportion or its Bias, but it has a Strong Effect on the Standard Deviation and thus the Variability of the distribution.**

9.1 Assignment
pg. 457 #9.1 to 9.4;
pg. 468 #9.9 and 9.10;
pg. 471 #9.13 and 9.14

9.2: Sampling Proportions

Normal Approximation of Sampling Proportion in a Binomial Setting

When an **unbiased survey** involves a binomial setting using a **categorical variable**, and the sample size along with the sampling proportion met the following conditions, we can calculate the mean and standard deviation, and use the normal approximation to determine the relative probability.

Conditions for Normal Approximation of a Sampling Proportion in a Binomial Setting.

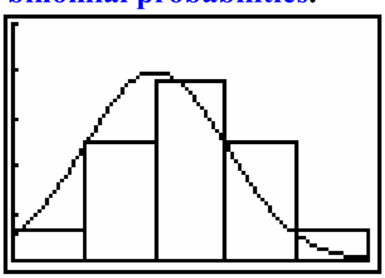
1. **The Population must be At LEAST 10 times the Sample Size ($n_{pop} \geq 10n_{sample}$).**
2. **To Ensure a good fit to the Normal Curve, $np \geq 10$ and $n(1-p) \geq 10$**

Mean and Standard Deviation of Normal Approximation of a Sampling Proportion in a Binomial Setting (Categorical Variable $n \in N$)

$$\mu_{\hat{p}} = p \qquad\qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

n = Sample Size and p = Population Proportion or Theoretical Probability

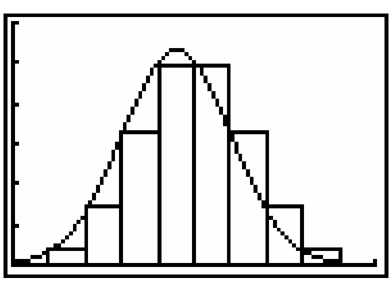
Using the bell curve to approximate a binomial distribution really depends on the number of trials, n. When n is small, there are very few bars on the binomial distribution and the bell curve does not fit the graph well. However, when n is large, the bell curve fits the binomial distribution much better. Therefore, we can use the area under normal bell curve to approximate the cumulative sum of the binomial probabilities.



$n = 4$ $\quad$ $p = 0.5$

$np = 4 \times 0.5 = 2$ (less than 10)
$n(1-p) = 4 \times (1-0.5) = 2$ (less than 10)

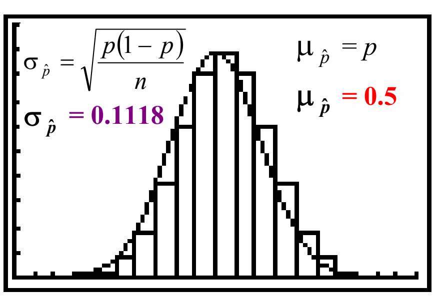
CANNOT use Normal Approximation
Bell-Curve does NOT fit the binomial distribution well.



$n = 9$ $\quad$ $p = 0.5$

$np = 9 \times 0.5 = 4.5$ (less than 10)
$n(1-p) = 9 \times (1-0.5) = 4.5$ (less than 10)

CANNOT use Normal Approximation. Bell-Curve does NOT fit the binomial distribution well. It's still not good enough.



$n = 20$ $\quad$ $p = 0.5$

$np = 20 \times 0.5 = 10$ (at least 10)
$n(1-p) = 20 \times (1-0.5) = 10$ (at least 10)

CAN use Normal Approximation. Bell-Curve fits the binomial distribution well. (MORE bars means better fit!)

Example 1: A multiple-choice test has 10 questions. Each question has 4 possible choices.

a. Determine whether the conditions for normal approximation are met.
b. Graph the resulting binomial distribution.
c. Find the probability that a student will score **exactly** 6 out of 10 on the test.
d. Calculate the probability that a student will **at least** pass the test.

a. Determining Conditions for Normal Approximation

$n = 10$ questions $\qquad p = \frac{1}{4} = 0.25$ (probability of guessing a question correct)

$np = 10 \times 0.25$ $\qquad n(1-p) = 10 \times (1 - 0.25)$

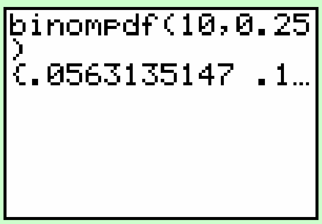
$np = 2.5$ **(less than 10)** $\qquad n(1-p) = 10 \times 0.75$

$n(1-p) = 7.5$ **(less than 10)**

Since the np and $n(1-p)$ conditions are NOT met, we CANNOT use the normal approximation.

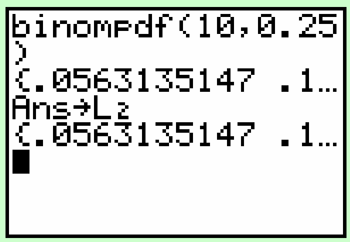
b. To Graph the Binomial Distribution:

1. binompdf (10, 0.25) **2. Store answer in L_2.** **3. Enter 0 to 10 in L_1.**



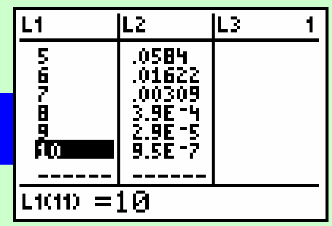
STO→

2nd **L2** **2**



STAT

ENTER



4. WINDOW Settings

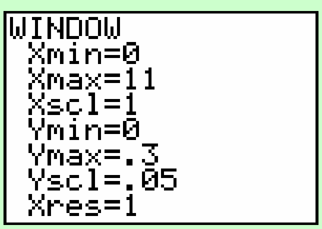
x: $[x_{min}, x_{max}, x_{scl}] = x$: **[0, 11, 1]**

y: $[y_{min}, y_{max}, y_{scl}] = y$: **[0, 0.30, 0.05]**

Select a number slightly higher than the maximum in L_2.

Must be 1 more than the number of trials.

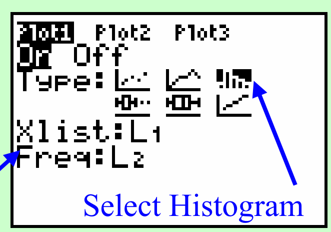
WINDOW



5. Select Histogram in STAT PLOT and Graph.

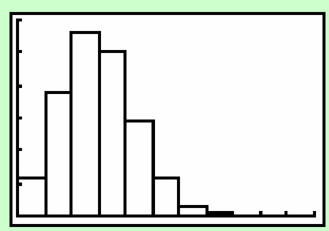
2nd **STAT PLOT** **Y=** **ENTER** **GRAPH**

Plot1 is ON.

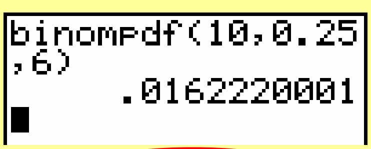


Type in L_2 as Frequency

Select Histogram



c. P (6) = binompdf (10, 0.25, 6)



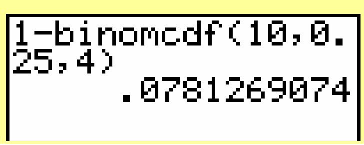
exactly 6 successes

P (6) = 0.01622

d. P (passing) = P (at least 5 out of 10) = P ($X \geq 5$)

$P(X \geq 5) = 1 - P(X \leq 4)$

$P(X \geq 5) = 1 -$ binomcdf (10, 0.25, 4)



P (passing) = 0.07813

Example 2: A multiple-choice test has 50 questions. Each question has 4 possible choices.

a. Determine whether the conditions for normal approximation are met.
b. Find the mean and standard deviation
c. Graph the resulting binomial distribution.
d. Find the probability that a student will score **exactly** 30 out of 50 on the test.
e. Calculate the probability that a student will fail the test.

a. Determining Conditions for Normal Approximation

n = 50 questions $\qquad p = \frac{1}{4} = 0.25$ (probability of guessing a question correct)

$np = 50 \times 0.25 \qquad n(1-p) = 50 \times (1-0.25)$

$np = 12.5$ **(greater than 10)** $\qquad n(1-p) = 50 \times 0.75 = 37.5$ **(greater than 10)**

Since both np and $n(1-p)$ condition are met, we **CAN** use the normal approximation.

b. Mean and Standard Deviation of Binomial Distribution (NOT Sampling Distribution)

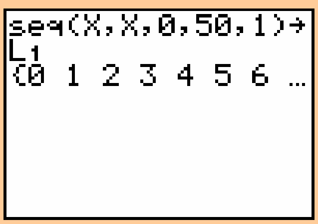
$$\mu = np = 50(0.25) = 12.5$$

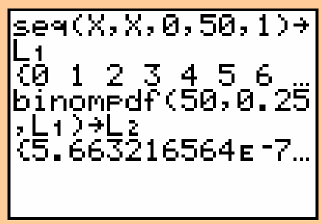
$$\sigma = \sqrt{np(1-p)} = \sqrt{(50)(0.25)(1-0.25)} = 3.061862178$$

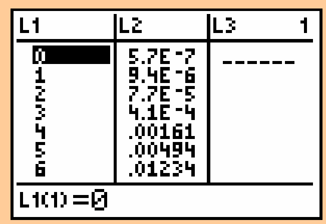
$\mu = 12.5$
$\sigma = 3.062$

c. To Graph the Binomial Distribution:

1. Enter 0 to 50 in L_1 using seq (X,X, 0, 50, 1). 2. binompdf (50, 0.25, L_1) and store answer in L_2.



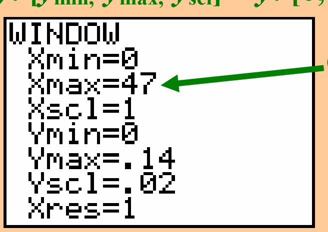




3. WINDOW Settings

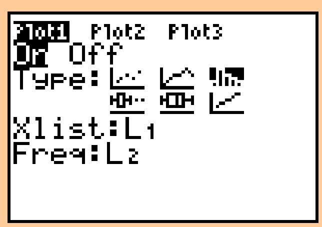
x: $[x_{min}, x_{max}, x_{scl}] = x$: [0, 47, 1]

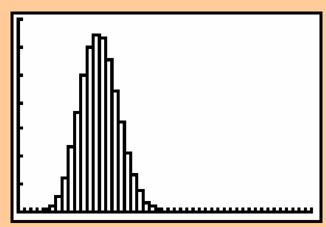
y: $[y_{min}, y_{max}, y_{scl}] = y$: [0, 0.14, 0.02]



Calculator can only handle 48 bars (0 to 47)

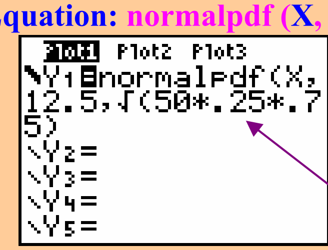
4. Select Histogram in STAT PLOT and Graph.





5. To Overlay the Binomial Distribution with a Normal Bell Curve [Enter Equation: normalpdf (X, μ, σ)]

Y=

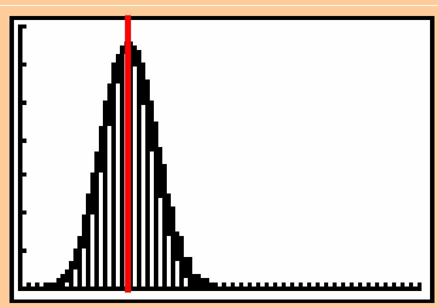


GRAPH

$\mu = 12.5$

$\sigma = \sqrt{50 \times 0.25 \times 0.75}$

use exact value for σ



$\mu = 12.5$

d. $P(30)$ = binompdf (50, 0.25, 30)

$P(30) = 1.296 \times 10^{-7}$

e. $P(\text{fail}) = P(0 \le X \le 24)$

Since Normal Approximation is allowed, we can use **normalcdf** to calculate area under the bell curve.

$P(0 \le X \le 24)$ = normalcdf (0, 24, 12.5, 3.062)

$P(0 \le X \le 24) = 0.99989$

We can also use **binomcdf (50, 0.25, 24) = 0.99987**

Example 3: Suppose that 85% of underage college students drink alcohol regularly.

a. What is the probability that in a simple random survey of 100 students the sample proportion with be within ±5 percentage points?
b. Determine the sample size needed to achieve the probability of 0.95 of the time the sample proportion will be within ±1 percentage point.

a. $p = 0.85$ $\quad n = 100$ $\quad$ Within 5% = $0.85 - 0.05 \le X \le 0.85 + 0.05$ $\quad$ $P(0.80 \le X \le 0.90) = ?$

First, we have to decide on the whether the conditions are met.

1. **It is likely that there are more than 1000 college students (10 × 100) in the population.**
2. $np = 100 \times 0.85 = 85$ **(greater than 10) and** $n(1 - p) = 100 \times (1 - 0.85) = 15$ **(greater than 10).**

Since the conditions are met,

$$\mu_{\hat{p}} = p$$
$$\mu_{\hat{p}} = 0.85$$

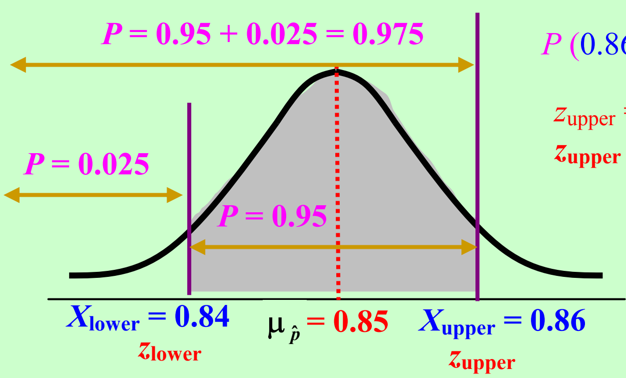
$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$
$$\sigma_{\hat{p}} = \sqrt{\frac{0.85(1-0.25)}{100}} = 0.0357071421$$

$$\mu_{\hat{p}} = 0.85 \qquad \sigma_{\hat{p}} = 0.03571$$

$P(0.80 \le X \le 0.90)$ = normalcdf (0.80, 0.90, 0.85, 0.0357071421) $\qquad$ $P(0.80 \le X \le 0.90) = 0.8386$

With the population proportion being 85%, using the sample size of 100, the sample proportion will fall within ±5 percentage points of the population proportion 83.86% of the time.

b. First we have to determine the z-scores corresponding with the mid 95% of the bell curve.



$P(0.86 \le X \le 0.87) = 0.95$ $\qquad$ $n = ?$

z_{upper} = invNorm (0.975)
$z_{\text{upper}} = 1.959963986$

Using $z = \dfrac{X - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$

$$\sigma_{\hat{p}} = \frac{X - \mu_{\hat{p}}}{z} = \frac{0.86 - 0.85}{1.959963986}$$
$$\sigma_{\hat{p}} = 0.0051021346$$

Since $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$

$$n = \frac{p(1-p)}{(\sigma_{\hat{p}})^2} = \frac{(0.85)(1-0.85)}{(0.0051021346)^2} = 4897.860004$$

n = 4898 people

9.2 Assignment
pg. 477–480 #9.15 to 9.25

9.3: Sampling Means

Parameter and Statistic Means and Standard Deviations of Quantitative Variables

When an **unbiased survey** on a **large population** involves **quantitative variable**, we can calculate the mean and standard deviation and use the normal approximation to determine the relative probability.

Symbols need to be distinguished between parameter and statistic measures because they are both quantitative variables.

Parameter Mean of Population (μ) **Parameter Standard Deviation of Population (σ)**
Statistic Mean of Sample ($\bar{x}$ or $\mu_{\bar{x}}$) **Statistics Standard Deviation of Sample (s or $\sigma_{\bar{x}}$)**

Mean and Standard Deviation of Normal Approximation of a Sample that uses Quantitative Variable ($X \in R$)

$$\bar{x} = \mu_{\bar{x}} = \mu \qquad s = \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \qquad n = \text{Sample Size}$$

Example 1: The average life span for all Americans in 2000 is 76.9 years of age with a standard deviation of 8.4 years. In a random sample 20 Americans who has died in 2000, calculate the mean and standard deviation of the sample.

$\mu = 76.9$ years $\sigma = 8.4$ years $n = 20$ $\mu_{\bar{x}} = ?$ $\sigma_{\bar{x}} = ?$

$\mu_{\bar{x}} = \mu$ $\mu_{\bar{x}} = 76.9$ years

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{8.4}{\sqrt{20}}$ $\sigma_{\bar{x}} = 1.878$ years

Central Limit Theorem: - states that **the sampling of any form of probability distributions will always result in a sampling distribution resembling the Normal Distribution** $N\left(\mu, \dfrac{\sigma}{\sqrt{n}}\right)$

- **the larger the sample size n, the smaller the standard deviation or the spread of the sample distribution, s. Hence, the sampling distribution gets closer to the sample mean (central limit).**

(Check out Central Limit Theorem Animations at:)
1. **Central Limit Theorem in Action:** http://www.rand.org/methodology/stat/applets/clt.html
2. **Uniform Probability Distribution:** http://www.statisticalengineering.com/central_limit_theorem.htm
3. **Various Form of Sampling Distributions:** http://www.ruf.rice.edu/~lane/stat_sim/sampling_dist/
4. **Sampling Exponential Distribution:** http://bcs.whfreeman.com/ips4e/cat_010/applets/CLT-SampleMean.html)

Examples of Exponential Distributions: the probability of a life time of a light bulb or any electronic component once it is turned on, and the amount of time to serve customer.

Example 2: In 2002, the Dow Jones Industrial Average was normally distributed and had an average lost of 16.1% with a standard deviation of 10.4%. If 15 trading days were randomly selected from all trading days in 2002,

a. what are the mean and standard deviation of the sample?
b. calculate the probability in the sample that the Dow will at least break even? How does this sample probability compare to the similar probability of the entire population?

a. $\mu = -16.1\%$ $\sigma = 10.4\%$ $n = 15$ $\mu_{\bar{x}} = ?$ $\sigma_{\bar{x}} = ?$

$\mu_{\bar{x}} = \mu$ $\mu_{\bar{x}} = -16.1\%$ $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10.4}{\sqrt{15}}$ $\sigma_{\bar{x}} = 2.685\%$

b. For the Sample:

$P(X \geq 0\%) = \text{normalcdf}(0, 1\times 10^{99}, \mu_{\bar{x}}, \sigma_{\bar{x}})$

$P(X \geq 0\%) = \text{normalcdf}\left(0, 1\times 10^{99}, -16.1, \dfrac{10.4}{\sqrt{15}}\right)$

$P(X \geq 0\%) = 1.02 \times 10^{-9}$

```
normalcdf(0,1E99
,-16.1,10.4/√(15
))
 1.016821335E-9
normalcdf(0,1E99
,-16.1,10.4)
     .0608019021
```

For the Population:

$P(X \geq 0\%) = \text{normalcdf}(0, 1\times 10^{99}, \mu, \sigma)$

$P(X \geq 0\%) = \text{normalcdf}(0, 1\times 10^{99}, -16.1, 10.4)$

$P(X \geq 0\%) = 0.0608$

Example 3: A call centre manager finds that the time it takes to wait for a call representative to a phone call is exponentially distributed with a mean of 2.8 minutes and a standard deviation of 0.8 minutes. If a sample of 5 calls were taken in a given day, what is the sample probability that a call will be answered within the first two minute?

b. $\mu = 2.8$ minutes $\sigma = 0.8$ minutes $n = 5$

$P(X \leq 2 \text{ min}) = \text{normalcdf}(0, 2, \mu_{\bar{x}}, \sigma_{\bar{x}})$

$P(X \leq 2 \text{ min}) = \text{normalcdf}\left(0, 2, 2.8, \dfrac{0.8}{\sqrt{5}}\right)$

$P(X \leq 2 \text{ min}) = 0.01267$

```
normalcdf(0,2,2.
8,0.8/√(5))
     .0126736174
```

Law of Large Number: - **states that as the number of sample size increases, the sample mean, $\bar{x}$ or $\mu_{\bar{x}}$ will get increasingly closer to the population mean, μ. This is the direct result of the central limit theorem due to $s = \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$.**

9.3 Assignment
pg. 485–486 #9.26 to 9.29;
pg. 491–493 #9.30 to 9.35;
pg. 494–495 #9.36 to 9.39

Chapter 9 Review
pg. 497–498 #9.41 to 9.47

